4-Neutrino mass schemes and the likelihood of (3+1)-mass spectra

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W. Grimus, TS, Eur. Phys. J. C20 (2001) 1
M. Maltoni, TS, J.W.F. Valle, in preparation

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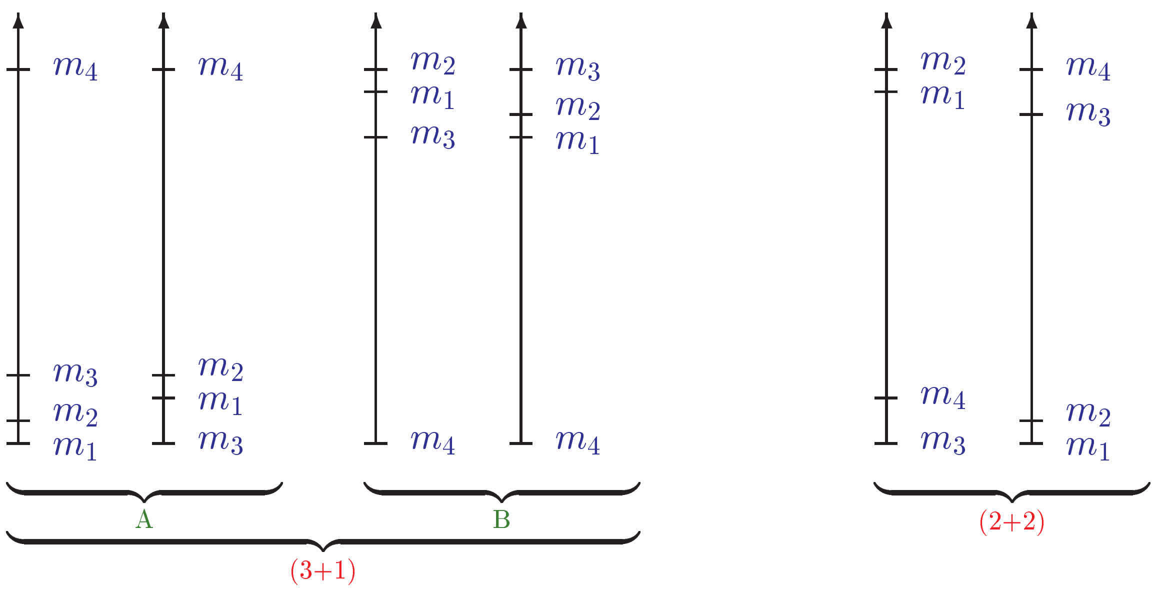
Introduction

At present there are three indications in favor of neutrino oscillations:

- solar neutrinos: $\Delta m^2_{\rm sol} \lesssim 10^{-4}\ {\rm eV}^2$
- atmospheric neutrinos: $\Delta m^2_{\rm atm} \sim 3 \times 10^{-3}\ {\rm eV}^2$
- LSND experiment: $\Delta m^2_{\rm LSND} \gtrsim 0.2\ {\rm eV}^2$

We need at least 4 neutrinos to obtain 3 mass squared differences of different orders of magnitude $\Rightarrow$ 6 possible 4-neutrino mass spectra:



(3+1)-mass spectra are disfavored by the data:

S.M. Bilenky, C. Giunti and W. Grimus, *Proc. of Neutrino '96*; Eur. Phys. J. C **1**, 247 (1998)

N. Okada and O. Yasuda, Int. J. Mod. Phys. A **12**, 3669 (1997)

V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Rev. D **58**, 093016 (1998)

S.M. Bilenky, C. Giunti, W. Grimus and TS, Phys. Rev. D **60**, 073007 (1999)

At *Neutrino 2000* a new LSND analysis was presented – the allowed region was shifted to smaller mass squared differences $\Rightarrow$
(3+1)-mass spectra *less disfavored*:

V. Barger, B. Kayser, J. Learned, T. Weiler, K. Whisnant, Phys. Lett. B **489**, 345 (2000)

C. Giunti and M. Laveder, JHEP **0102**, 001 (2001)

O.L.G. Peres and A.Yu. Smirnov, Nucl.Phys. B **599**, 3 (2001)

SBL oscillation probabilities in (3+1)-spectra

$$\Delta m^2_{\rm sol} \approx 0\,, \quad \Delta m^2_{\rm atm} \approx 0\,, \quad \Delta m^2_{\rm LSND} \equiv \Delta m^2$$

4-neutrino unitary mixing matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

SBL $\overset{(-)}{\nu}_{\mu} \to \overset{(-)}{\nu}_{e}$ transition probability (LSND, KARMEN):

$$P_{\nu_\mu \to \nu_e} = P_{\bar\nu_\mu \to \bar\nu_e} = A_{\mu;e} \sin^2 \frac{\Delta m^2 L}{4E}$$

with

$$A_{\mu;e} = 4\,|U_{e4}|^2 |U_{\mu 4}|^2$$

SBL disappearance probability (Bugey: $\alpha = e$, CDHS: $\alpha = \mu$):

$$P_{\nu_\alpha \to \nu_\alpha} = P_{\bar\nu_\alpha \to \bar\nu_\alpha} = 1 - 4\,|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2) \sin^2 \frac{\Delta m^2 L}{4E}$$

we define: $d_e = |U_{e4}|^2$ and $d_\mu = |U_{\mu 4}|^2$ $\Rightarrow$ $A_{\mu;e} = 4\,d_e d_\mu$

3 parameters: d_e, d_μ, Δm^2

Data used in our analysis

- $\bar{\nu}_e$ disappearance experiments:
  - Bugey: total of 60 bins in positron energy for the ratios of the number of observed to expected events
  - CHOOZ: $P_{\text{CHOOZ}} = 1.01 \pm 2.8\% \pm 2.8\%$

  taking into account the disappearance of solar ν_e:

$$d_e \lesssim 10^{-2}$$

- $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ appearance experiments:
  - KARMEN: number of observed and expected events in 9 positron energy bins
  - $\nu_\mu \to \nu_e$ oscillation search at NOMAD: number of observed and expected events in 14 positron energy bins

upper bound on $A_{\mu;e}$

- ν_μ disappearance experiments:
  - CDHS: 15 bins for the ratio of the number of events in detectors at 130 m and 885 m away from the neutrino source

  taking into account the disappearance of atmospheric ν_μ:

$$d_\mu \lesssim 0.03 \quad (\text{for} \quad 0.7\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10\,\text{eV}^2)$$

- atmospheric neutrinos:
  - 4-neutrino fit to SK and MACRO data performed in M.C. Gonzalez-Garcia, M. Maltoni and C. Peña-Garay, hep-ph/0105269

$$d_\mu \leq 0.082\,(0.117) \quad \text{at} \quad 90\%\,(99\%)\,\text{CL}$$

We combine these data and calculate an upper bound on the amplitude $A_{\mu;e}$, which can be compared to the allowed value obtained in the LSND experiment.

Statistical method

The data is combined by using a likelihood function:

$$\begin{aligned}\mathcal{L}_{\rm osc}(d_e, d_\mu, \Delta m^2) &= \mathcal{L}_{\rm Bugey}(d_e, \Delta m^2)\,\mathcal{L}_{\rm CDHS}(d_\mu, \Delta m^2)\,\mathcal{L}_{\rm KARMEN}(d_e d_\mu, \Delta m^2) \\ &\times\ \mathcal{L}_{\rm NOMAD}(d_e d_\mu, \Delta m^2)\,\mathcal{L}_{\rm atm}(d_\mu)\,\mathcal{L}_{\rm CHOOZ}(d_e)\end{aligned}$$

Bayes' Theorem:

$$p(d_e, d_\mu) \propto \mathcal{L}_{\rm osc}(d_e, d_\mu, \Delta m^2)\,\pi(d_e, d_\mu)$$

$\Rightarrow$ probability density function in d_e and d_μ for a *fixed value* of Δm^2.
We use a *flat prior* in d_e and d_μ in the physical region:

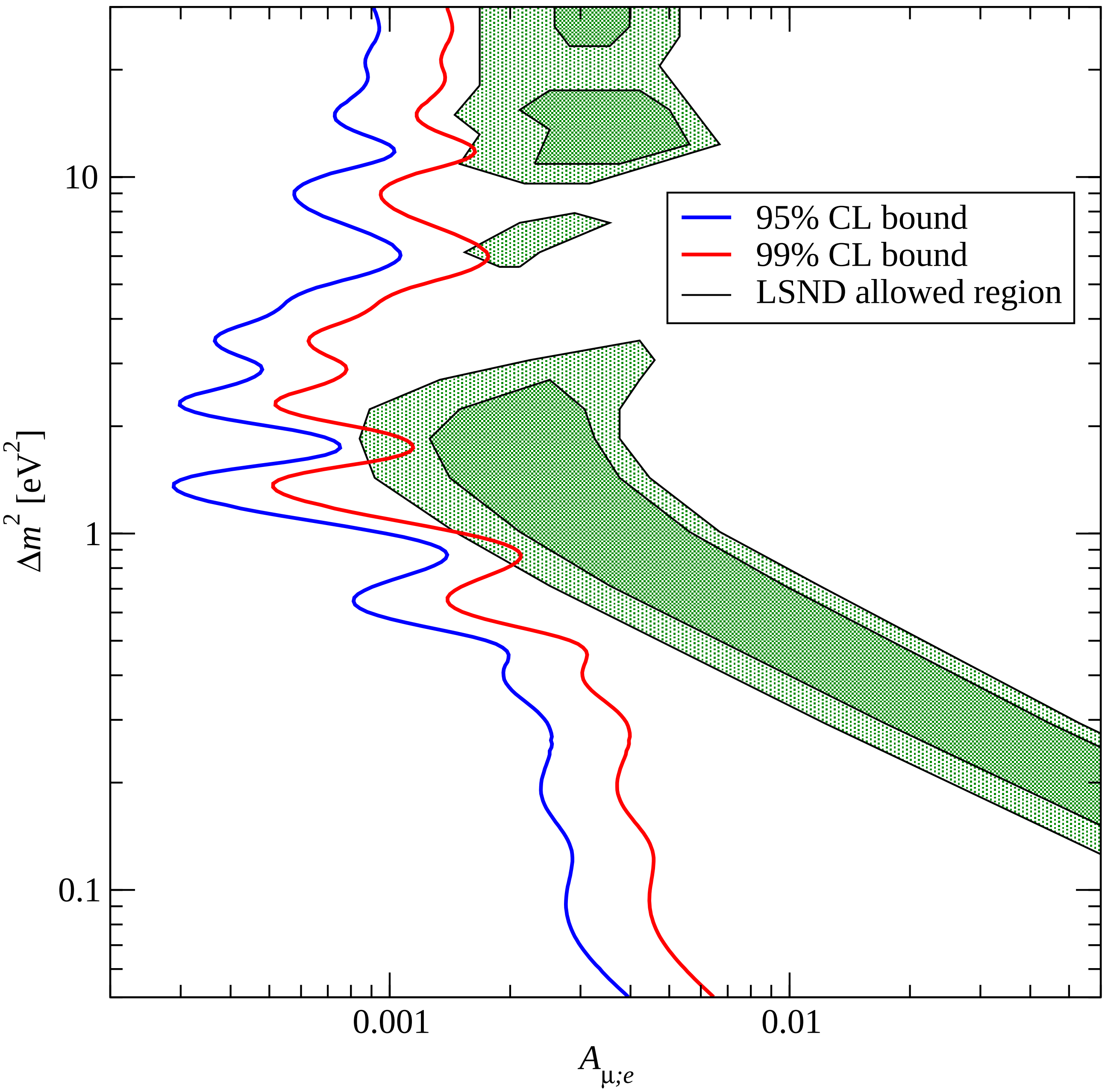
$$\pi(d_e, d_\mu) = \begin{cases} \text{const} & \text{if} \quad d_e \geq 0,\, d_\mu \geq 0 \quad \text{and} \quad d_e + d_\mu \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

We calculate an upper bound on the LSND amplitude

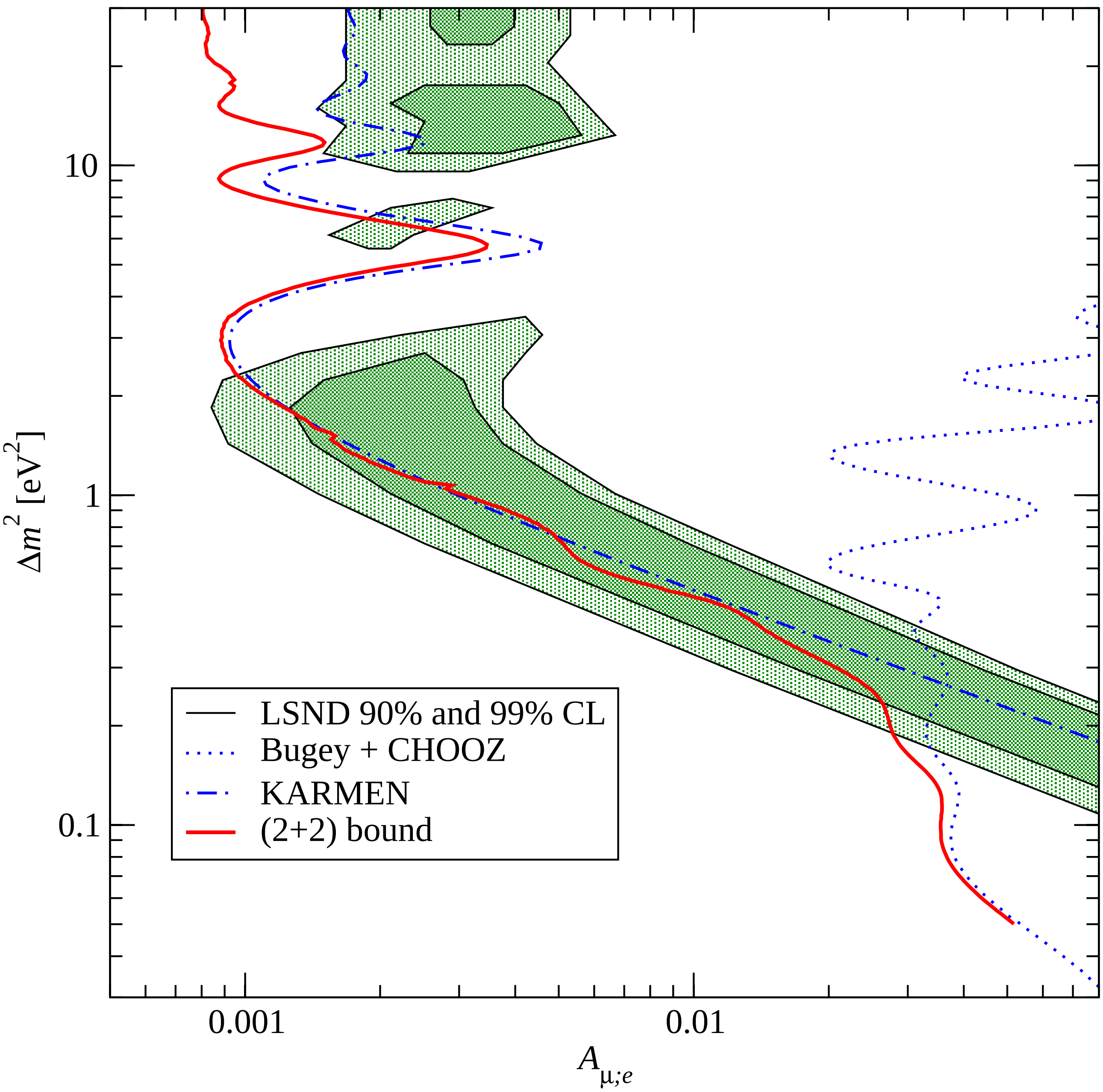
$$A_{\mu;e} = 4\,d_e d_\mu \leq A_\beta \quad \text{at} \quad 100\beta\%\ \text{CL}$$

by the prescription

$$\int_{4 d_e d_\mu \leq A_\beta} \mathrm{d}d_e\ \mathrm{d}d_\mu\ p(d_e, d_\mu) = \beta$$



Bounds on the LSND amplitude $A_{\mu;e}$ in (3+1)-mass spectra from the experiments Bugey, CDHS, KARMEN, NOMAD, CHOOZ and atmospheric neutrino data.



90% CL bound on $A_{\mu;e}$ in (2+2)-mass spectra. Also shown are the 90% CL bounds from KARMEN and from the $\bar{\nu}_e$ disappearance experiments Bugey and CHOOZ.

Including tritium β-decay

We assume that the lowest neutrino mass is much smaller than the sensitivity of the tritium experiments. This gives the weakest restriction on Δm^2.

Analysis of tritium β-decay in
Y. Farzan, O.L.G. Peres and A.Yu. Smirnov, hep-ph/0105105:

$$\begin{array}{ll} (3+1)\text{A:} & m^2_{\text{eff}} \ll 1\,\text{eV}^2 \\ (3+1)\text{B:} & m^2_{\text{eff}} \approx \Delta m^2 \end{array}$$

We use the likelihood function

$$\mathcal{L}_{\text{tot}}(d_e, d_\mu, \Delta m^2) = \mathcal{L}_\beta(\Delta m^2)\, \mathcal{L}_{\text{osc}}(d_e, d_\mu, \Delta m^2)$$

where

$$\mathcal{L}_\beta(\Delta m^2) \propto \begin{cases} \text{const} & \text{for } (3+1)\text{A} \\ \exp\left[-\frac{1}{2}\sum_i \left(\frac{(m^2_{\text{eff}})_i - \Delta m^2}{\sigma_i}\right)^2\right] & \text{for } (3+1)\text{B} \end{cases}$$

with the values given at *Neutrino 2000*:

$$\text{Troitsk:} \quad m^2_{\text{eff}} = -1.0 \pm 3.0 \pm 2.1\,\text{eV}^2$$

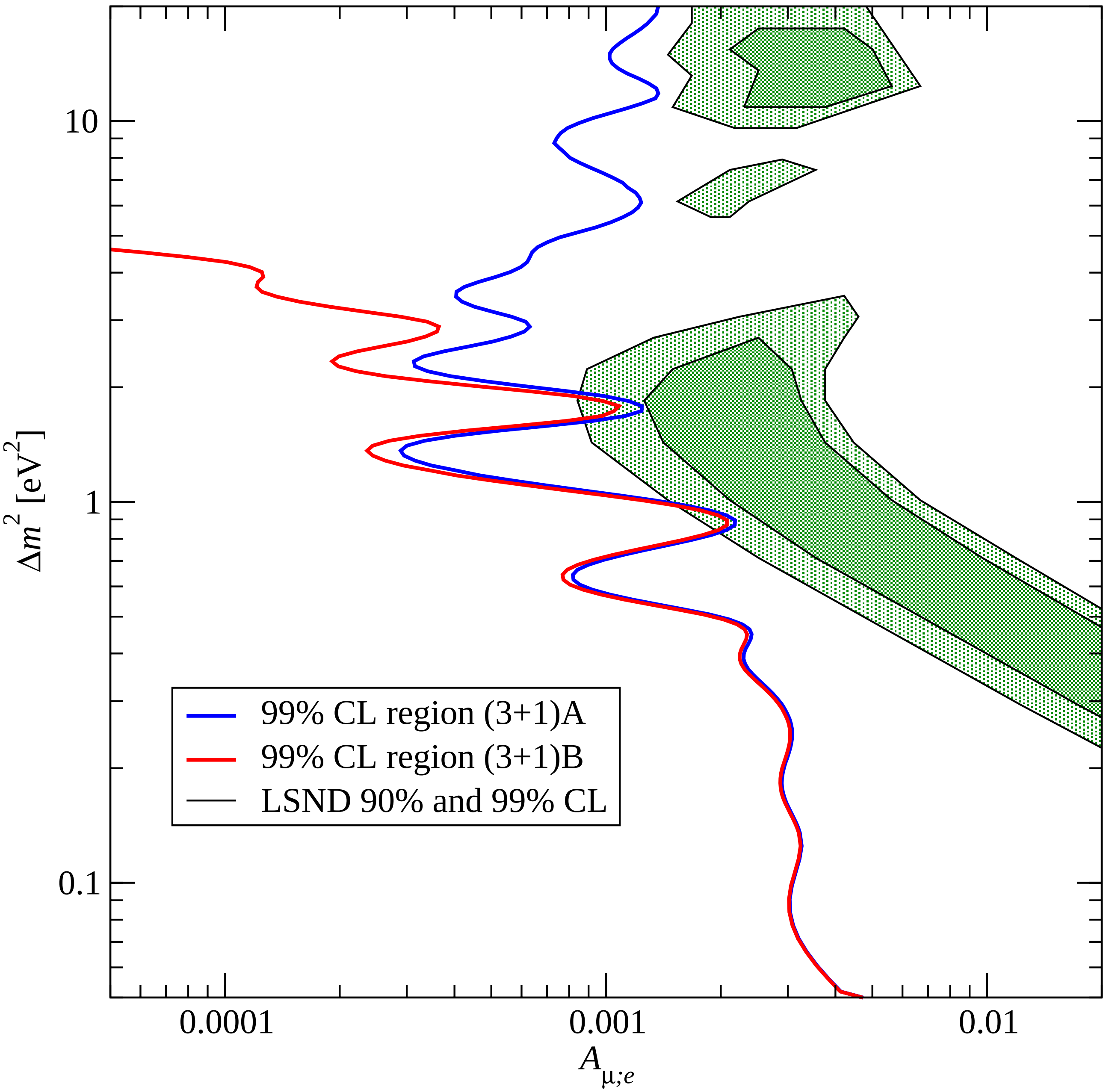
$$\text{Mainz:} \quad m^2_{\text{eff}} = \begin{cases} +0.6 \pm 2.8 \pm 2.1\,\text{eV}^2 \\ -1.6 \pm 2.5 \pm 2.1\,\text{eV}^2 \end{cases}$$

Bayes' Theorem with a flat prior density in d_e and d_μ in the physical region, and a flat prior density in $\log \Delta m^2$ $\Rightarrow$ $p(A_{\mu;e}, \log \Delta m^2)$

We calculate an allowed region at the $100\beta\%$ CL by demanding

$$\int \mathrm{d}A_{\mu;e}\, \mathrm{d}(\log \Delta m^2)\, p(A_{\mu;e}, \log \Delta m^2) = \beta\,.$$

The boundary in the $A_{\mu;e} - \Delta m^2$ plane is determined such that the value of $p(A_{\mu;e}, \log \Delta m^2)$ along this line is constant.



Allowed regions in the $A_{\mu;e} - \Delta m^2$ plane at 99% CL for spectra of the types (3+1)A and (3+1)B including tritium β-decay.

Conclusions

- We have performed an analysis of neutrino oscillation data for the (3+1)-mass spectra in a Bayesian statistical framework.

- We use data from the experiments Bugey, CDHS, KARMEN, CHOOZ, NOMAD and atmospheric neutrino experiments to calculate an upper bound on the LSND amplitude $A_{\mu;e}$:
  - Our bound on $A_{\mu;e}$ at 95% CL has no overlap with the region allowed by LSND at 99% CL.
  - Our bound on $A_{\mu;e}$ at 99% CL has small overlaps with the region allowed by LSND at 99% CL around $\Delta m^2 \sim 0.9, 2$ and $6\,\mathrm{eV}^2$.

- We perform a different statistical analysis including also results from tritium β-decay experiments:
  - Our allowed regions at 95% CL have no overlap with the region allowed by LSND at 99% CL.
  - Our allowed regions at 99% CL have small overlaps with the region allowed by LSND at 99% CL only around $\Delta m^2 \sim 0.9$ and $2\,\mathrm{eV}^2$.

We conclude that the (3+1)-class of 4-neutrino mass spectra is very unlikely