4-NEUTRINO SCHEMES AND THE LIKELIHOOD OF (3+1) MASS SPECTRA[a]

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ABSTRACT

We consider the status 4-neutrino schemes in the light of short-baseline oscillation data, taking into account also the most recent atmospheric neutrino data, as well as long-baseline and tritium β-decay data. We show that the oscillation interpretation of the LSND experiment in (3+1) neutrino schemes is severely disfavoured, in contrast to (2+2) schemes, which are in good agreement with all short-baseline data.

1. Introduction

Reconciling the data on solar[1)], atmospheric[2)] and the LSND[3)] neutrino experiments in terms of neutrino oscillations requires three different scales of mass-squared differences, and hence at least 4 neutrinos, including a sterile one[4)]. All possible 4-neutrino mass spectra are shown in Fig. 1.

Based on semi-quantitative considerations it was realized some time ago that spectra of the type (3+1) are disfavoured by short-baseline (SBL) neutrino data[5)]. However, in a new analysis of the LSND experiment[6)] the allowed region in the parameter space was slightly shifted to smaller mass-squared differences, which makes the (3+1) spectra somewhat less disfavoured. Moreover, it is a prediction of the (2+2) spectra that the sterile neutrino is important in solar or in atmospheric neutrino oscillations or in both, which seems to be disfavoured by the recent data, although a combined fit[7)] is still acceptable. These developments lead to some recent attempts to revive the (3+1) schemes[8)].

This was the motivation to perform a combined analysis of the data of the SBL experiments Bugey[9)], CDHS[10)], KARMEN[11)] and NOMAD[13)], of the long-baseline experiment CHOOZ[12)], of atmospheric neutrino experiments[2)] and of tritium β-decay experiments[14)] to examine the status of 4-neutrino schemes in a well defined statistical framework. The details of our calculations and further references can be found in Refs.[15,16)].

2. Notations

The oscillation probabilities relevant in SBL appearance and disappearance ex-

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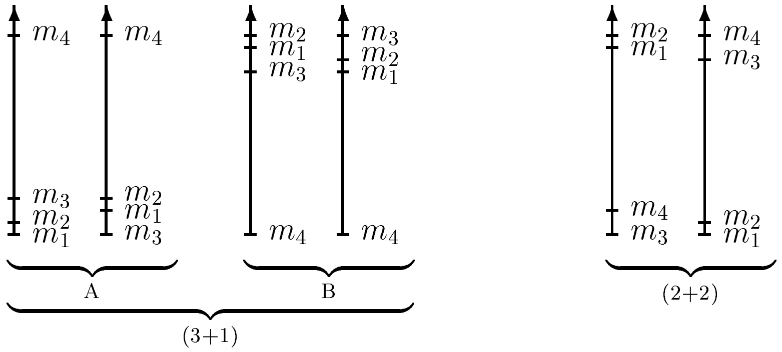

Figure 1: The six types of 4-neutrino mass spectra.

periments in 4-neutrino schemes are given by very simple two-neutrino-like formulas:

$$P_{\nu_\mu\to\nu_e} = P_{\bar\nu_\mu\to\bar\nu_e} = A_{\mu;e}\ \sin^2\left(\Delta m^2 L/4E\right)\,, \tag{1}$$
$$P_{\nu_\alpha\to\nu_\alpha} = P_{\bar\nu_\alpha\to\bar\nu_\alpha} = 1 - 4\, d_\alpha(1-d_\alpha)\sin^2\left(\Delta m^2 L/4E\right)\,. \tag{2}$$

Here L is the distance between source and detector, E is the neutrino energy and $\alpha = e, \mu$. With our labeling of the neutrino masses indicated in Fig. 1 we have $\Delta m^2_{\rm SBL} \equiv \Delta m^2 \approx |m_4^2 - m_1^2|$. In Eqs. (1) and (2) we used the following definitions:

$$\begin{array}{lll} d_\alpha = |U_{\alpha 4}|^2\,, & A_{\mu;e} = 4\, d_e d_\mu & \text{for} \quad \text{(3+1) spectra,} \\ d_\alpha = \sum_{j=1,2} |U_{\alpha j}|^2\,, & A_{\mu;e} = 4 \left|\sum_{j=3,4} U_{ej} U_{\mu j}^*\right|^2 & \text{for} \quad \text{(2+2) spectra,} \end{array} \tag{3}$$

where U is the PMNS lepton mixing matrix.

The results of the Bugey and CHOOZ experiments in combination with solar ν_e-disappearance require $d_e \lesssim 10^{-2}$ for (3+1) and $1 - d_e \lesssim 10^{-2}$ for (2+2) spectra. The CDHS experiment together with atmospheric ν_μ-disappearance implies $d_\mu \lesssim 0.03$ for $0.7\ \text{eV}^2 \lesssim \Delta m^2 \lesssim 10\ \text{eV}^2$. The 4-neutrino fit to the full atmospheric neutrino data[7)] allows to set the additional constraint $d_\mu \leq 0.090\,(0.13)$ at 90% (99%) CL. Together with the bounds on $A_{\mu;e}$ from the experiments KARMEN and NOMAD we calculate an upper bound on $A_{\mu;e}$ by combining all the data and compare this bound to the region in the $(\Delta m^2, A_{\mu;e})$ plane allowed by LSND.

3. Results

Let us consider at first the (3+1) mass spectra. In this case we combine all the oscillation data using the likelihood function

$$\begin{aligned} \mathcal{L}_{\rm osc}(d_e, d_\mu, \Delta m^2) = {} & \mathcal{L}_{\rm Bugey}(d_e, \Delta m^2)\, \mathcal{L}_{\rm CDHS}(d_\mu, \Delta m^2)\, \mathcal{L}_{\rm atm}(d_\mu)\, \mathcal{L}_{\rm CHOOZ}(d_e) \\ & \times \mathcal{L}_{\rm KARMEN}(d_e d_\mu, \Delta m^2)\, \mathcal{L}_{\rm NOMAD}(d_e d_\mu, \Delta m^2)\,. \end{aligned} \tag{4}$$

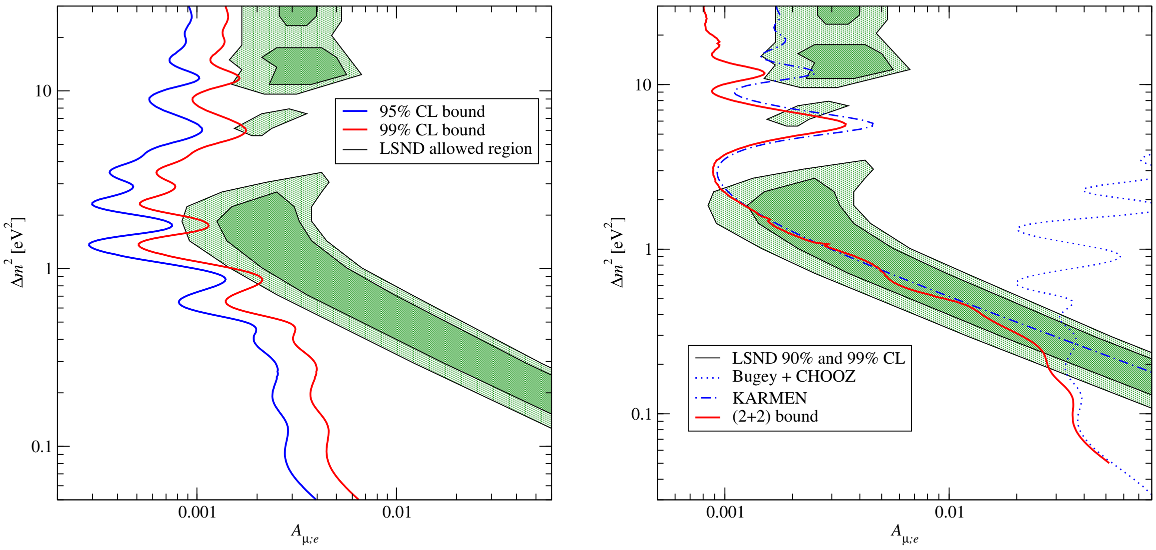

Figure 2: Upper bounds on $A_{\mu;e}$ for (3+1) spectra at 95% and 99% CL (left panel) and for (2+2) spectra (right panel) at 90% CL. The shaded regions are allowed by LSND at 90% and 99% CL[6)].

Details concerning the individual likelihood function of each of the experiments can be found in Refs.[15,16)]. For a *fixed* value of Δm^2 the likelihood function Eq. (4) is transformed into a probability distribution $p(d_e, d_\mu)$ by applying Bayes' theorem and assuming a flat prior in the physically allowed region $d_e, d_\mu \geq 0$ and $d_e + d_\mu \leq 1$. Choosing a CL β, we find the corresponding upper bound A^0_β on $A_{\mu;e}$ by the prescription

$$\int\limits_{4d_e d_\mu \leq A^0_\beta} \mathrm{d}d_e\, \mathrm{d}d_\mu\, p(d_e, d_\mu) = \beta\,. \tag{5}$$

The bounds at 95% and 99% CL are shown in the left panel of Fig. 2 together with the regions allowed by LSND at 90% and 99% CL. We find that there is no overlap of the region allowed by our bound at 95% CL with the region allowed by LSND at 99% CL. If we take our bound at 99% CL there are marginal overlaps with the 99% CL LSND allowed region at $\Delta m^2 \sim 0.9$ and 2 eV^2, and a very marginal overlap region still exists around 6 eV^2.[b]

Let us now come to the (2+2) mass spectra. Because of the more complicated relation between the SBL parameters d_e, d_μ and $A_{\mu;e}$ given in Eq. (3) in this case the statistical analysis is more involved. The details of our calculation can be found in Ref.[15)]. Here we present only the results, which are shown in the right panel of

[b]We note that for $\Delta m^2 \gtrsim 10$ eV^2 there are additional constraints on the amplitude $A_{\mu;e}$ from the experiments BNL E776[17)] and CCFR[18)], which are not included in our analysis. Therefore, the (in any case very marginal) overlap region at $\Delta m^2 \sim 10$ eV^2 is irrelevant.

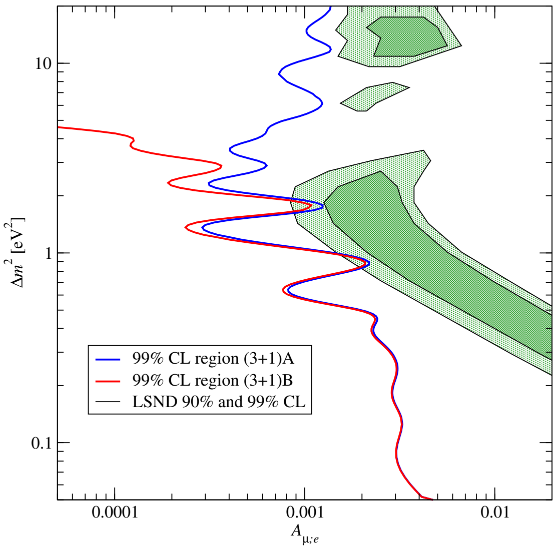

Figure 3: Allowed regions at 99% CL for spectra of the types (3+1)A and (3+1)B including tritium β-decay. The shaded regions are allowed by LSND at 90% and 99% CL[6)].

Fig. 2. The solid line represents our upper bound on $A_{\mu;e}$ at 90% CL. It follows the bound from the reactor experiments Bugey and CHOOZ for small Δm^2 and the bound from KARMEN for intermediate values of Δm^2. Even at 90% CL there is a large overlap with the region allowed by LSND. Therefore, the (2+2) schemes are in good agreement with all SBL experiments.

4. Implications of tritium β-decay

For (3+1) mass spectra we have also done an analysis taking into account the information on the effective electron neutrino mass m_β obtained in tritium β-decay experiments. Assuming a lightest neutrino mass $\ll 1$ eV one finds[19)] that there is no significant effect in tritium decay experiments for spectra of type (3+1)A, however in the case of (3+1)B one obtains $m_\beta^2 \approx \Delta m^2$. We include this result in our analysis by using the likelihood function

$$\mathcal{L}_\beta(\Delta m^2) \propto \exp\left[-\frac{1}{2}\sum_i \left(\frac{(m_\beta^2)_i - \Delta m^2}{\sigma_i}\right)^2\right] \tag{6}$$

for the spectrum (3+1)B. The sum is over the experimental values[14)] $m_\beta^2 = -1.0 \pm 3.0 \pm 2.1$ eV^2 obtained by the Troitsk experiment and the two values presented by the Mainz collaboration, obtained from two different analyses of their data: $m_\beta^2 = +0.6 \pm 2.8 \pm 2.1\,\mathrm{eV}^2$ and $m_\beta^2 = -1.6 \pm 2.5 \pm 2.1\,\mathrm{eV}^2$; σ_i is the corresponding error (statistical and systematic errors added in quadrature).

Now we perform an analysis with the two parameters $A_{\mu;e}$ and Δm^2, in contrast to the previous section, where the analysis is performed only with the parameter $A_{\mu;e}$ for each value of Δm^2 (for details see Ref.[16)]). The total likelihood function is transformed into a probability distribution $p(A_{\mu;e}, \Delta m^2)$ by using Bayes' theorem. We assume a flat prior distribution for d_e and d_μ in the physical region and a flat prior distribution for $\log \Delta m^2$. This ensures that we introduce no bias concerning the order of magnitude of Δm^2, a priori all scales are equally probable. We calculate an allowed region at the 100β% CL by demanding

$$\int \mathrm{d}A_{\mu;e}\mathrm{d}(\log \Delta m^2)\, p(A_{\mu;e}, \Delta m^2) = \beta\,. \tag{7}$$

The integration boundary in the $(A_{\mu;e}, \Delta m^2)$ plane is determined such that the value of $p(A_{\mu;e}, \Delta m^2)$ along this line is constant.

In Fig. 3 we show the allowed regions at 99% CL for spectra of the types (3+1)A and (3+1)B. In the case of (3+1)B spectra tritium β-decay rules out values of $\Delta m^2 \gtrsim 5$ eV^2. In both cases (3+1)A and B only two marginal overlaps with the 99%CL LSND allowed region survive at $\Delta m^2 \sim 0.9$ and 2 eV^2.

5. Conclusions

We have performed a combined analysis of the SBL experiments Bugey, CDHS, KARMEN and NOMAD, together with atmospheric neutrino data and information from the CHOOZ experiment and tritium β-decay in the framework of 4-neutrino mass schemes. We find that for spectra of the type (3+1) the oscillation interpretation of the LSND experiment is severely disfavoured, whereas spectra of the type (2+2) are in good agreement with all SBL data.

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