Majorana Neutrinos, Neutrino Mass Spectrum and CP-Violation in the Lepton Sector

S. T. Petcov

SISSA/INFN - sezione di Trieste, Trieste, Italy, and
INRNE, Bulgarian Academy of Sciences, Sofia, Bulgaria

Les Houches
EuroConference
18-23 June, 2001

EVIDENCES FOR ν-OSCILLATIONS:

- ν_{ATM} : SK UP-DOWN ASYMMETRY (ZENITH ANGLE DEPENDENCE) MULTI-GeV μ-LIKE SAMPLE

  DOMINANT $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\tau$

  K2K; MINOS, CNGS.

- $\nu_\odot$: HOMESTAKE, KAMIOKANDE, SAGE, GALLEX/GNO, SUPER-KAMIOKANDE, SNO

  DOMINANT $\nu_e \to \nu_{\mu,\tau}$

  KAMLAND; BOREXINO, ...

- LSND

  $\bar{\nu}_\mu \to \bar{\nu}_e$ MINIBOONE

$$\nu_{\ell L} = \sum_{j=1} U_{\ell j}\, \nu_{jL}, \qquad \ell = e, \mu, \tau$$

ν- FACTORIES : 3-ν MIXING, LMA MSW

$L \sim (3000 - 7000)$ km.

THE QUESTION OF THE NATURE OF MASSIVE NEUTRINOS EMERGES AS ONE OF THE FUNDAMENTAL QUESTIONS IN THE STUDIES OF ν - MIXING.

MASSIVE DIRAC ν'S:

$$L = L_e + L_\mu + L_\tau = \text{const.}, \text{ or}$$

$$L' = L_e - L_\mu - L_\tau = \text{const.} \qquad \text{STP '82}$$

$$L^{(\prime)}(\nu) \neq L^{(\prime)}(\bar{\nu})$$

MASSIVE MAJORANA ν'S:

$$\Delta L \neq 0, \ldots$$

TRUELY NEUTRAL PARTICLES ≡ ANTIPARTICLES:

$$\nu \equiv \bar{\nu}$$

THE NATURE OF MASSIVE ν'S IS RELATED TO THE FUNDAMENTAL SYMMETRIES OF PARTICLE INTERACTIONS.

SEE-SAW MECHANISM $\Rightarrow$ ν_j - MASSIVE MAJORANA ν'S.

IF ν_j - MAJORANA PARTICLES.

BILENKY ET AL. '80
DOI ET AL. '81

U_{PMNS} - CONTAINS 3ν MIXING
$\left\{\begin{array}{l}\delta \text{ - DIRAC} \\ \alpha_{21}, \alpha_{31} \text{ - MAJORANA} \\ \text{PHYSICAL CP-VIOLATING PHASES}\end{array}\right.$

ν - OSCILLATIONS $\overset{(-)}{\nu}_l \rightleftarrows \overset{(-)}{\nu}_{l'}$, $l, l' = e, \mu, \tau$

BILENKY ET AL. '80; LANGACKER ET AL. '87

- ARE NOT SENSITIVE TO THE NATURE OF ν_j;
- PROVIDE INFORMATION ON $\Delta m^2_{jk} = m_j^2 - m_k^2$, $j>k=1,2,3$ BUT NOT ON THE ABSOLUTE VALUES OF NEUTRINO MASSES m_j.

HOW CAN ONE OBTAIN INFORMATION ON

- THE NATURE OF ν_j ?
- m_1, m_2, m_3, i.e., ON THE ν-MASS SPECTRUM?
- ON THE CP-VIOLATION IN THE LEPTON SECTOR, INDUCED BY THE MAJORANA CP-VIOLATING PHASES?

THE MAJORANA NATURE OF ν_j CAN MANIFEST ITSELF IN THE EXISTENCE OF PROCESSES

$\Delta L = \pm 2$: $K^+ \to \pi^- \mu^+ \mu^+$
$\mu^- + (A, Z) \to \mu^+ + (A, Z-2), \ldots$

THE MOST SENSITIVE PROCESS - $(\beta\beta)_{0\nu}$ - decay OF CERTAIN EVEN-EVEN NUCLEI:

STANDARD PARAMETRIZATION:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\alpha_{21}/2} & U_{e3}\,e^{i\frac{\alpha_{31}}{2}} \\ -s_{12}c_{23}-c_{12}s_{23}U^*_{e3} & (c_{12}c_{23}-s_{12}s_{23}U^*_{e3})e^{i\frac{\alpha_{21}}{2}} & s_{23}c_{13}e^{i\alpha_{31}/2} \\ s_{12}s_{23}-c_{12}c_{23}U^*_{e3} & (-c_{12}s_{23}-s_{12}c_{23}U^*_{e3})e^{i\frac{\alpha_{21}}{2}} & c_{23}c_{13}e^{i\alpha_{31}/2} \end{pmatrix}$$

$$U_{e3} = s_{13}e^{-i\delta}$$

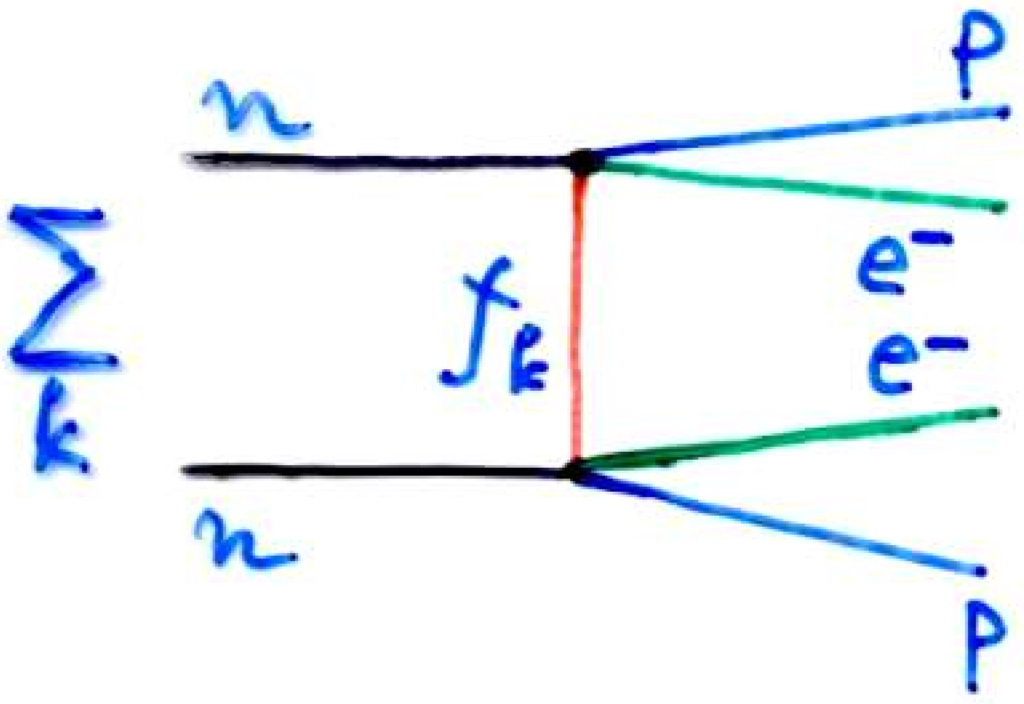
α_{21}, α_{31} – MAJORANA CP-VIOLATING PHASES

S.M. BILENKY ET AL. '80

IF ν_j ARE MAJORANA PARTICLES, CP-SYMMETRY CAN BE VIOLATED EVEN IN THE CASE OF $n=2$ FAMILIES OF LEPTONS.

$(\beta\beta)_{0\nu}$ - decay:

$$(A,Z) \to (A,Z+2) + e^- + e^-$$



$$\nu_{eL}(x) = \sum_k U_{ek}\, \underbrace{\nu_{kL}(x)}_{m_{\nu_k}}$$

$$\mathcal{H}_W^{\beta} = \frac{G_F}{\sqrt{2}}\, 2\left(\bar{e}_L(x)\gamma_\alpha \nu_{eL}(x)\right) j_\alpha^{h}(x) + h.c.$$

$$S^{(2)} = -\frac{(-i)^2}{2}\, 4\left(\frac{G_F}{\sqrt{2}}\right)^2 \int dx_1\, dx_2 \times T_{\alpha\beta}(x_1,x_2) \times$$

$$\times N\left[\bar{e}_L(x_1)\gamma_\alpha\, \nu_{eL}(x_1)\, \nu_{eL}^T(x_2)\,\gamma_\beta^T\, \bar{e}_L^T(x_2)\right]$$

$$A(\beta\beta)_{0\nu} \sim \langle m_\nu \rangle ,$$

$$\langle m_\nu \rangle \underset{\substack{m_k \lesssim \text{few MeV} \\ CP\text{-inv.}}}{=} \sum_k |U_{ek}|^2 \frac{1}{i}\, \eta^{CP}(\nu_k)\, m_k$$

Data: ^{76}Ge

$$\boxed{|\langle m_\nu \rangle| < (1 \div 2)\ \text{eV}}$$

MAJORANA ν'S:

MAJORANA 1936 : χ_j, $C\bar{\chi}_j^T(x) = \chi_j(x)$

RACAH 1937 : $\nu = \chi \Rightarrow (\beta\beta)_{0\nu}$ - DECAY ALLOWED

PONTECORVO 1946 : ν'S CAN BE DETECTED !
(REACTOR, SUN)
Cl-AR METHOD

DAVIS 1955 : $\bar{\nu}$ (REACTOR) + $^{37}Cl \to {}^{37}Ar + e^-$
(AT THAT EPOCH : $\nu \equiv \chi \equiv \bar{\nu}$)

PONTECORVO 1957 : $\nu = \dfrac{\chi_1 + \chi_2}{\sqrt{2}}$

$\bar{\nu} = \dfrac{\chi_1 - \chi_2}{\sqrt{2}}$ $\quad \nu \rightleftarrows \bar{\nu}$

PONTECORVO 1967 : ν_e - OSCILLATIONS $\Rightarrow$

$\nu_\odot$ - FLUX REDUCED BY ~ 2;
STERILE ν'S.

GRIBOV + PONTECORVO 1969 : ν_{eL}, $\nu_{\mu L}$ GENERAL MAJORANA MASS TERM

$\nu_{eL} = \chi_{1L} \cos\theta + \chi_{2L} \sin\theta$
$\nu_{\mu L} = \perp$

MAKI-NAKAGAWA-SAKATA 1962 : $\nu_e = \nu_1 \cos + \nu_2 \sin\theta$
$\nu_\mu = \perp$

PONTECORVO 1971: $\nu_{eL} = \sum_{j=1}^{n} U_{ej}\, \nu_{jL}$

BILENKY + PONTECORVO 1976: $\nu_{\ell L}, \nu_{\ell R}, \underbrace{\ell = e, \mu, \tau, \sigma, \ldots}_{n}$

DIRAC + MAJORANA MASS TERM (USED IN THE SEE-SAW MECHANISM)

$$\nu_{\ell L} = \sum_{j=1}^{2n} U_{\ell j}\, \chi_{jL}$$

$$\nu^{c}_{\ell L} = \sum_{j=1}^{2n} \bar{U}_{\ell j}\, \chi_{jL}$$

$$V_{PMNS} \equiv \begin{pmatrix} U \\ \bar{U} \end{pmatrix}$$

<u>MAJORANA ν's</u>: χ_j

$$C \bar{\chi}_j^{T}(x) = \xi_j\, \chi_j(x)$$

$$|\xi_j|^2 = 1$$

- CP-PARITY: $\eta_{CP}(\chi_j) = \pm i$
- $\mu(\chi_j) = 0$; $d(\chi_j) = 0$
- $\bar{\chi}(x)\, \gamma_\mu\, \chi(x) = 0$
- $\overbrace{\chi_\alpha(x)\, \chi_\beta}(y) \neq 0$, $\overbrace{\bar{\chi}_\alpha(x)\, \bar{\chi}_\beta}(y) \neq 0$

The Majorana nature of the massive neutrinos can manifest itself in the existence of $L \neq 0$, $\Delta L = 2$, processes. The process most sensitive to the existence of massive Majorana neutrinos (coupled to the electron) is the neutrinoless double β ($(\beta\beta)_{0\nu}-$) decay of certain even-even nuclei

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-. \tag{1}$$

If the $(\beta\beta)_{0\nu}-$ decay is generated *only by the left-handed (LH) charged current weak interaction through the exchange of virtual massive Majorana neutrinos*, $A((\beta\beta)_{0\nu})$ is proportional to the so-called "effective Majorana mass"

$$A((\beta\beta)_{0\nu}) \sim \, <m> \equiv \sum_{j=1} U_{ej}^2 \, m_j, \qquad m_j \lesssim \text{ few MeV}, \tag{2}$$

where m_j is the mass of the Majorana neutrino ν_j and U_{ej} is the element of neutrino (lepton) mixing matrix U.

A large number of experiments are searching for $(\beta\beta)_{0\nu}-$decay of different nuclei at present. No indications that this process takes place were found. ^{76}Ge Heidelberg-Moscow experiment:

$$|<m>| \, < 0.35 \text{ eV}, \quad 90\% \text{ C.L.} \tag{3}$$

Taking into account a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element

$$|<m>| \, < (0.35 \div 1.05) \text{ eV}, \quad 90\% \text{ C.L.}. \tag{4}$$

The IGEX collaboration has obtained [30]:

$$|<m>| \, < (0.33 \div 1.35) \text{ eV}, \quad 90\% \text{ C.L.} \tag{5}$$

Considerably higher sensitivity to the value of $|<m>|$ is planned to be reached in several $(\beta\beta)_{0\nu}-$decay experiments of a new generation:

- the NEMO3 experiment scheduled to start in 2001, will search for $(\beta\beta)_{0\nu}-$decay of ^{100}Mo and ^{82}Se; will reach a sensitivity to $|<m>| \, \cong 0.1$ eV.

- CUORE: a similar sensitivity is planned to be reached with the cryogenic detector CUORE; will search for the $(\beta\beta)_{0\nu}-$decay of ^{130}Te.

- GENIUS: sensitivity to $|<m>| \, \cong 10^{-2}$ eV, is planned to be achieved utilizing one ton of enriched ^{76}Ge.

- EXO: proposal to study the $(\beta\beta)_{0\nu}-$decay of ^{136}Xe in a background-free experiment with detection of the two e^- and the $^+$Ba atom in the final state; the estimated sensitivity of this experiment is $|<m>| \, \cong (1-5) \times 10^{-2}$ eV.

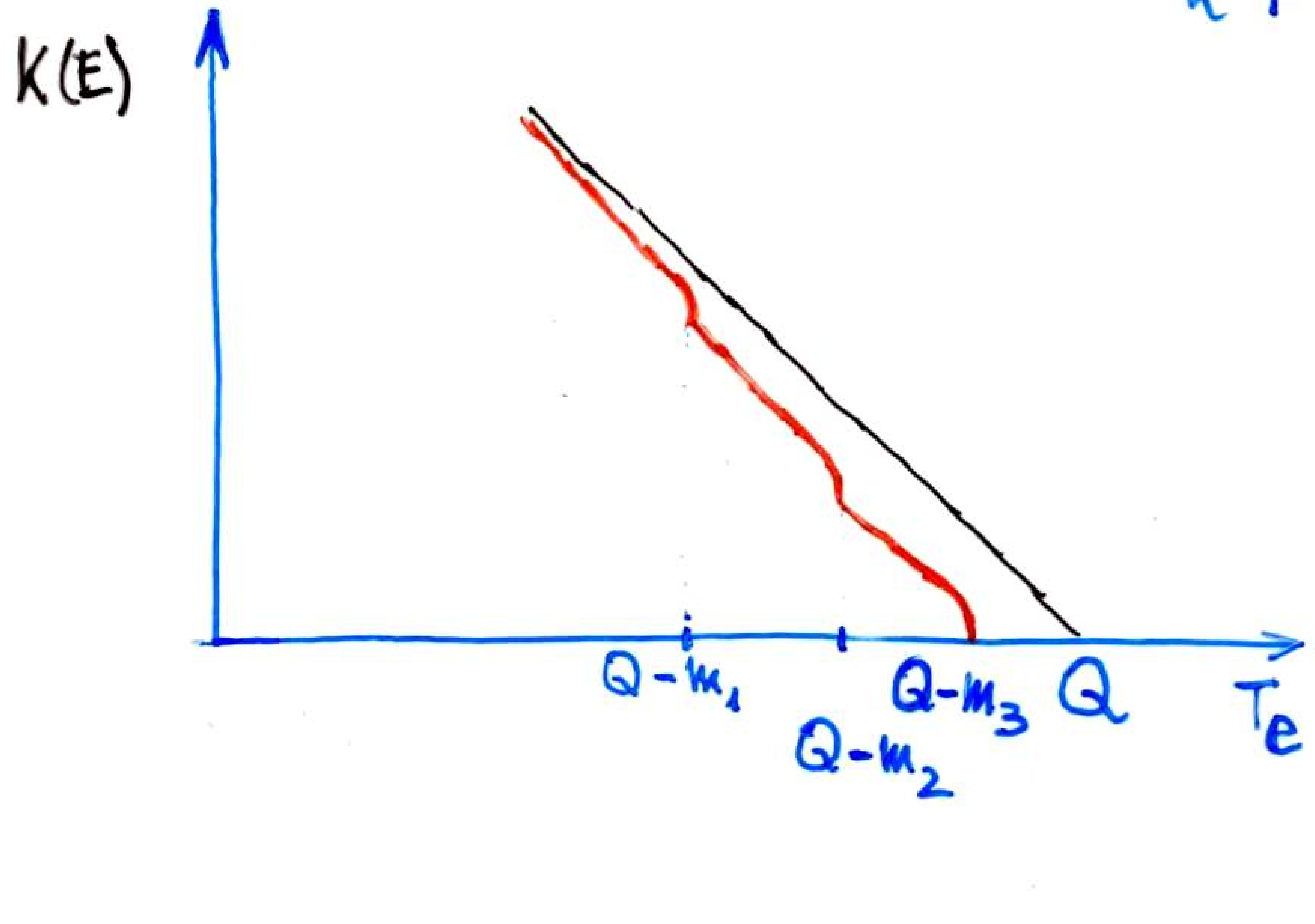
INFORMATION ABOUT ν-MASS SPECTRUM

$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$ FERMI 1934

e^- - SPECTRUM

$$\frac{dN_e}{dE} = \sum_{j=1} \underbrace{|U_{ej}|^2}\ \underbrace{W(E, m_j^2)}$$

$$W(E, m_j^2) = c\, p_e E\, (Q - T_e) \sqrt{(Q-T_e)^2 - m_j^2} \propto$$
$$\propto F(E)$$



The Troitzk [67] and Mainz [68] ^{3}H β-decay experiments, studying the electron spectrum, provide information on the electron (anti-)neutrino mass m_{ν_e}. The data contain features which require further investigation (e.g., a peak in the end-point region which varies with time [67]). The upper bounds given by the authors (at 95% C.L.) read:

$$m_{\nu_e} < 2.5 \text{ eV} \quad [67], \quad m_{\nu_e} < 2.9 \text{ eV} \quad [68]. \tag{15}$$

There are prospects to increase the sensitivity of the ^{3}H β-decay experiments,

$$\text{KATRIN}: \qquad m_{\nu_e} \sim (0.3 - 0.4) \text{ eV}.$$

Cosmological and astrophysical data provide information on the sum of the neutrino masses. The current upper bound reads (see, e.g., [70] and the references quoted therein):

$$\sum_j m_j \lesssim 5.5 \text{ eV} . \tag{16}$$

The future experiments MAP and PLANCK can be sensitive to [71]

$$\sum_j m_j \cong 0.4 \text{ eV} . \tag{17}$$

ν-OSCILLATION DATA:

$$0.03\,\text{eV} \lesssim \sum_{j=1}^{3} m_j < 7.5\,\text{eV}$$

As is well known, the explanation of the atmospheric and solar neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\nu_{j\mathrm{L}}, \tag{5}$$

where ν_{lL}, $l = e, \mu, \tau$, are the three left-handed flavour neutrino fields, $\nu_{j\mathrm{L}}$ is the left-handed field of the neutrino ν_j having a mass m_j and U is a 3×3 unitary mixing matrix - the Pontecorvo-Maki-Nakagawa-Sakata neutrino (lepton) mixing matrix. If ν_j are Majorana neutrinos,

$$C(\bar{\nu}_j)^T = \nu_j,\ j = 1, 2, 3,$$

C is the charge conjugation matrix, and for $m_j \lesssim$ few MeV,

$$|<m>| = \left|m_1 U_{\mathrm{e}1}^2 + m_2 U_{\mathrm{e}2}^2 + m_3 U_{\mathrm{e}3}^2\right| \tag{6}$$

$$= \left|m_1 |U_{\mathrm{e}1}|^2 + m_2 |U_{\mathrm{e}2}|^2\ e^{i\alpha_{21}} + m_3 |U_{\mathrm{e}3}|^2\ e^{i\alpha_{31}}\right| \tag{7}$$

where

$$U_{ej} = |U_{ej}|\ e^{\frac{i\alpha_j}{2}}$$

$$\alpha_{21} \equiv (\alpha_2 - \alpha_1),\quad \alpha_{31} \equiv (\alpha_3 - \alpha_1)$$

are two CP-violating phases. If CP-invariance holds, one has

$$\alpha_{21} = k\pi,\quad \alpha_{31} = k'\pi,\qquad k, k' = 0, 1, 2, ...$$

In this case

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \tag{8}$$

represent the relative CP-parities of the neutrinos ν_1 and ν_2, and ν_1 and ν_3, respectively.

We can numerate (without loss of generality) the neutrino masses in such a way that $m_1 < m_2 < m_3$. If we denote by $\theta_\odot$ and θ respectively the mixing angles constrained by the solar neutrino data and the data from the CHOOZ experiment, then depending on the type of the neutrino mass spectrum one has either

$$|U_{\mathrm{e}1}| = \cos\theta_\odot \sqrt{1 - |U_{\mathrm{e}3}|^2},\quad |U_{\mathrm{e}2}| = \sin\theta_\odot \sqrt{1 - |U_{\mathrm{e}3}|^2},\quad |U_{\mathrm{e}3}|^2 = \sin^2\theta, \tag{9}$$

or

$$|U_{\mathrm{e}2}| = \cos\theta_\odot \sqrt{1 - |U_{\mathrm{e}1}|^2},\quad |U_{\mathrm{e}3}| = \sin\theta_\odot \sqrt{1 - |U_{\mathrm{e}1}|^2},\quad |U_{\mathrm{e}1}|^2 = \sin^2\theta. \tag{10}$$

Relations (9) are valid for the hierarchical neutrino mass spectrum, while those in eq. (10) are realized for the neutrino mass spectrum with inverted hierarchy.

The neutrino oscillation experiments provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$ $(j > k)$. In the case of 3-neutrino mixing as an independent set of three neutrino mass parameters one can choose

$$m_1, \quad \sqrt{\Delta m^2_{21}}, \quad \sqrt{\Delta m^2_{32}}.$$

Then :

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}\ , \tag{11}$$

$$m_3 = \sqrt{m_1^2 + \Delta m^2_{21} + \Delta m^2_{32}}\ . \tag{12}$$

The Δm^2 inferred from the atmospheric neutrino data,

$$\Delta m^2_{\rm atm} = \Delta m^2_{31} = \Delta m^2_{21} + \Delta m^2_{32}\ , \tag{13}$$

while for the one deduced from the solar neutrino data, $\Delta m^2_{\odot}$, we have two possibilities:

$$\Delta m^2_{\odot} \equiv \Delta m^2_{32} \quad \text{or} \quad \Delta m^2_{\odot} \equiv \Delta m^2_{21}\ . \tag{14}$$

Depending on the relative magnitudes of m_1, $\sqrt{\Delta m^2_{21}}$ and $\sqrt{\Delta m^2_{32}}$, one recovers the different possible types of neutrino mass spectrum:

1. if $m_1 \ll \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{32}}$, one has $m_1 \ll m_2 \ll m_3$, i.e., hierarchical (H) neutrino mass spectrum;
2. $m_1 \ll \sqrt{\Delta m^2_{32}} \ll \sqrt{\Delta m^2_{21}}$ implies $m_1 \ll m_2 \simeq m_3$, i.e., neutrino mass spectrum with inverted hierarchy (IH);
3. for $\sqrt{\Delta m^2_{21}}, \sqrt{\Delta m^2_{32}} \ll m_1$, we have $m_1 \simeq m_2 \simeq m_3$, i.e., quasi-degenerate (QD) neutrino mass spectrum;
4. if $\sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{32}} \sim \boldsymbol{O}(m_1)$, one finds $m_1 \simeq m_2 < m_3$, i.e., spectrum with "partial mass hierarchy"; interpolates between H and QD.
5. for $\sqrt{\Delta m^2_{32}} \ll \sqrt{\Delta m^2_{21}} \sim \boldsymbol{O}(m_1)$, we have $m_1 < m_2 \simeq m_3$, i.e., spectrum with "partial inverted mass hierarchy"; interpolates between IH and QD.

Given the values of $\Delta m^2_{\odot}$, $\theta_{\odot}$, $\Delta m^2_{\rm atm}$ and of θ,

$$|{<}m{>}| = |{<}m{>}|\,(m_1, \alpha_{21}, \alpha_{31}; S), \qquad S = H,\ IH$$

The knowledge of m_1 would allow to determine the neutrino mass spectrum.

GIVEN THE VALUES OF Δm^2_{atm} AND $\Delta m^2_{\odot}$, INFERRED FROM THE DATA, ONE HAS:

- $\underline{m_1 \ll 0.02\,eV}$ - HIERARCHICAL OR INVERTED HIERARCHY

- $\underline{0.02 \lesssim m_1 \lesssim 0.2\,eV}$ - PH OR PIH

- $\underline{m_1 > 0.2\,eV}$ - QD

ν - MASS SPECTRUM

4 Hierarchical Neutrino Mass Spectrum

The hierarchical neutrino mass spectrum is characterized by

$$m_1 \ll m_2 \ll m_3. \tag{42}$$

This type of neutrino mass spectrum is predicted by the standard versions of the see-saw mechanism of neutrino mass generation. The pattern corresponds to

$$m_1 \ll \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{32}}\ . \tag{43}$$

Using (42) and (43) it is possible to make the identification

$$\begin{aligned}
\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\mathrm{atm}} &\equiv \Delta m^2_{32}, \\
|U_{\mathrm{e}1}|^2 &= \cos^2\theta_{\odot}(1 - |U_{\mathrm{e}3}|^2), \\
|U_{\mathrm{e}2}|^2 &= \sin^2\theta_{\odot}(1 - |U_{\mathrm{e}3}|^2), \\
|U_{\mathrm{e}3}|^2 &\equiv \sin^2\theta < 0.09 \qquad \text{(CHOOZ)}.
\end{aligned} \tag{44}$$

We will suppose that $\Delta m^2_{\mathrm{atm}}$ lies in the interval (5) or (13), $\Delta m^2_{\odot}$ and $\theta_{\odot}$ take values in the regions given in Tables 1 and 2, and that $|U_{\mathrm{e}3}|^2$ satisfies the CHOOZ upper bound. Equations (42) and (44) further imply:

$$m_2 \simeq \sqrt{\Delta m^2_{\odot}}\,, \qquad m_3 \simeq \sqrt{\Delta m^2_{\mathrm{atm}}}\ . \tag{45}$$

One finds

$$|<\!m\!>| \ \simeq \left|\sqrt{\Delta m^2_{\odot}}\,(1 - |U_{\mathrm{e}3}|^2)\sin^2\theta_{\odot} + \sqrt{\Delta m^2_{\mathrm{atm}}}\,|U_{\mathrm{e}3}|^2 e^{i(\alpha_3-\alpha_2)}\right| \tag{46}$$

where we have neglected the contribution of the term $\sim m_1$. Although in this case one of three massive Majorana neutrinos effectively "decouples" and does not give a contribution to $|<\!m\!>|$, the value of $|<\!m\!>|$ still depends on the Majorana CP-violating phase $\alpha_{32} = \alpha_3 - \alpha_2$. This reflects the fact that in contrast to the case of massive Dirac neutrinos (or quarks), CP-violation can take place in the mixing of only two massive Majorana neutrinos.

The possibility of partial or complete cancellation between the two terms in $|\langle m \rangle|$ is extremely important in view of the future searches for the $(\beta\beta)_{0\nu}$-decay. Barring finite m_1 corrections which can be relevant in the case of the SMA solution of the solar neutrino problem, the cancellation can take place if $|U_{e3}|^2$ has a value given by:

$$|\langle m \rangle| = 0: \qquad |U_{e3}|_0^2 = \frac{\sqrt{\Delta m^2_{\odot}}\ \sin^2\theta_{\odot}}{\sqrt{\Delta m^2_{\odot}}\ \sin^2\theta_{\odot} + \sqrt{\Delta m^2_{atm}}}\ . \tag{61}$$

For the ranges of allowed values of $\Delta m^2_{\odot}$, $\sin^2\theta_{\odot}$ for the three solutions of the $\nu_{\odot}-$problem, and of Δm^2_{atm} , found in [58], the corresponding values of $|U_{e3}|_0^2$ for which the cancellation can occur belong to the intervals:

$$\begin{array}{lllll} 8.0\ (8.0) \times 10^{-3} & \leq |U_{e3}|_0^2 & \leq & 1.3\ (3.7) \times 10^{-1} & \text{LMA,} \\ 1.0\ (0.3) \times 10^{-5} & \leq |U_{e3}|_0^2 & \leq & 7.0\ (20) \times 10^{-5} & \text{SMA,} \\ 8.0\ (0.7) \times 10^{-4} & \leq |U_{e3}|_0^2 & \leq & 0.5\ (13) \times 10^{-2} & \text{LOW-QVO.} \end{array} \tag{62}$$

The cancellation cannot occur for the best fit value of $|U_{e3}|^2 = 0.005$ (LMA).

The equation for $|\langle m \rangle|$ permits to express the cosine of the CP-violating phase $(\alpha_3 - \alpha_2)$ in terms of measurable quantities:

$$\cos(\alpha_3 - \alpha_2) = \frac{|\langle m \rangle|^2 - \Delta m^2_{\odot}\ \sin^4\theta_{\odot}(1 - |U_{e3}|^2)^2 - \Delta m^2_{atm}\ (|U_{e3}|^2)^2}{2\sqrt{\Delta m^2_{\odot}}\ \sqrt{\Delta m^2_{atm}}\ \sin^2\theta_{\odot}|U_{e3}|^2(1 - |U_{e3}|^2)}. \tag{63}$$

Thus, if $|\langle m \rangle|$ and $|U_{e3}|^2$ will be found to be nonzero and their values (together with the values of $\Delta m^2_{\odot}$, Δm^2_{atm} and $\sin^2\theta_{\odot}$) will be determined experimentally with sufficient precision, one can get direct information about the CP-violation in the lepton sector, caused by Majorana CP-violating phases.

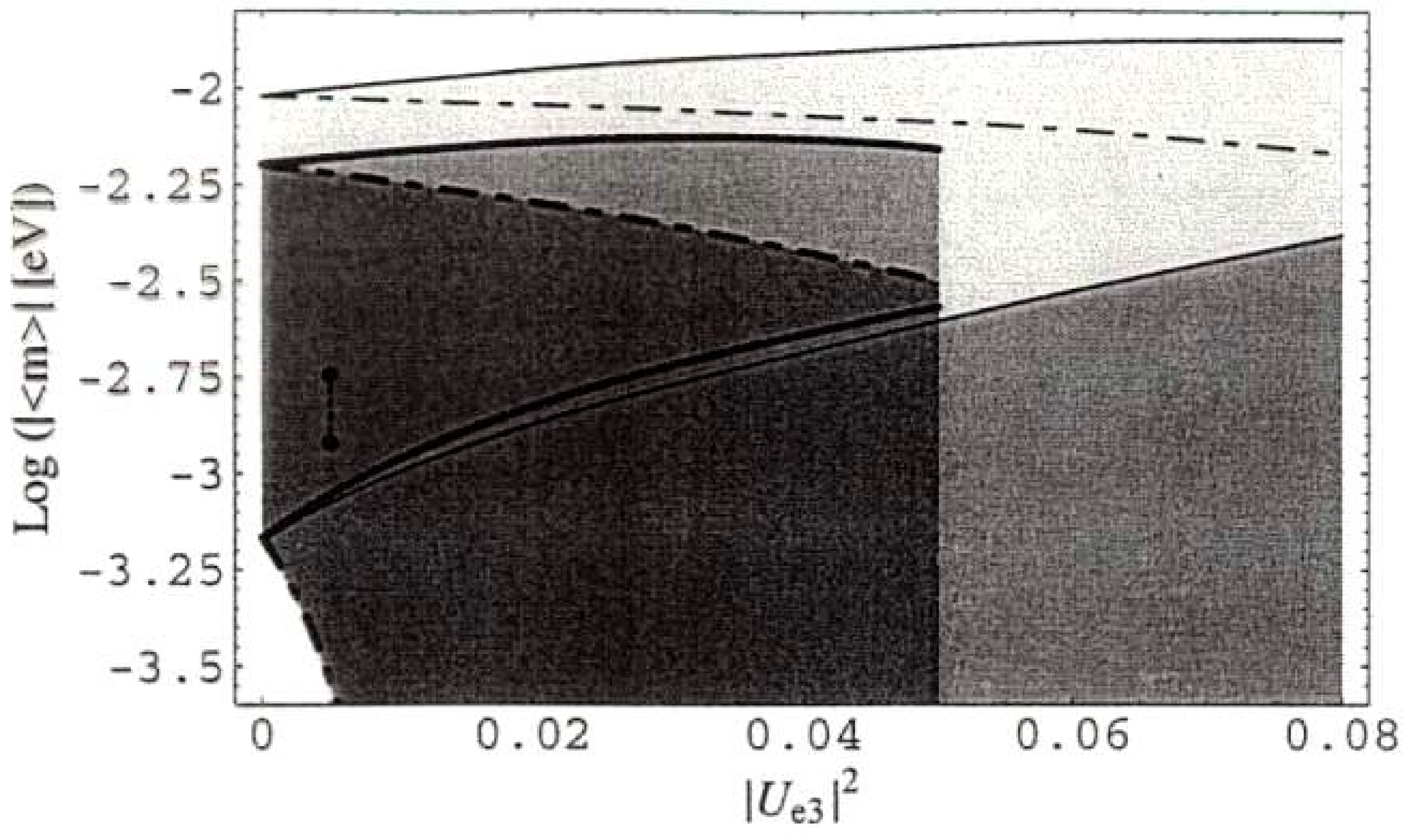


Figure 2: The effective Majorana mass $|<m>|$, allowed by the data from the solar and atmospheric neutrino and CHOOZ experiments, as a function of $|U_{e3}|^2$ in the case of hierarchical neutrino mass spectrum, eq. (40). The values of $|<m>|$ are obtained for $\Delta m^2_\odot$, $\sin^2\theta_\odot$ from the LMA MSW solution region, and $\Delta m^2_{\rm atm}$ and $|U_{e3}|^2$, derived in [58] at 90% C.L. (medium grey and dark grey regions at $|U_{e3}|^2 < 0.05$) and at 99% C.L. (light, medium and dark grey regions at $|U_{e3}|^2 < 0.08$). The 90% (99%) C.L. allowed regions located *i)* between the two thick (thin) solid lines and *ii)* between the thick (thin) dashed lines and the horizontal axis, correspond to the two cases of CP conservation: *i)* $\phi_2 = \phi_3$, and *ii)* $\phi_2 = -\phi_3$, $i\phi_{2,3}$ being the CP-parities of $\nu_{2,3}$. The values of $|<m>|$, calculated for the best fit values of the input parameters, are indicated by thick dots (CP-conservation) and a dotted line (CP-violation). In the case of CP-violation all the regions marked with different grey colour scales are allowed.

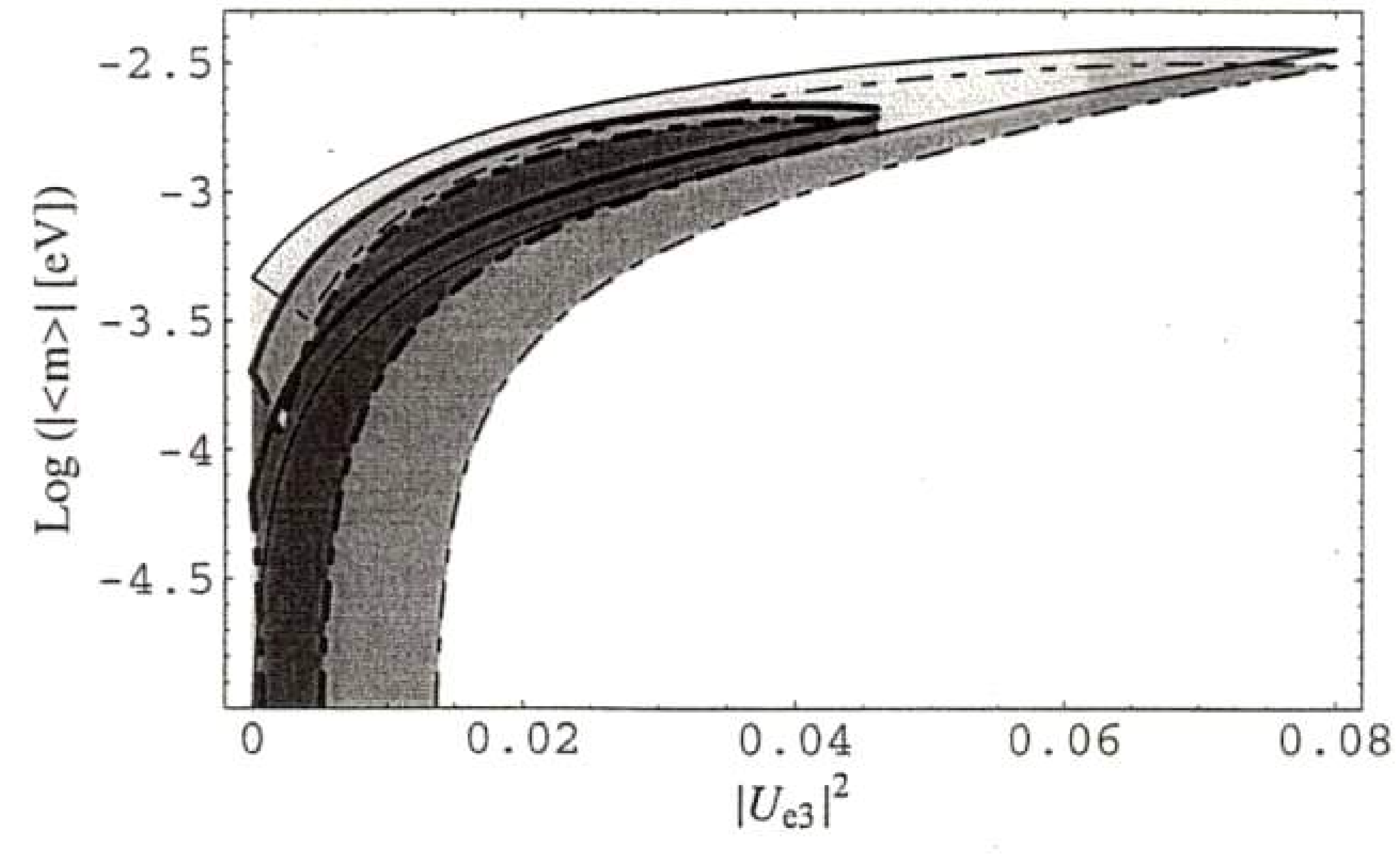


Figure 3: The same as Fig. 2 but for $\Delta m^2_\odot$ and $\sin^2\theta_\odot$ from the 90% (99%) C.L. region of the LOW-QVO solution of the $\nu_\odot$-problem [58]. The regions limited by the two thick (thin) solid and dashed lines correspond to the two cases of CP-conservation: *i)* $\phi_2 = \phi_3$, (regions within the solid lines) and *ii)* $\phi_2 = -\phi_3$ (regions within the dashed lines). In the case of CP-violation, $(\alpha_3 - \alpha_2) \neq k\pi$, $k = 0, 1, ...,$ all the regions marked by different grey color scales are allowed.

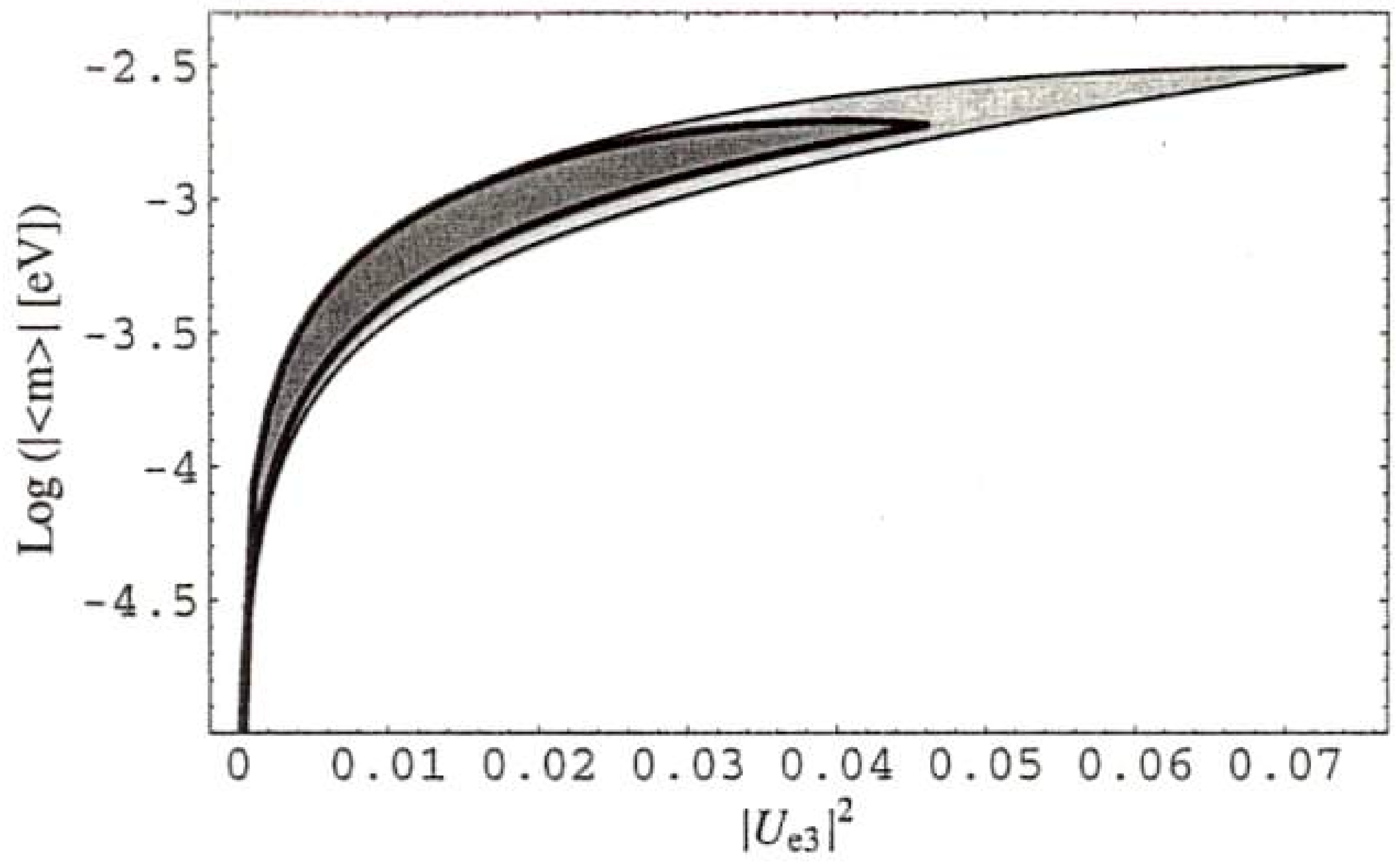


Figure 4: The same as Fig. 2 but for $\Delta m^2_{\odot}$ and $\sin^2\theta_{\odot}$ from the region of the SMA solution of the $\nu_{\odot}$-problem [58] obtained at 90% C.L. (region within the thick solid lines) and at 99% C.L. (region within the thin solid lines) . The results shown are derived assuming $m_1 \ll 10^{-4}$ eV . For the range of values of $|<m>|$ in the figure, one has in this case $|<m>| \sim \sqrt{\Delta m^2_{\rm atm}}\,|U_{e3}|^2$ and thus $|<m>|$ does not depend on the CP-violating phase $(\alpha_3 - \alpha_2)$.

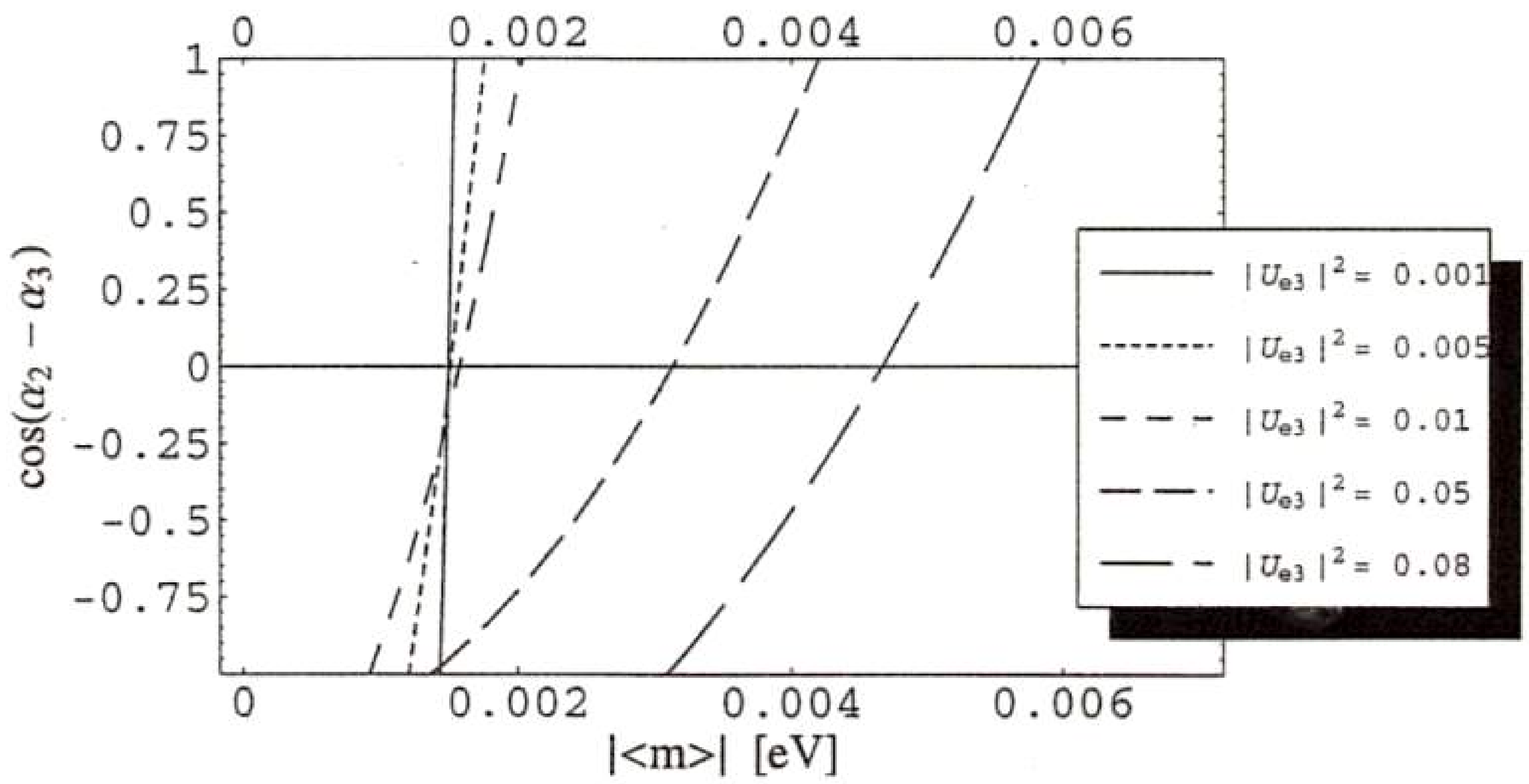


Figure 5: The CP-violation factor $\cos(\alpha_2 - \alpha_3)$, eq. (40) (hierarchical neutrino mass spectrum), as a function of $|<m>|$ for different values of $|U_{e3}|^2$ and for the best fit values of the parameters $\Delta m^2_{\odot}$, $\sin^2\theta_{\odot}$ and $\Delta m^2_{\rm atm}$, found in the analysis [58]. Values of $\cos(\alpha_2 - \alpha_3) = 0, \pm 1$, correspond to CP-invariance.

5 Inverted Mass Hierarchy Spectrum

The inverted mass hierarchy spectrum is characterized by

$$m_1 \ll m_2 \simeq m_3. \tag{66}$$

The identification with the neutrino oscillation parameters probed in the solar and atmospheric neutrino experiments and in CHOOZ reads

$$\begin{aligned}
\Delta m^2_{\odot} &\equiv \Delta m^2_{32}, \\
\Delta m^2_{\mathrm{atm}} &\equiv \Delta m^2_{31} \simeq \Delta m^2_{21}, \\
|U_{\mathrm{e}1}|^2 &= \sin^2\theta < 0.09 \quad \text{(CHOOZ)}, \\
|U_{\mathrm{e}2}|^2 &= \cos^2\theta_{\odot}(1 - |U_{\mathrm{e}1}|^2), \\
|U_{\mathrm{e}3}|^2 &= \sin^2\theta_{\odot}(1 - |U_{\mathrm{e}1}|^2).
\end{aligned} \tag{67}$$

We also have:

$$m_2 \simeq m_3 \simeq \sqrt{\Delta m^2_{\mathrm{atm}}}. \tag{68}$$

The inverted mass hierarchy spectrum can also be defined by the inequalities

$$m_1 \ll (<) \sqrt{\Delta m^2_{32}} \ll \sqrt{\Delta m^2_{21}}. \tag{69}$$

The term $m_1|U^2_{\mathrm{e}1}|$ in the expression for $|{<}m{>}|$ can be neglected, since $m_1 \ll m_{2,3}$ and $|U^2_{\mathrm{e}1}| \ll 1$. This approximation would be valid as long as the sum of the two other terms in eq. (21) exceeds $\sim 5.0 \times 10^{-3}$ eV. Under the assumption that $m_1|U^2_{\mathrm{e}1}|$ gives a negligible contribution to $|{<}m{>}|$ we have

$$|{<}m{>}| \simeq \left||U_{\mathrm{e}2}|^2 m_2 + |U_{\mathrm{e}3}|^2 m_3 e^{i(\alpha_3-\alpha_2)}\right| \tag{70}$$

$$= m_{2,3}\sqrt{1 - 4|U_{\mathrm{e}2}|^2|U_{\mathrm{e}3}|^2 \sin^2\left(\frac{\alpha_3-\alpha_2}{2}\right)} \tag{71}$$

$$= \sqrt{\Delta m^2_{\mathrm{atm}}}\,(1 - |U_{\mathrm{e}1}|^2)\sqrt{1 - \sin^2 2\theta_{\odot} \sin^2\left(\frac{\alpha_3-\alpha_2}{2}\right)}, \tag{72}$$

Even though one of the three massive Majorana neutrinos "decouples", the value of $|{<}m{>}|$ depends on the Majorana CP-violating phase $(\alpha_3 - \alpha_2)$.

Obviously, $|{<}m{>}|$ satisfies

$$\sqrt{\Delta m^2_{\mathrm{atm}}}\,(1 - |U_{\mathrm{e}1}|^2)|\cos 2\theta_{\odot}| \le |{<}m{>}| \le \sqrt{\Delta m^2_{\mathrm{atm}}}\,(1 - |U_{\mathrm{e}1}|^2). \tag{73}$$

The upper and the lower limits correspond respectively to the CP-conserving cases $\phi_2 = \phi_3$ ($\alpha_3 - \alpha_2 = 0$, or $\alpha_{21} = \alpha_{31} = 0, \pm\pi$) and $\phi_2 = -\phi_3$ ($\alpha_3 - \alpha_2 = \pm\pi$, or $\alpha_{21} = \alpha_{31} + \pi = 0, \pm\pi$).

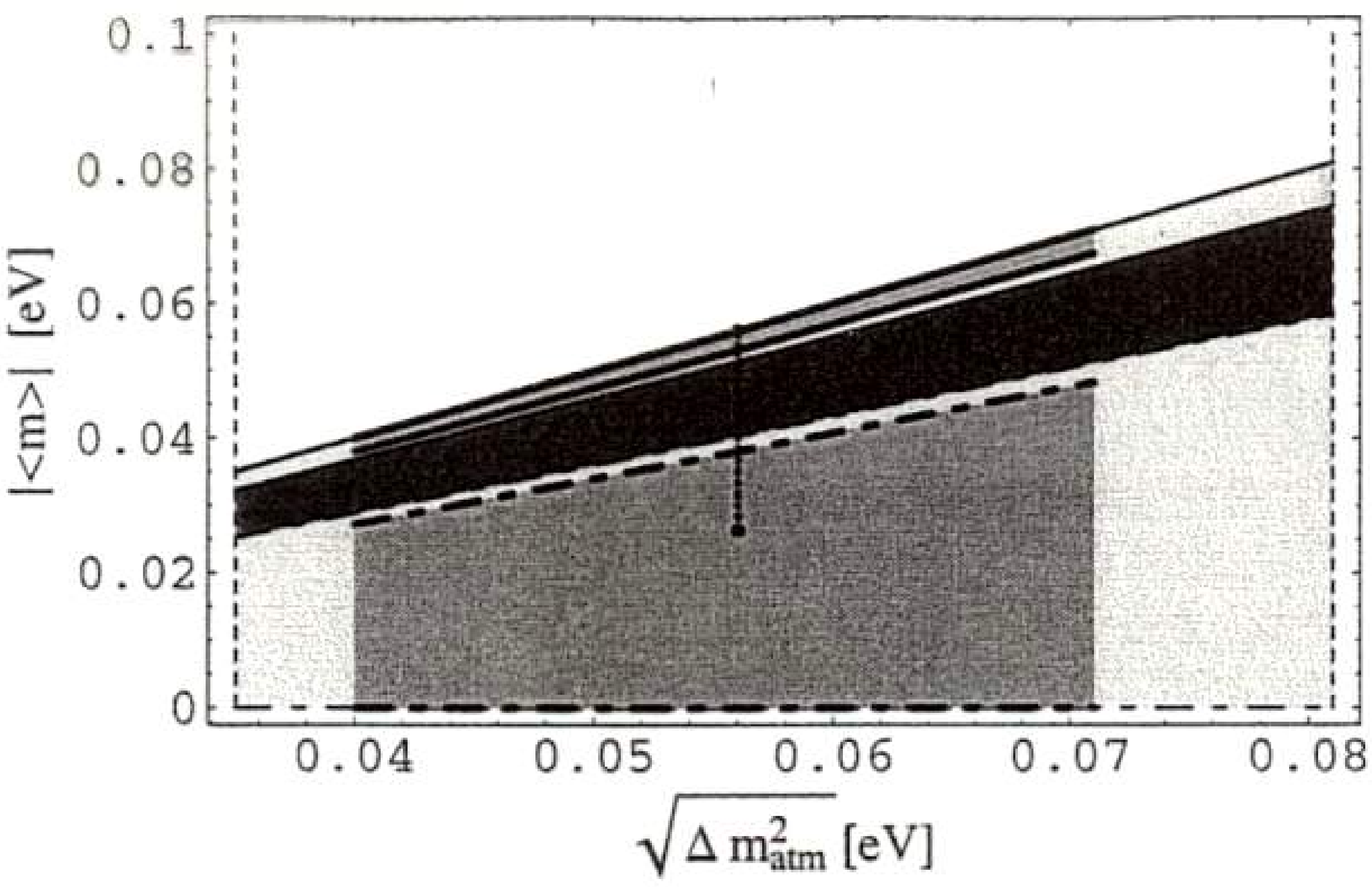


Figure 6: The effective Majorana mass $|<m>|$ as a function of $\sqrt{\Delta m^2_{\rm atm}}$ for the neutrino mass spectrum of the inverted hierarchy type, eq. (61). The allowed regions (in grey) correspond to the LMA solution of ref. [58]. In the case of CP-invariance and for the 90% (99%) C.L. results for the solution region, $|<m>|$ can have values *i)* for $\phi_2 = \phi_3$ - in the medium grey (light grey and medium grey) upper region, limited by the doubly thick (thick and doubly thick) solid lines and *ii)* for $\phi_2 = -\phi_3$ - in the medium grey (light grey and medium grey) region limited by the doubly thick (thick and doubly thick) dash-dotted lines . If CP is not conserved, $|<m>|$ can lie in any of the regions marked by different grey scales The dark-grey region corresponds to "just-CP-violation": $|<m>|$ can have value in this region *only if the CP-parity is not conserved.* The values of $|<m>|$ corresponding to the best fit values of the input parameters, found in [58], are denoted by dots in the CP-conserving cases and by a dotted line in the CP-violating one.

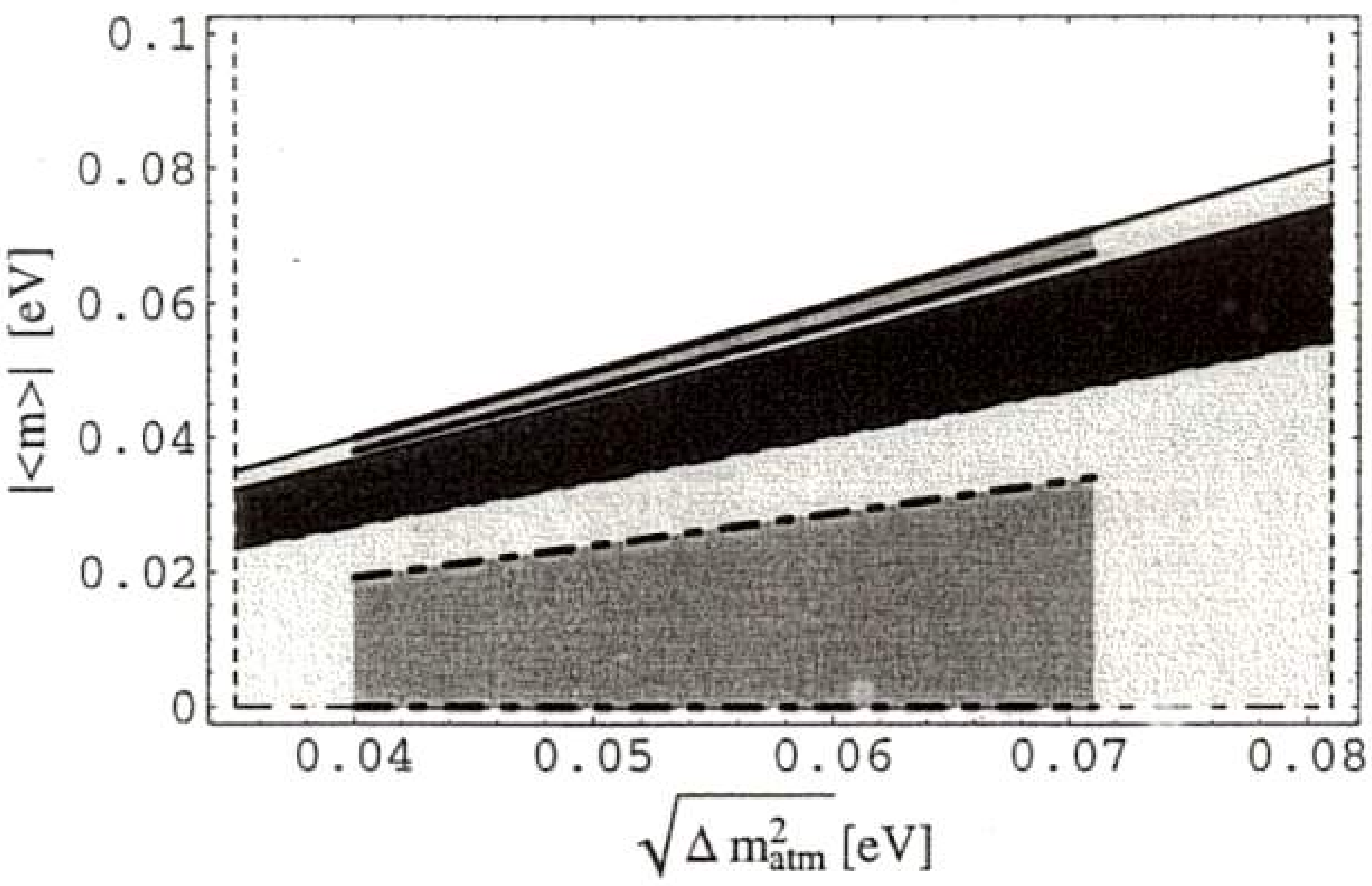


Figure 7: The same as in Fig. (6) for the 90% (99%) C.L. LOW-QVO solution of ref. [58]. The medium grey (light-grey and medium grey) region, bounded by the the thick (thick and doubly thick) solid lines, and medium grey (light grey and medium grey) lower region, bounded by the thick (thin) dash-dotted lines, correspond to the two cases of CP-invariance, $\phi_2 = \phi_3$ and $\phi_2 = -\phi_3$, respectively. If CP is not conserved, $|<m>|$ can lie in any of the regions marked by different grey scales. The "just-CP-violation" region is shown in dark-grey color: $|<m>|$ can have value in this region *only if the CP-symmetry is violated.*

The expression for $|\!<\!m\!>\!|$ permits to relate the value of $\sin^2(\alpha_3 - \alpha_2)/2$ to the experimentally measured quantities $|\!<\!m\!>\!|$, $\Delta m^2_{\rm atm}$ and $\sin^2 2\theta_\odot$:

$$\sin^2 \frac{\alpha_3 - \alpha_2}{2} = \left(1 - \frac{|\!<\!m\!>\!|^{\,2}}{\Delta m^2_{\rm atm}\, (1 - \,|U_{\rm e1}|^2\,)^2} \right) \frac{1}{\sin^2 2\theta_\odot}. \tag{75}$$

A more precise determination of $\Delta m^2_{\rm atm}$ and $\theta_\odot$ and a sufficiently accurate measurement of $|\!<\!m\!>\!|$ could allow to get information about the value of $(\alpha_3 - \alpha_2)$, provided the neutrino mass spectrum is of the inverted hierarchy type.

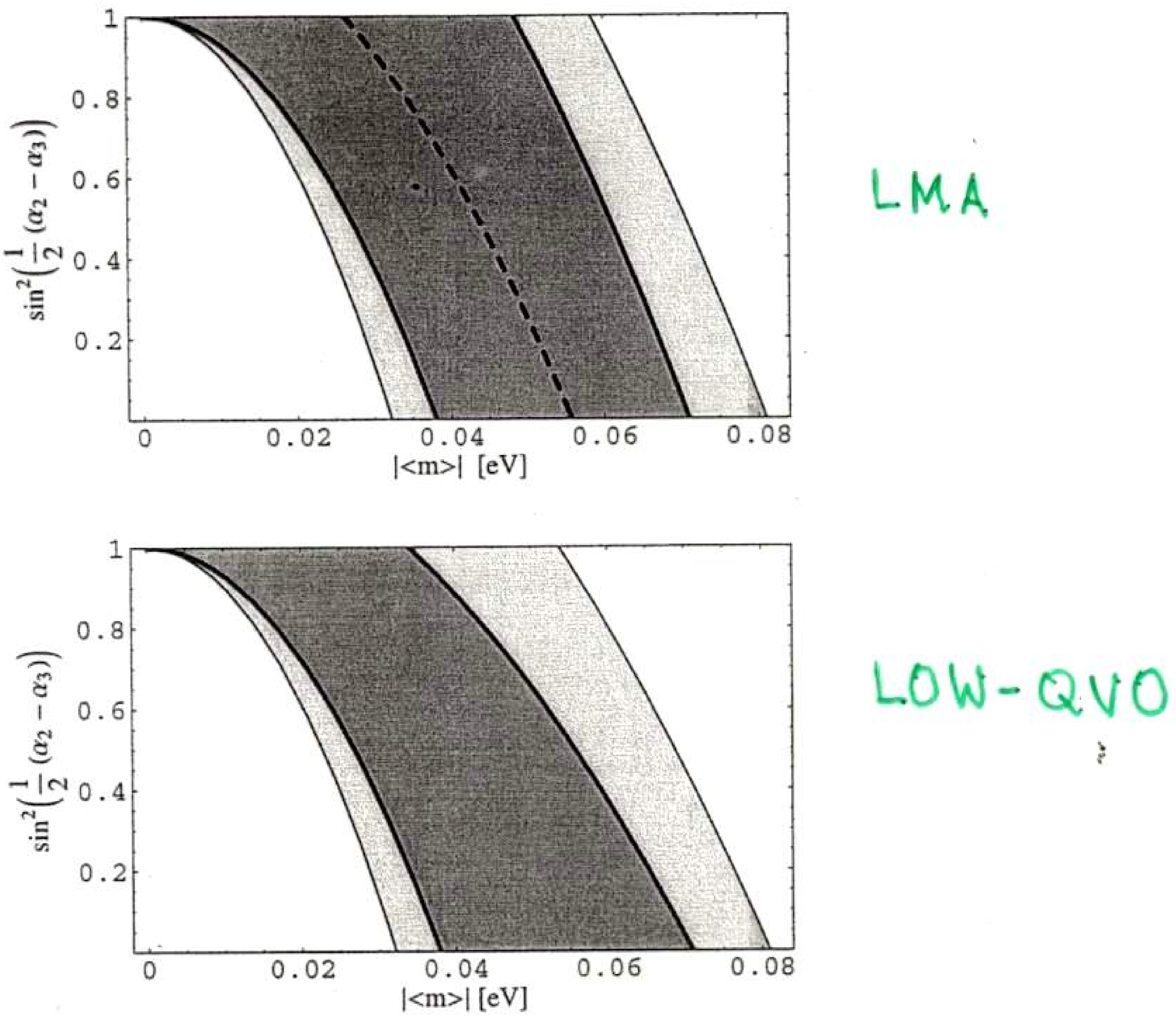


Figure 11: The CP-violation factor $\sin^2(\alpha_2 - \alpha_3)/2$ as a function of $|<m>|$ in the case of inverted mass hierarchy spectrum, eq. (61), and the LMA (upper panel) and LOW-QVO (lower panel) solutions of ref. [58], obtained at 90% C.L. (medium grey region with thick contours) and 99% C.L. (light grey and medium grey region limited by ordinary solid lines). The values of $|<m>|$, corresponding to the best fit values of the input parameters, found in [58], are indicated by the dashed line. A value of $\sin^2(\alpha_2 - \alpha_3)/2 \neq 0, 1$, would signal CP-violation.

6 The Case of Three Quasi-Degenerate Neutrinos

The neutrinos $\nu_{1,2,3}$ are quasi-degenerate in mass if

$$m_1 \simeq m_2 \simeq m_3 \equiv m, \tag{76}$$

and

$$m \gg \sqrt{\Delta m^2_{\rm atm}}\ . \tag{77}$$

When (76) holds but $m \sim O(\sqrt{\Delta m^2_{\rm atm}}\,)$, we have a partial hierarchy or partial inverted hierarchy between the neutrino masses.

As for the hierarchical neutrino mass spectrum we have:

$$\begin{aligned} \Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} &\equiv \Delta m^2_{32}, \\ |U_{\rm e1}|^2 &= \cos^2\theta_{\odot}(1 - |U_{\rm e3}|^2\,), \\ |U_{\rm e2}|^2 &= \sin^2\theta_{\odot}(1 - |U_{\rm e3}|^2\,), \\ |U_{\rm e3}|^2 &= \sin^2\theta < 0.09 \quad \text{(CHOOZ)}. \end{aligned} \tag{78}$$

Equation (78) allows to express eqs. (76) and (77) in the compact form

$$\sqrt{\Delta m^2_{21}} \ll\ (<)\ \sqrt{\Delta m^2_{32}} \ll m_1. \tag{79}$$

The mass scale m effectively coincides with the electron (anti-)neutrino mass m_{ν_e} measured in the ^{3}H β-decay experiments:

$$m = m_{\nu_e}. \tag{80}$$

Thus, the experimental upper bounds in eq. (15) lead to $m < 2.5$ eV .

The QD neutrino mass spectrum under discussion is actually realized for values of the neutrino mass m, which is measured in the ^{3}H $\beta-$decay experiments, $m = m_{\nu_e} \gtrsim (0.2-0.3)$ eV. The new ^{3}H $\beta-$decay experiment KATRIN is planned to have a record sensitivity of 0.35 eV to the neutrino mass m_{ν_e}. The realization of this project could be crucial for the test of the possibility of three quasi-degenerate neutrinos.

The effective Majorana mass $|<m>|$ can be approximated by

$$|<m>| \ \simeq m\left|\cos^2\theta_{\odot}(1 - |U_{\rm e3}|^2\,)e^{i\alpha_1} + \sin^2\theta_{\odot}(1 - |U_{\rm e3}|^2\,)e^{i\alpha_2} + |U_{\rm e3}|^2 e^{i\alpha_3}\right|. \tag{81}$$

If CP is conserved, we have the following possibilities, depending on the relative CP-parities of the neutrinos ν_j.

Case A. For $\phi_1 = \phi_2 = \pm\phi_3$ (i.e., $\alpha_{21} = 0,\ \alpha_{31} = 0, \pm\pi$), we get a very simple expression for $|<m>|$:

$$|<m>| \simeq m\left(1 - |U_{e3}|^2 \pm |U_{e3}|^2\right), \tag{82}$$

i.e., $|<m>|$ does not depend on $\Delta m^2_{\odot}$, $\theta_{\odot}$ and $\Delta m^2_{\rm atm}$. Note that if $\phi_1 = \phi_2 = \phi_3$, $|<m>| = m$. Using the CHOOZ limit [64] on $|U_{e3}|^2$ one finds the range of allowed values of $|<m>|$:

$$0.8\ m \leq |<m>| \leq 1.0\ m. \tag{83}$$

The above result and the bound on $|<m>|$ obtained in the Heidelberg-Moscow experiment imply

$$m < (0.4 \div 1.2)\ \text{eV}\ , \tag{84}$$

which is compatible with the ^{3}H beta-decay limit on m.

Case B. If $\phi_1 = -\phi_2 = \pm\phi_3$ (i.e., $\alpha_{21} = \pm\pi,\ \alpha_{31} = 0, \pm\pi$), $|<m>|$ is given by:

$$|<m>| \simeq m\left|\cos 2\theta_{\odot}(1 - |U_{e3}|^2) \pm |U_{e3}|^2\right|. \tag{85}$$

Now $|<m>|$ depends $\cos 2\theta_{\odot}$ and thus varies with the solution of the solar neutrino problem.

Table 4: Values of $|<m>|\ /m$ for the quasi-degenerate neutrino mass spectrum, CP-conservation and $\phi_1 = -\phi_2 = \pm\phi_3$.

Data from		$\lvert<m>\rvert\ /m$	
		$\phi_1 = -\phi_2 = \phi_3$	$\phi_1 = -\phi_2 = -\phi_3$
Ref. [7]	LMA	$0.21 \div 0.68$	$0.10 \div 0.65$
(95% C.L.)	LOW-QVO	$0.3 \div 0.58$	$0.18 \div 0.54$
Ref. [56]	LMA	0.25 (0) $\div$ 0.68 (0.70)	0.12 (0) $\div$ 0.65 (0.67)
(90% (99%) C.L.)	SMA	(1.0)	(0.8 $\div$ 1.0)
	LOW-QVO	0.05 (0) $\div$ 0.33 (0.49)	0 (0) $\div$ 0.26 (0.42)
Ref. [58]	LMA	0 (0) $\div$ 0.70 (0.74)	0 (0) $\div$ 0.68 (0.72)
(90% (99%) C.L.)	SMA	1.0 (1.0)	0.9 (0.8) $\div$ 1.0 (1.0)
	LOW-QVO	0 (0) $\div$ 0.50 (0.69)	0 (0) $\div$ 0.48 (0.67)

Several remarks are in order.

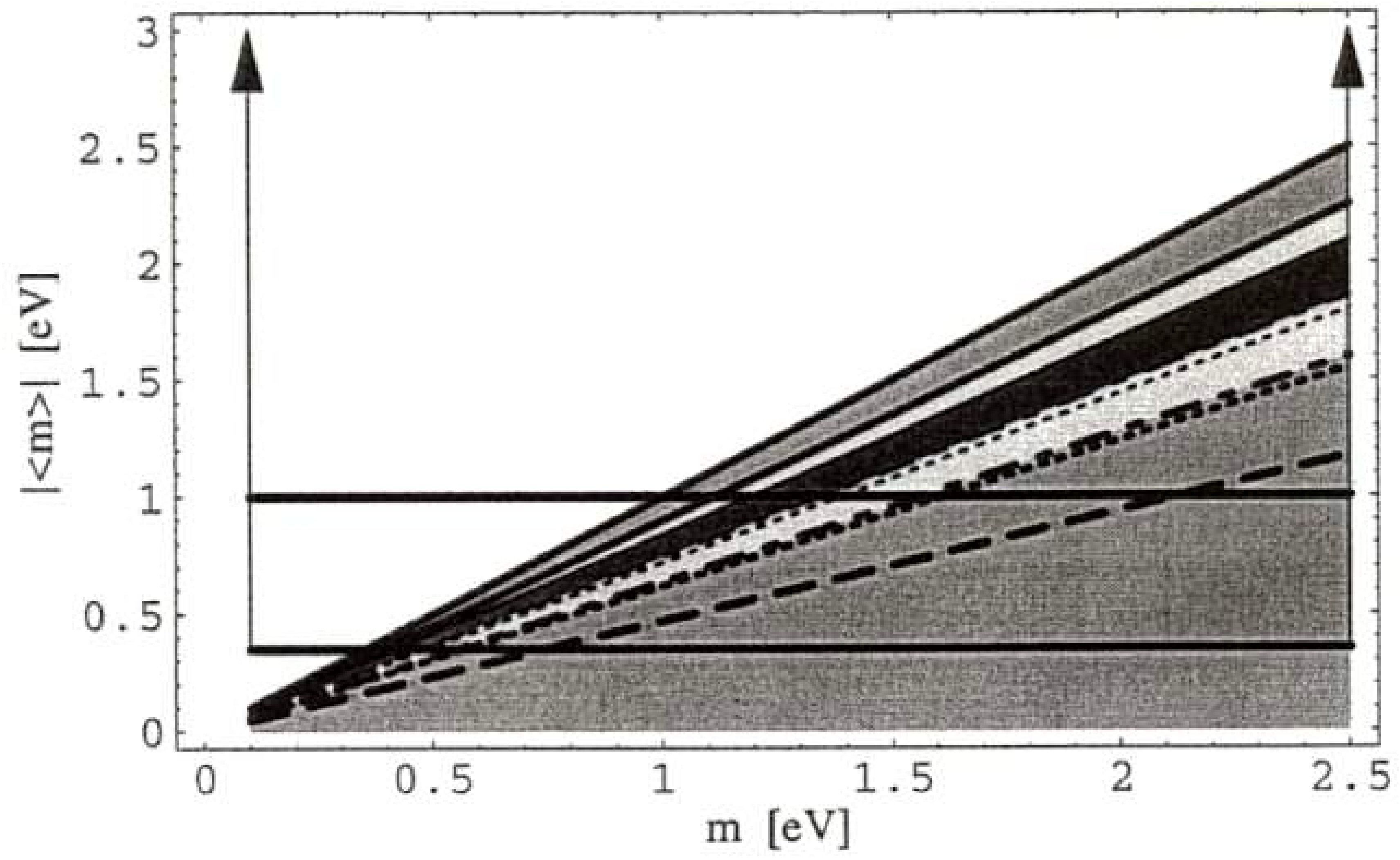


Figure 12: $|{<}m{>}|$ as a function of the neutrino mass m for the quasi-degenerate neutrino mass spectrum, eqs. (71) - (72), and the LMA solution of the $\nu_{\odot}-$problem of ref. [58] at 90% C.L. (99% C.L.). The regions allowed in the cases of CP-conservation are marked by *i)* $\phi_1 = \phi_2 = \phi_3$ - the thick solid (non-horizontal) line $|{<}m{>}| = m$, *ii)* $\phi_1 = \phi_2 = -\phi_3$ - medium grey color triangular region between the two doubly thick solid lines (light grey and medium grey color triangular region between the doubly thick and thick solid lines, *iii)* $\phi_1 = -\phi_2 = \pm\phi_3$ (two cases) - medium grey color triangular regions between the doubly thick dashed-dotted line and the horizontal axes and between the doubly thick dotted line and the horizontal axes (light grey and medium grey region between the thin dashed-dotted line and the horizontal axes and between the thin dotted line and the horizontal axes). The "just-CP-violation" region is denoted by dark-grey color. The doubly thick solid line corresponding to $|{<}m{>}| = m$ and the doubly thick long dashed line indicate the "best fit lines" of values of $|{<}m{>}|$ for the i)-ii) and the iii) cases, respectively. The two horizontal (doubly thick) lines show the upper limits [29], quoted in eq. (3).

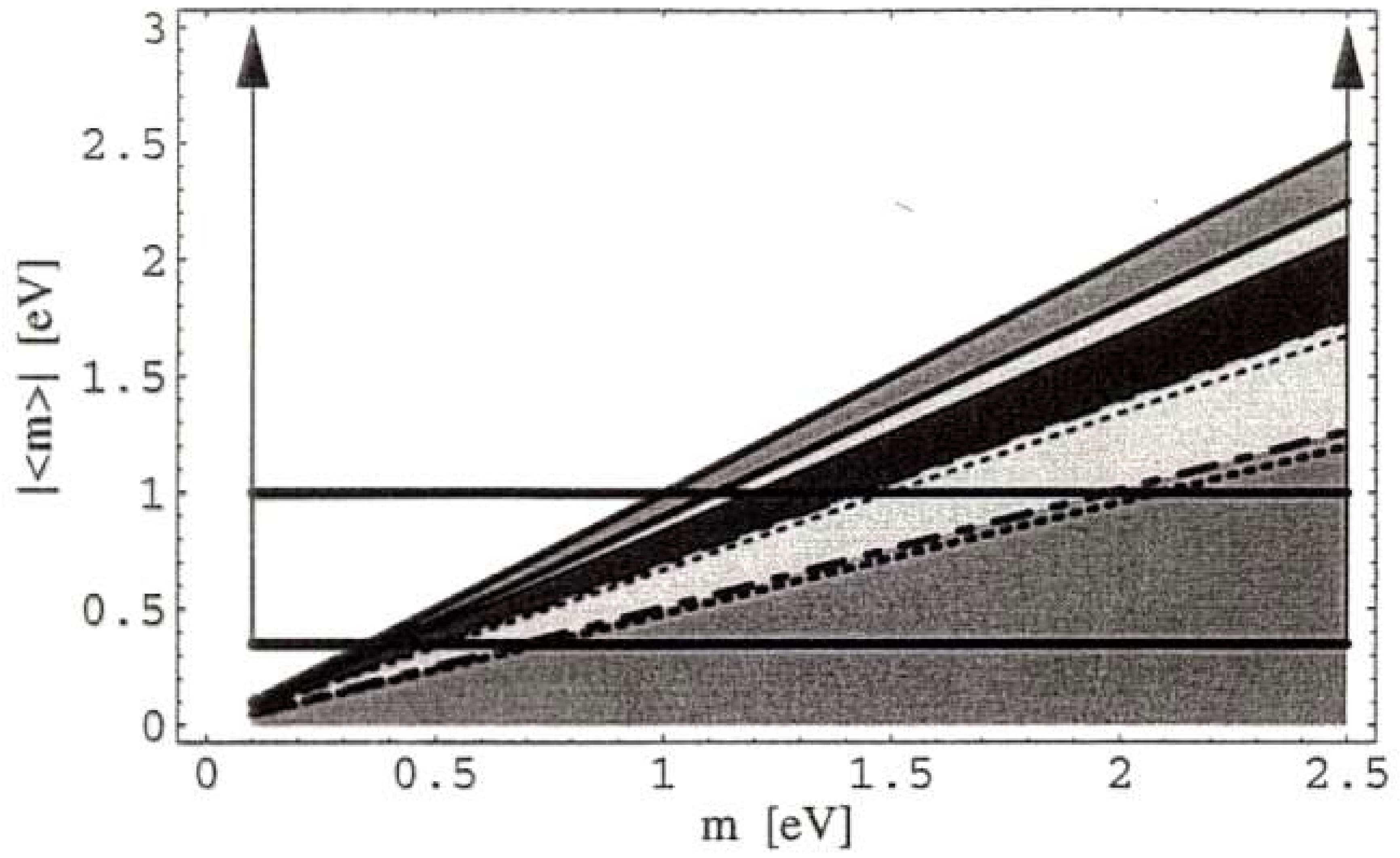


Figure 13: The same as in Fig.(12) for the LOW-QVO solution of the $\nu_{\odot}-$problem [58]. If CP is conserved and at 90% C.L. (99 % C.L.) [58], $|<m>|$ should lie in the two medium grey (the two light grey and medium grey) triangular regions: i) for $\phi_1 = \phi_2 = \phi_3$ - on the line $|<m>| = m$, ii) for $\phi_1 = \phi_2 = -\phi_3$ - in the upper medium grey (light grey and medium grey) triangular region, iii) if $\phi_1 = -\phi_2 = \pm\phi_3$ - in the lower medium grey (light grey and medium grey) triangular region with dash-dotted ($\phi_1 = +\phi_3$) and dotted ($\phi_1 = -\phi_3$) contours. The "just-CP-violation" region is denoted by dark-grey color.

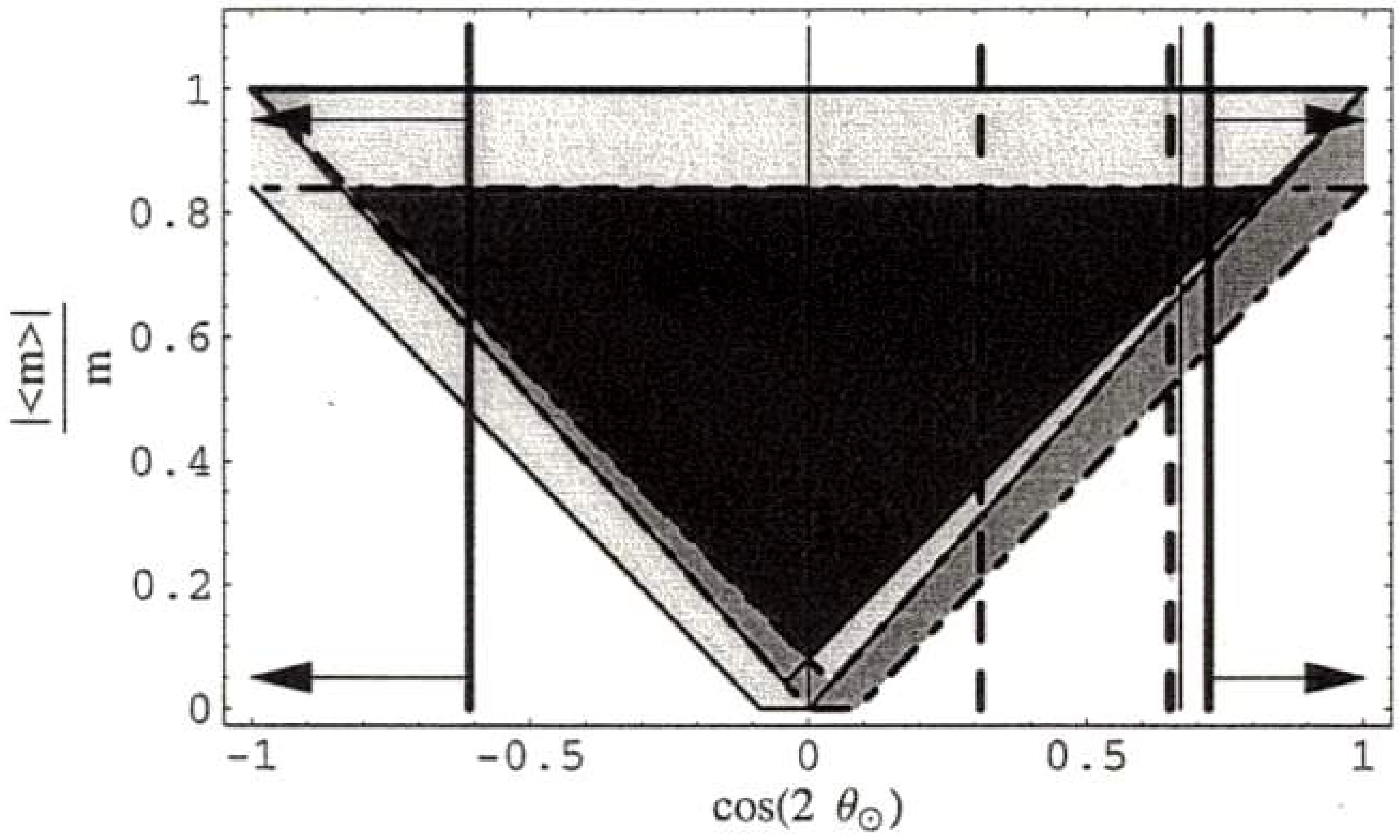


Figure 14: The dependence of $|<m>|$ /m on $\cos 2\theta_{\odot}$ for the quasi-degenerate neutrino mass spectrum, eqs. (71) - (72), and the LMA solution of the $\nu_{\odot}-$problem. If CP-invariance holds, the values of $|<m>|$ /m lie: i) for $\phi_1 = \phi_2 = \phi_3$ - on the line $|<m>|\ /m = 1$, ii) for $\phi_1 = \phi_2 = -\phi_3$ - in the region between the thick horizontal solid and dash-dotted lines (in light grey and medium grey colors), iii) for $\phi_1 = -\phi_2 = +\phi_3$ - in the light grey polygon with solid-line contours and iv) for $\phi_1 = -\phi_2 = -\phi_3$ - in the medium grey polygon with the dash-dotted-line contours. The "just-CP-violation" region is denoted by dark-grey color. The values of $\cos 2\theta_{\odot}$ between the doubly thick solid, the normal solid and the doubly thick dashed lines correspond to the 99%, 99% and 95% C.L. LMA solution regions in refs. [58], [56] and [7], respectively.

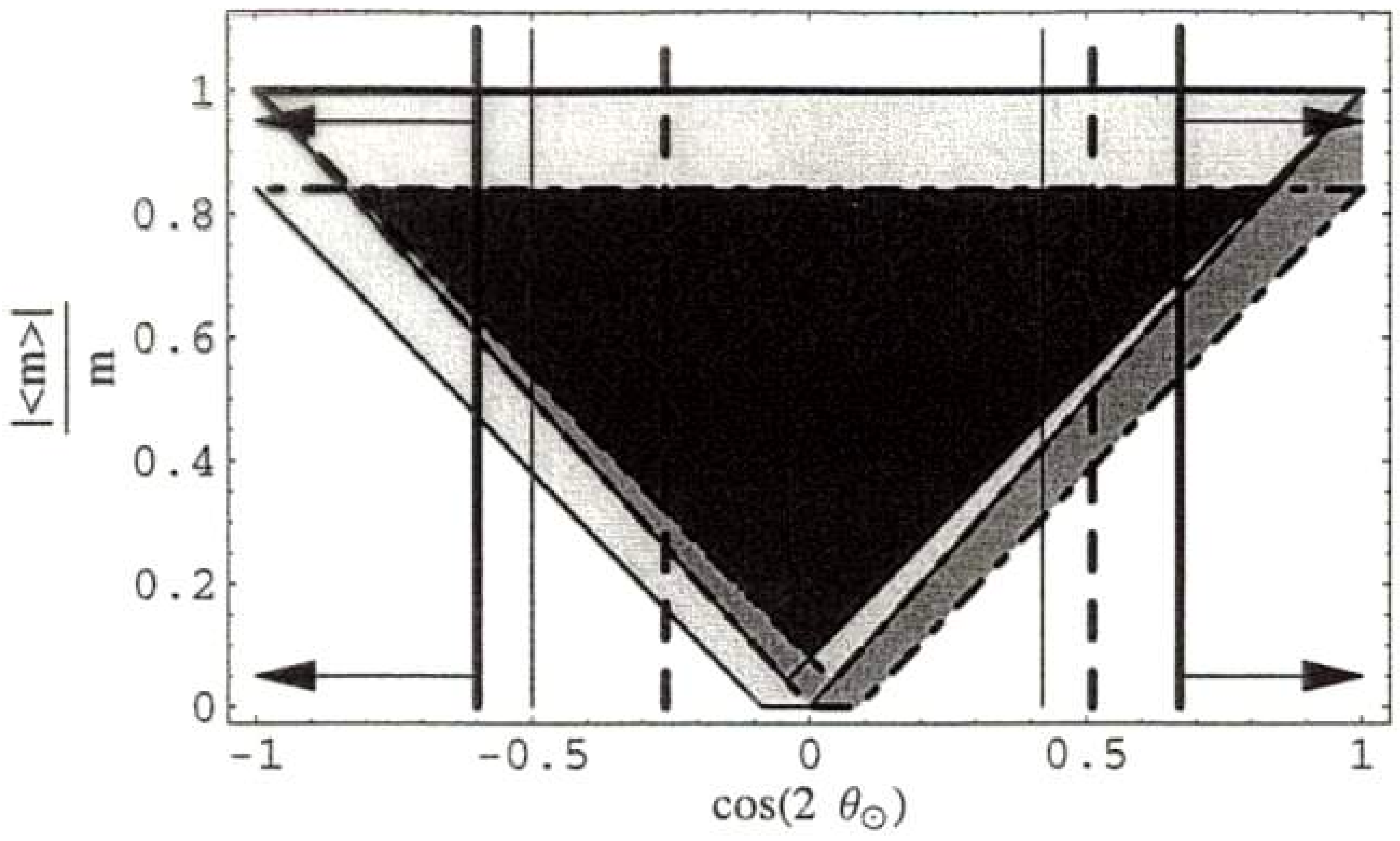


Figure 15: The same as in Fig. (14) for the LOW-QVO solution of the solar neutrino problem. The "just-CP-violating" region, in particular, is denoted by dark-grey color.

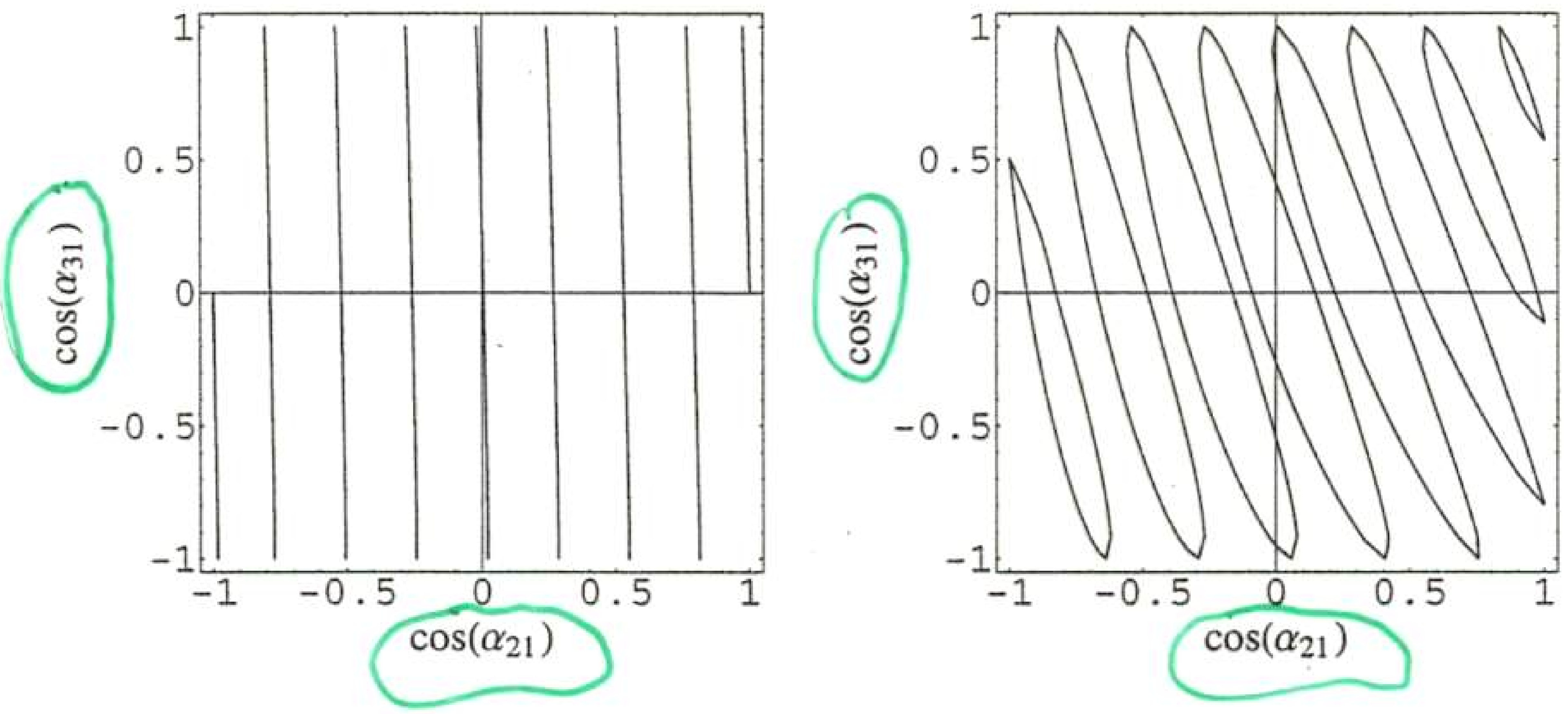


Figure 16: The interdependence of the two CP-violating phases, α_{21} and α_{31}, for a given value of the ratio $|<m>|$ /m in the case of quasi-degenerate neutrino mass spectrum. The figures are obtained for $|<m>|$ /m $= \sqrt{0.2+0.1n}$ eV with $n = 0, 1 \ldots 8$ (with increasing $|<m>|$ from left to right) and the best fit values of the solar and atmospheric neutrino oscillation parameters from [58], quoted in Section 2 (left-hand plot), and for $|<m>|$ /m $= \sqrt{0.24+0.10n}$ eV with $n = 0, 1 \ldots 7$ (with increasing $|<m>|$ from left to right), $|U_{e3}|^2 = 0.08$ and the best fit values of the other solar and atmospheric neutrino oscillation parameters [58] (right-hand plot). The values of $\cos\alpha_{21,31} = 0, \pm 1$, correspond to CP-invariance.

BILENKY, PASCOLI, S.T.P., 2001
(hep-ph/0102265)

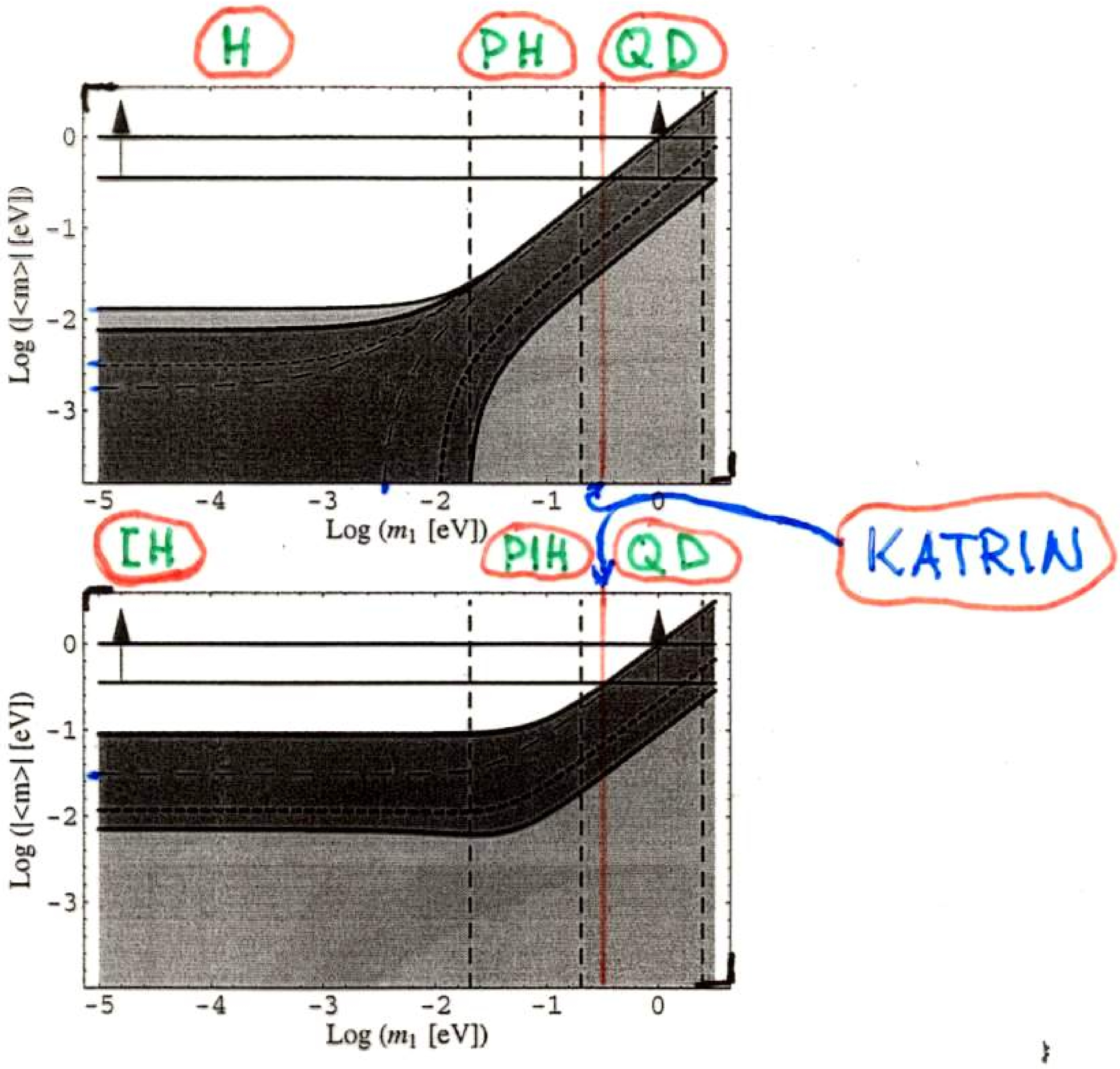


Figure 27: The dependence of $|<m>|$ on m_1 in the case of $\Delta m^2_\odot = \Delta m^2_{21}$ (upper panel) and of $\Delta m^2_\odot = \Delta m^2_{32}$ (lower panel). For $\Delta m^2_\odot = \Delta m^2_{21}$ (upper panel), the allowed values of $|<m>|$ are obtained *i)* using the 99% C.L. results of ref. [58] for the LMA solution of the $\nu_\odot$−problem (light grey and dark grey region between the thick solid line and the axes), for the LOW-QVO solution (light grey and dark grey region between the thin dotted line and the axes), and for the SMA one (the dark grey region between the thin dashed lines) and *ii)* using the results of ref. [7] for the LMA (dark grey region between the two doubly thick solid lines) and for the LOW-QVO (dark grey region between the upper doubly thick solid line and the doubly thick dotted line) solutions. For $\Delta m^2_\odot = \Delta m^2_{32}$ (lower panel), the allowed values are obtained *i)* using the 99% C.L. results of the analysis of ref. [58] for the LMA and LOW-QVO solutions (light grey and dark grey region between the doubly thick line and the axes) and the SMA one (the dark grey region between the upper doubly thick line and the thin dashed one) and *ii)* using the results of ref. [7] for the LMA (dark grey region between the two doubly thick solid lines) and the LOW-QVO (dark grey region between the upper doubly thick solid line and the doubly thick dotted line) solutions. The regions divided by the vertical dashed lines on the upper (lower) panel correspond to *i)* $m_1 \ll 0.02$ eV , and if $m_1 \ll \sqrt{\Delta m^2_\odot}$ ($m_1 < \sqrt{\Delta m^2_\odot} \ll \sqrt{\Delta m^2_{atm}}$) - to a hierarchical (inverted hierarchy) neutrino mass spectrum, *ii)* 0.02 eV $\leq m_1 \leq 0.2$ eV , i.e., spectrum with partial hierarchy (partial inverted hierarchy), and to *iii)* $m_1 \geq 0.2$ eV , i.e., quasi-degenerate neutrinos. For both cases the upper bounds from ref. [29], eq. (4), are shown by the horizontal upper doubly thick solid lines.

[58] GONZALEZ-GARCIA ET AL.
[7] SK COLLABORATION

S. PASCOLI, S.T.P., L. WOLFENSTEIN, '01

QUESTIONS:

Q1: SUPPOSE $\Delta m^2_{\odot}$, Δm^2_{atm}, $\theta_{\odot}$, θ ARE DETERMINED WITH HIGH PRECISION. CAN ONE PREDICT THE VALUE OF $|<m>|$?

Q2: SUPPOSE $|<m>|$ IS MEASURED AND $|<m>| \neq 0$, $|<m>| \gtrsim 5 \cdot 10^{-3}$ eV OR 10^{-2} eV. WHAT CAN ONE DEDUCE FROM THE MEASUREMENT (m_1, SPECTRUM, ...) ?

Q3: ARE THERE CASES WHERE A VALUE OF $|<m>|$ WOULD IMPLY CP-VIOLATION ?

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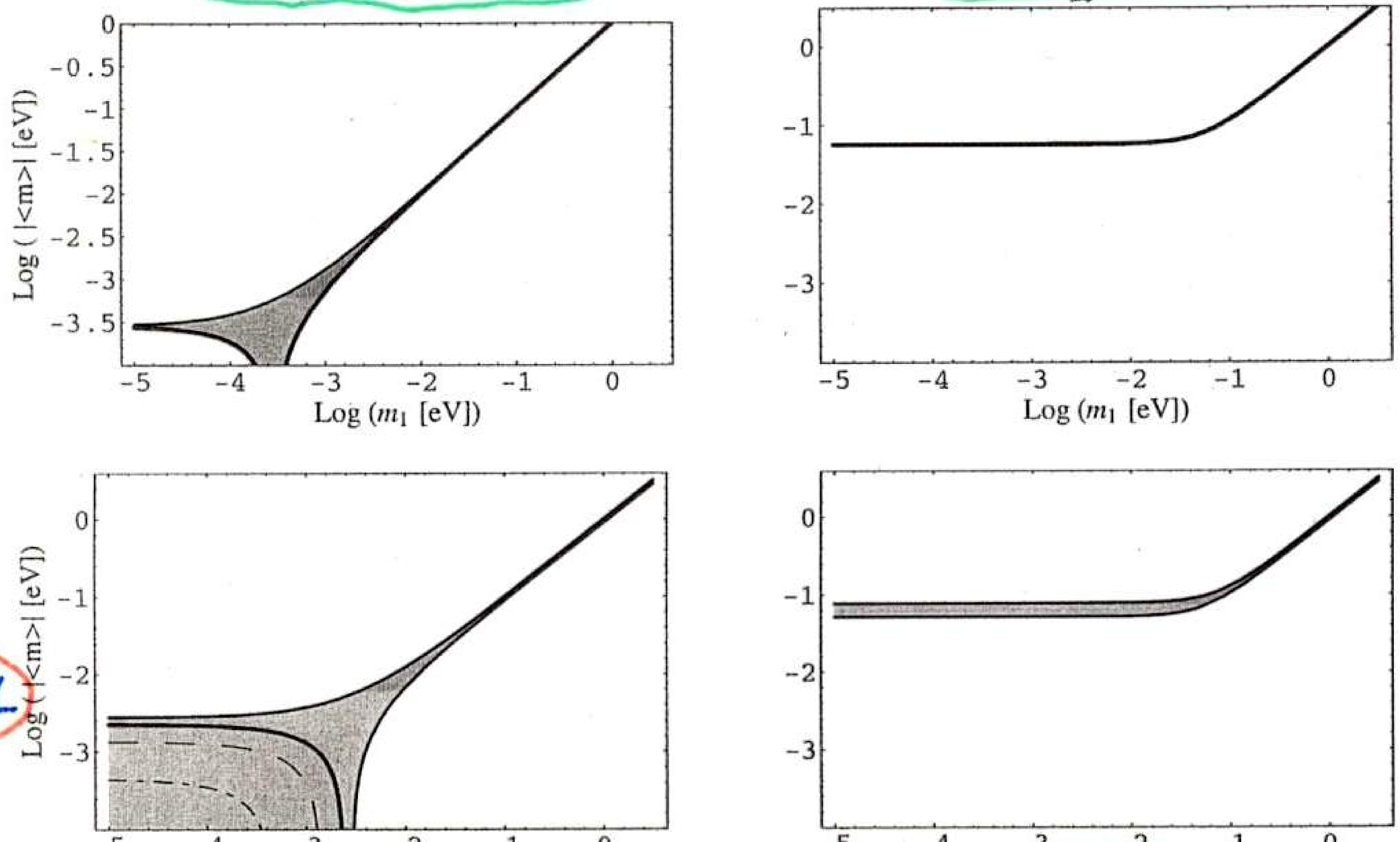
$\Delta m^2_{\odot} = \Delta m^2_{21}$ $\qquad$ $\Delta m^2_{\odot} = \Delta m^2_{32}$

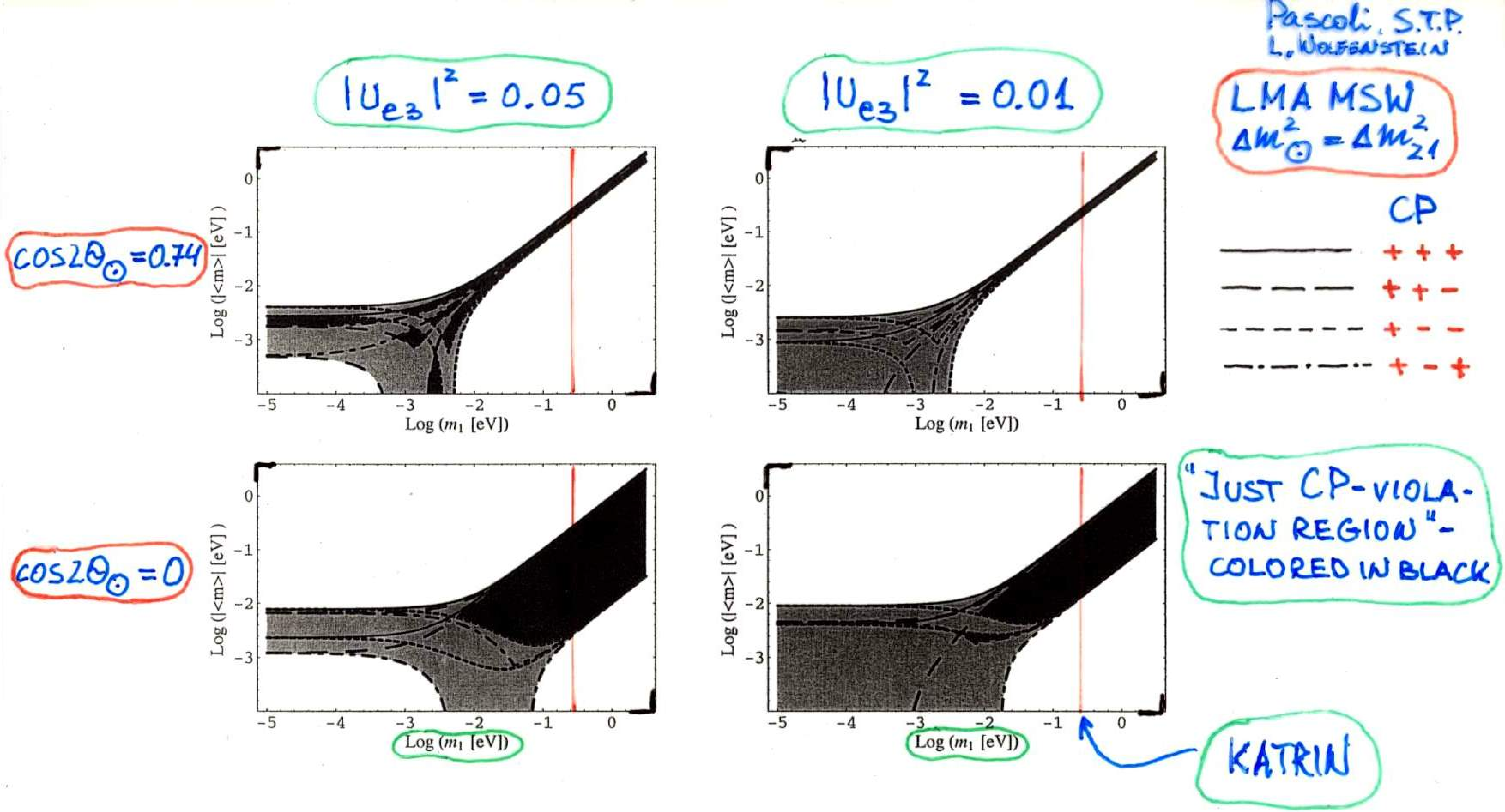
SMA MSW

NO INFORMATION ON CP-VIOLATION CAN BE OBTAINED.

B.F.

90% C.L.





Pascoli, S.T.P.
L. Wolfenstein
$|U_{e3}|^2 = 0.05$
$|U_{e3}|^2 = 0.01$
LMA MSW
$\Delta m^2_{\odot} = \Delta m^2_{21}$
CP
+ + +
+ + −
+ − −
+ − +
$\cos 2\theta_{\odot} = 0.74$
$\cos 2\theta_{\odot} = 0$
Log (|<m>| [eV])
Log (m_1 [eV])
"JUST CP-VIOLATION REGION" - COLORED IN BLACK
KATRIN

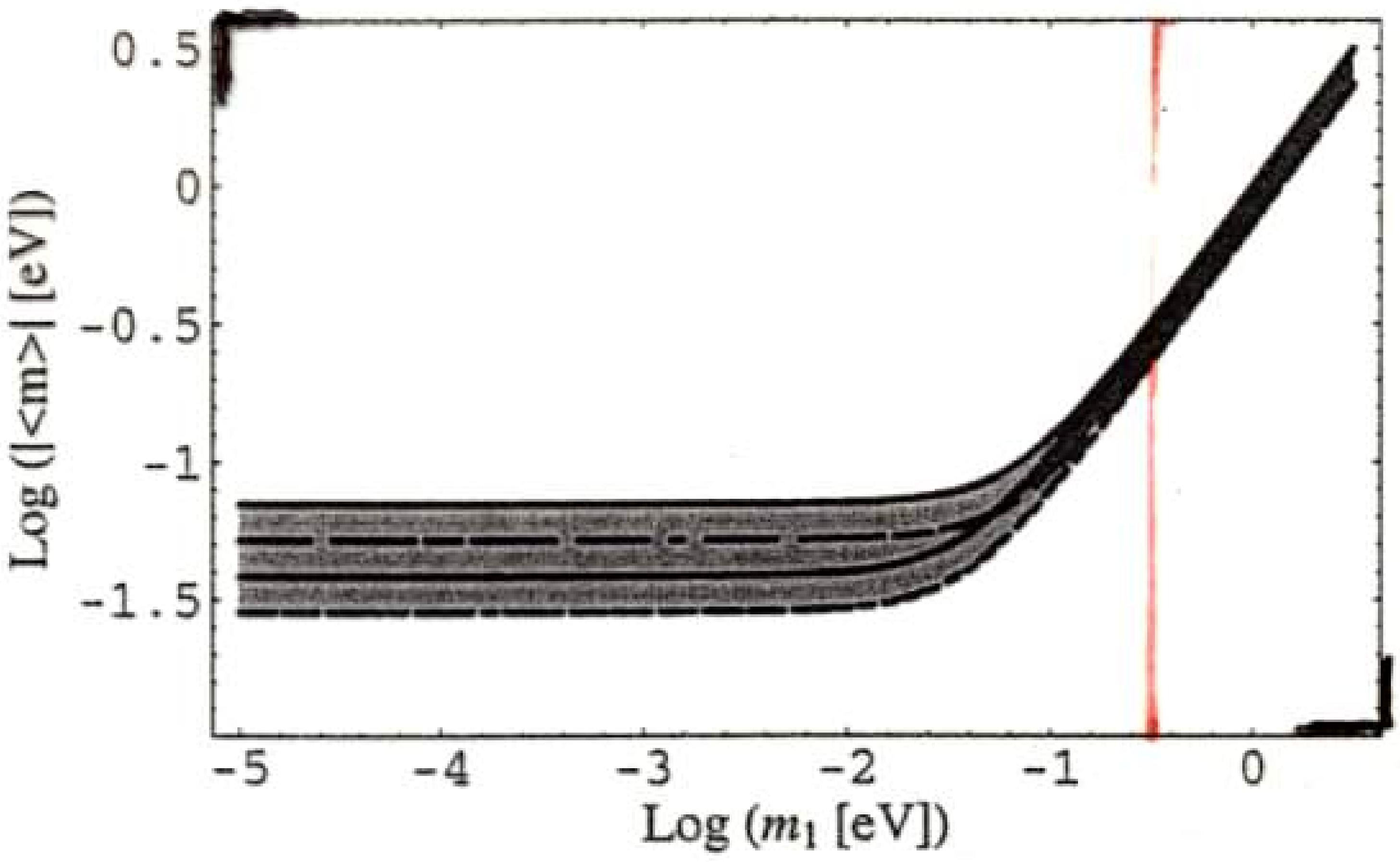
Pascoli, S.T.P., L. Wolfenstein

$\cos 2\theta_{\odot}$ $|U_{e1}|^2 = 0.01$

LMA MSW
$\Delta m^2_{\odot} = \Delta m^2_{32}$

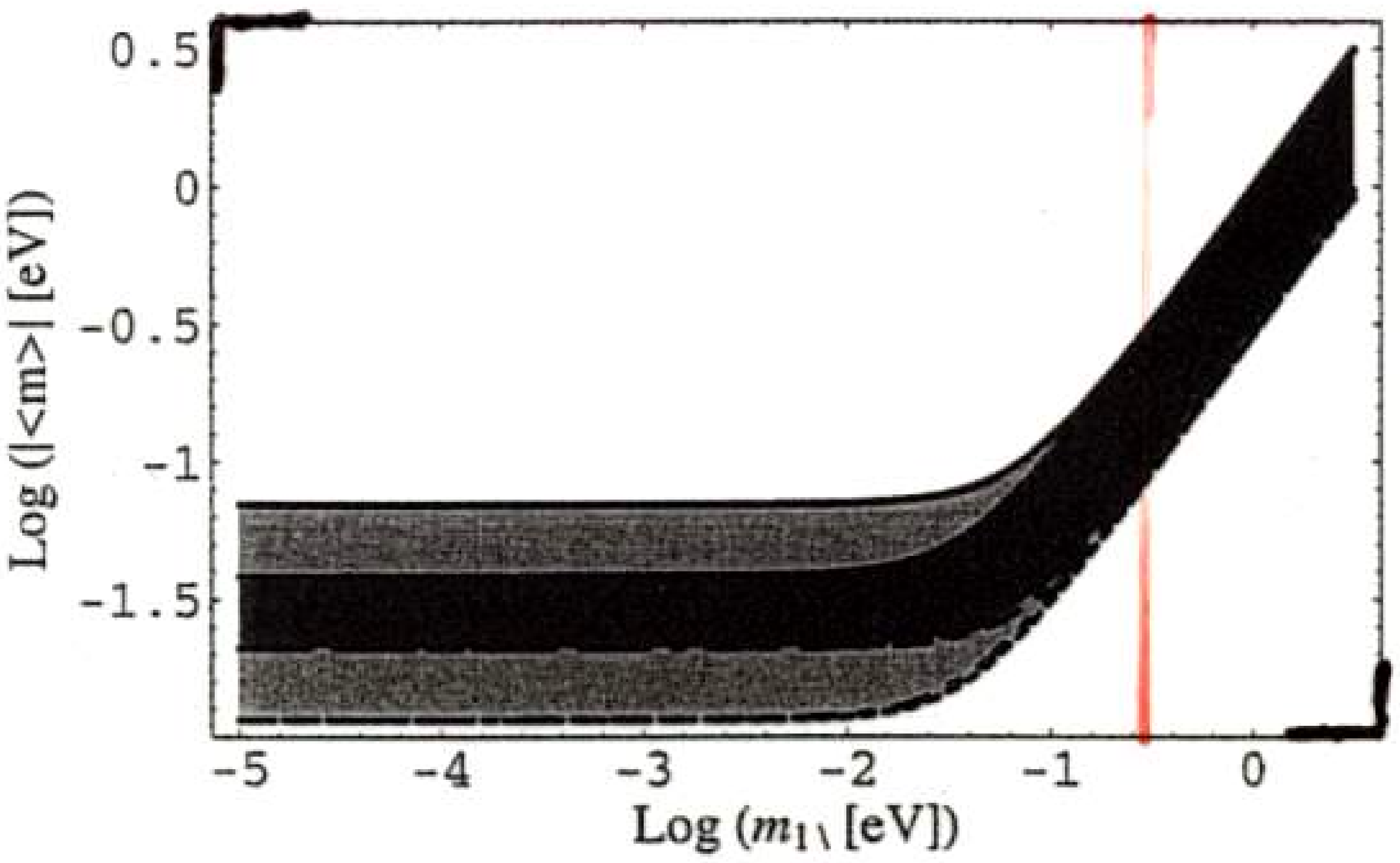
CP-PARITIES

0.74



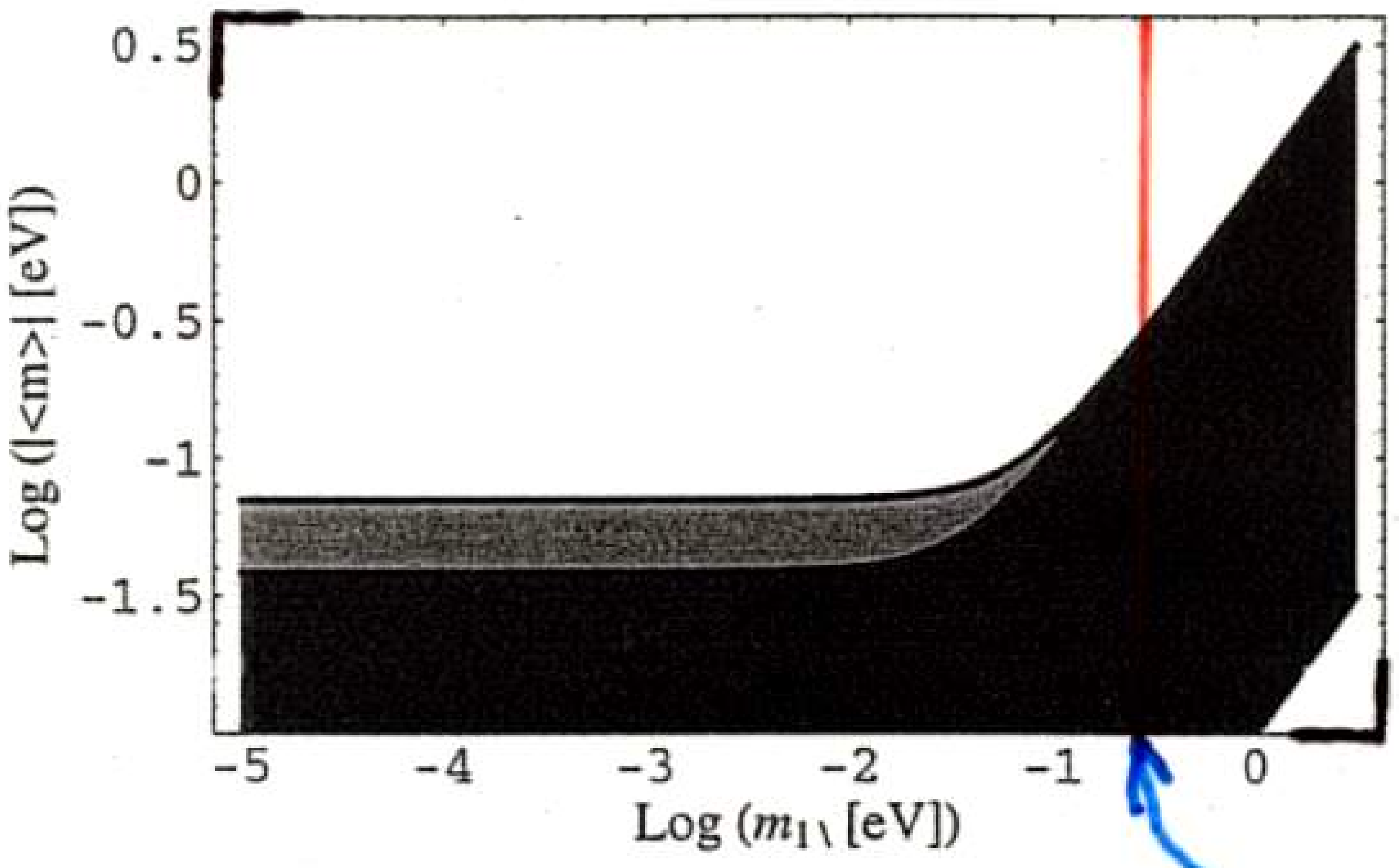
$\pm$ + +
$\pm$ + −

0.3



"JUST CP-VIOLATION REGIONS" - COLORED IN BLACK

0.0



KATRIN

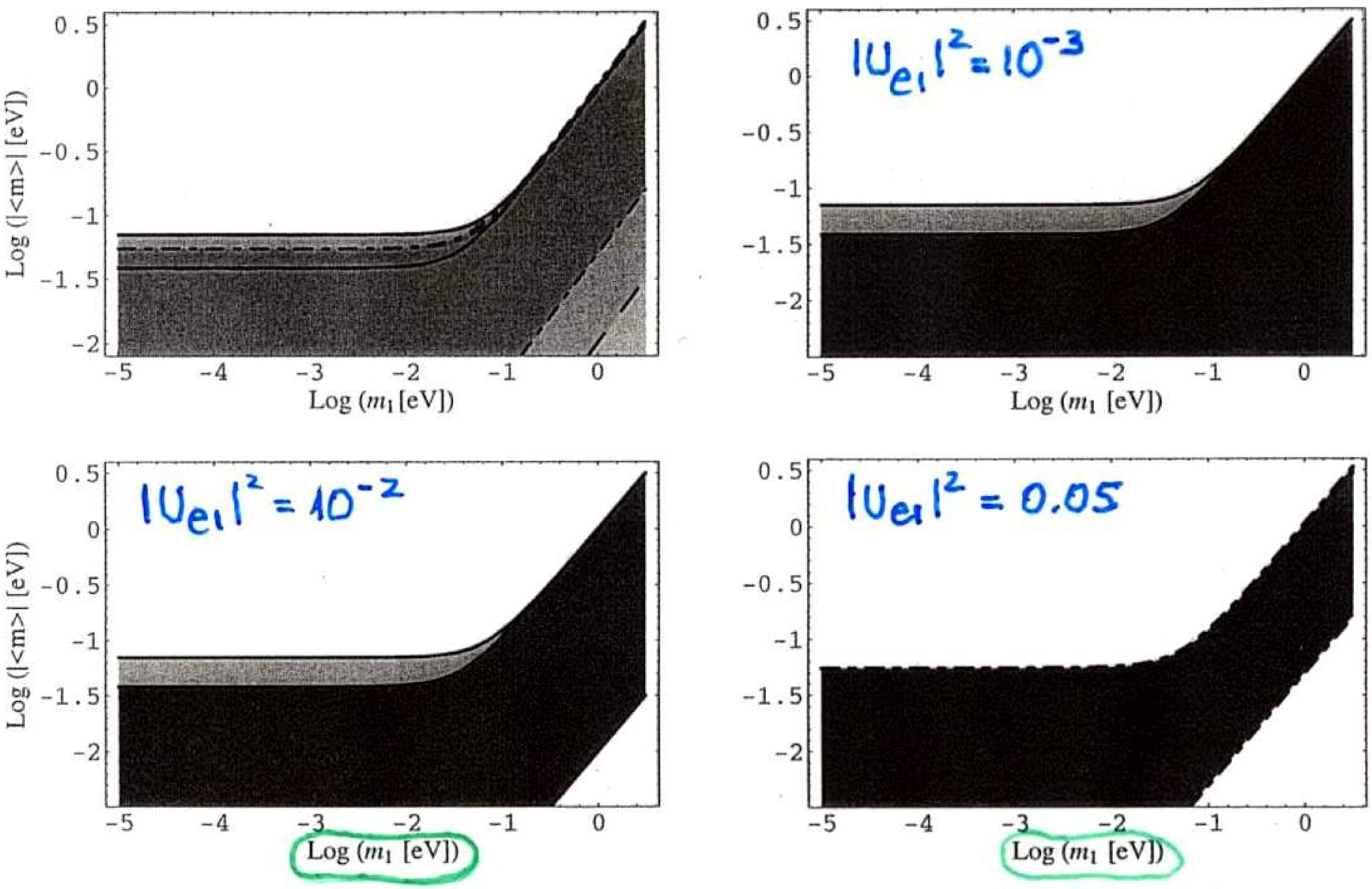
90% C.L.

$\cos 2\theta_{\odot} = 0$

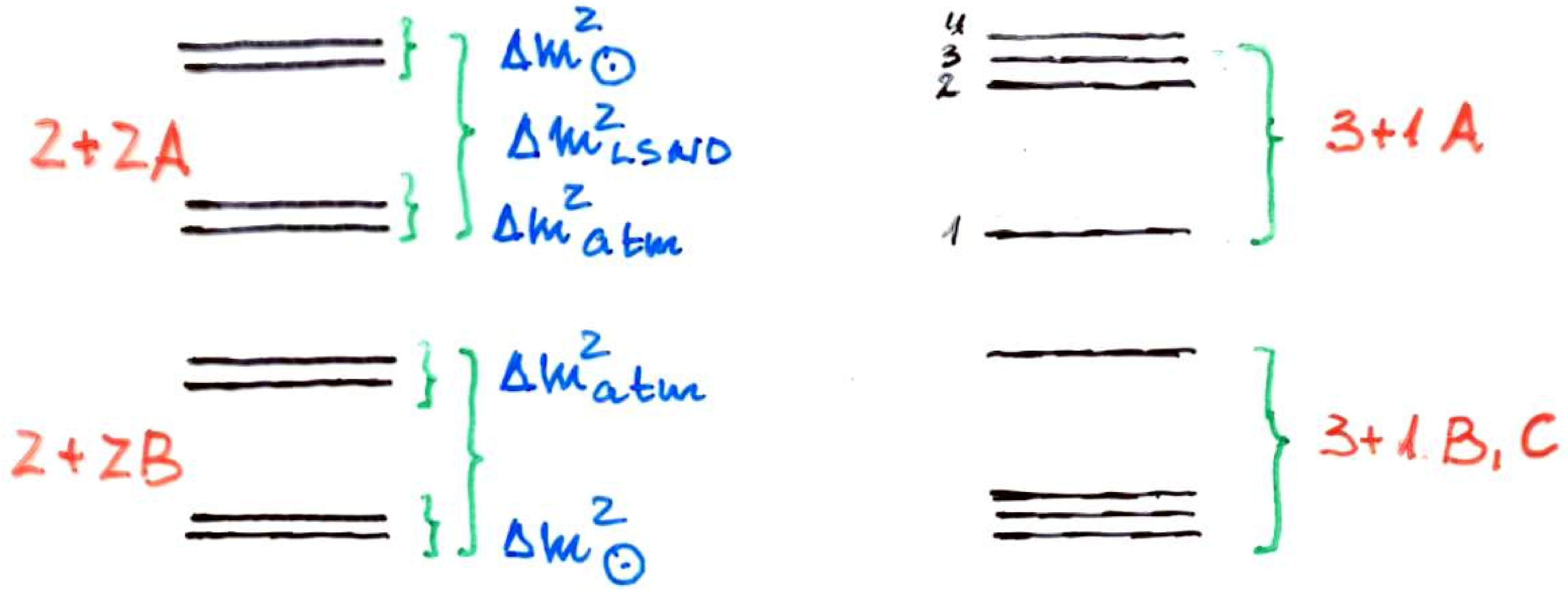
S. Pascoli, S.T.P., L. Wolfenstein

LMA MSW
$\Delta m^2_{\odot} = \Delta m^2_{32}$

"JUST CP-VIOLATION" REGIONS ARE SHOWN IN BLACK.



4-ν Mixing



TWO ADDITIONAL MIXING PARAMETERS:
θ_{23}, θ_{24}.

CHARACTERIZE THE $\nu_e^\odot \to \nu_s$ AND

$\nu_\mu^{atm} \to \nu_s$ TRANSITIONS.

$$\nu_e^\odot \to \nu_s c_{23} c_{24} + \sqrt{1 - c_{23}^2 c_{24}^2}\,(\nu_\mu \cos\theta' + \nu_\tau \sin\theta')$$

$$\cos\theta' = -s_{23}/\sqrt{1 - c_{23}^2 c_{24}^2}, \quad \sin\theta' = -s_{24} c_{23}/\sqrt{}$$

$\nu_\mu^{atm} : \nu_\beta \to \nu_\gamma \qquad \nu_\beta = s_{23} c_{24} \nu_s + c_{23} \nu_\mu - s_{23} s_{24} \nu_\tau$

$\nu_\gamma = s_{24} \nu_s + c_{24} \nu_\tau$

We set $s_{23} = 0$: $\nu_\mu^{atm} \to s_{24} \nu_s + c_{24} \nu_\tau$

$\nu_e^\odot \to \nu_s : c_{24}^2 = \cos^2\xi$

$\nu_\mu^{atm} \to \nu_s : s_{24}^2 = \sin^2\xi \qquad \cos^2\xi = 0.3; 0.5$

The effective Majorana mass parameter can be expressed as:

$$|<m>| \equiv \left| m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 + m_4 U_{e4}^2 \right| , \tag{33}$$

where U_{ei} are the entries of the lepton mixing matrix U, m_i is the mass of the ν_i massive neutrino. We have

$$U_{ej} = |U_{ej}| \, e^{\frac{i\alpha_j}{2}}, \tag{34}$$

where α_j, $j = 1, 2, 3, 4$, are real phases. Only the phase differences $(\alpha_j - \alpha_k) \equiv \alpha_{jk}$ $(j > k)$ can play a physical role. The CP-invariance constraint on the elements of the lepton mixing matrix of interest reads

$$U_{ej}^* = \eta_j^{CP} \, U_{ej}, \tag{35}$$

where $\eta_j^{CP} = i\phi_j = \pm i$ is the CP-parity of the Majorana neutrino ν_j with mass $m_j > 0$. In this case $|<m>|$ is given by:

$$|<m>| \equiv \left| \sum_{j=1}^{4} \eta_j^{CP} |U_{ej}|^2 m_j \right| = \left| \sum_{j=1}^{4} \phi_j |U_{ej}|^2 m_j \right| . \tag{36}$$

The neutrino oscillation experiments provide information on $\Delta m_{jk}^2 = m_j^2 - m_k^2$ $(j > k)$. In the case of 4-neutrino mixing (16) there are three independent Δm^2 parameters. The four neutrino masses m_j, $j = 1, 2, 3, 4$, can be expressed in terms of these three parameters and, e.g., of m_1. We have:

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2} \, , \tag{37}$$

$$m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2} \, . \tag{38}$$

$$m_4 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{43}^2} \, . \tag{39}$$

The mass-squared difference inferred from the neutrino oscillation interpretation of the LSND data, $\Delta m_{\rm SBL}^2$, is equal to Δm_{41}^2,

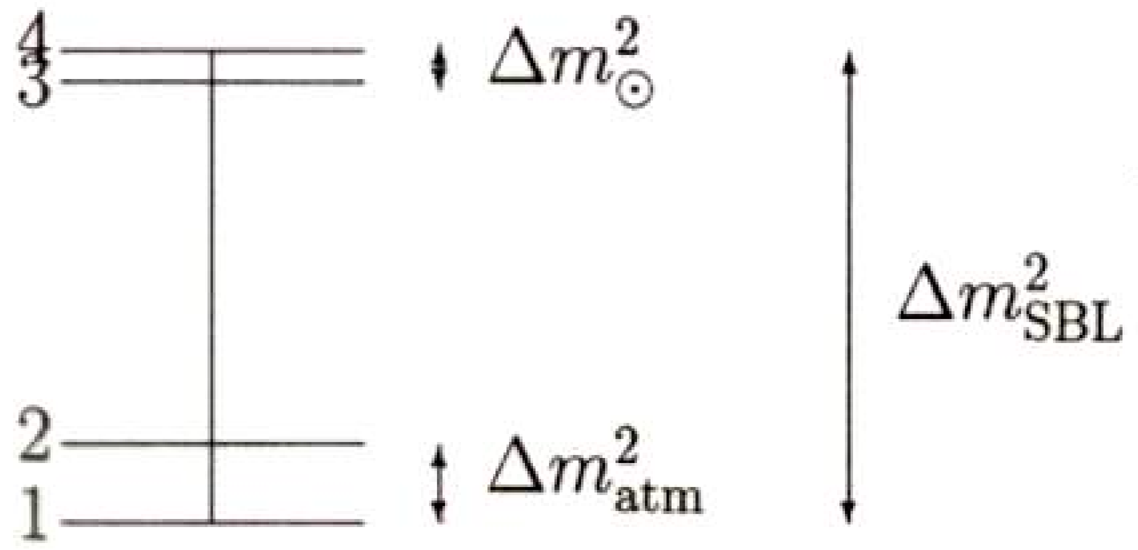
$$\Delta m_{\rm SBL}^2 = \Delta m_{41}^2 = \Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{43}^2 \, , \tag{40}$$

For the ones deduced from the solar and atmospheric neutrino data, $\Delta m^2_{\odot}$ and Δm^2_{atm} , we have several possibilities:

- in the case of the 2+2 type of neutrino mass spectrum we can identify two sub-cases: i) 2+2A - with $\Delta m^2_{\odot} = \Delta m^2_{43}$ and $\Delta m^2_{\text{atm}} = \Delta m^2_{21}$, and ii) 2+2B - with $\Delta m^2_{\odot} = \Delta m^2_{21}$ and $\Delta m^2_{\text{atm}} = \Delta m^2_{43}$;
- for the 3+1A type of neutrino mass spectrum characterized by one neutrino being much lighter than the other three which are nearly degenerate, we have $\Delta m^2_{\odot} = \Delta m^2_{43}$, $\Delta m^2_{\text{atm}} = \Delta m^2_{32}$, or $\Delta m^2_{\text{atm}} = \Delta m^2_{32}$, $\Delta m^2_{\odot} = \Delta m^2_{43}$;
- for the 3+1B and in the 3+1C neutrino mass spectra with one neutrino being much heavier than the other three, one finds, respectively, $\Delta m^2_{\odot} = \Delta m^2_{21}$, $\Delta m^2_{\text{atm}} = \Delta m^2_{32}$, and $\Delta m^2_{\odot} = \Delta m^2_{32}$, $\Delta m^2_{\text{atm}} = \Delta m^2_{31}$.

Getting information on the lightest neutrino mass m_1 would be crucial for determining the values of $m_{2,3,4}$. The neutrino oscillation data and cosmological arguments suggest that $m_1 \lesssim 1$ eV.

4 The 2+2A Neutrino Mass Spectrum



This pattern corresponds to the inequalities $m_1 < m_2 < \ (\ll) \ m_3 \simeq m_4$, or equivalently to:

$$\begin{aligned} m_1 &< \ (\ll) \ \sqrt{\Delta m^2_{41}}, \\ \sqrt{\Delta m^2_{43}} \ll \sqrt{\Delta m^2_{21}} &\ll \sqrt{\Delta m^2_{41}}. \end{aligned} \tag{41}$$

One can make the identification:

$$\Delta m^2_{21} \equiv \Delta m^2_{\rm atm}, \quad \Delta m^2_{43} \equiv \Delta m^2_\odot, \quad \Delta m^2_{41} \equiv \Delta m^2_{\rm SBL}. \tag{42}$$

Then

$$\begin{aligned} m_2 &= \sqrt{m_1^2 + \Delta m^2_{\rm atm}} \ , \\ m_3 \simeq m_4 &= \sqrt{m_1^2 + \Delta m^2_{\rm SBL}} \ . \end{aligned} \tag{43}$$

The condition $m_1, m_2 \ll m_3, m_4$, which could be valid for the neutrino mass spectrum under discussion, is satisfied for

$$m_1 < 0.25\sqrt{\Delta m^2_{\rm SBL}} \quad \Rightarrow \quad m_1 < 0.1 \text{ eV} \ . \tag{44}$$

For the elements U_{ej} we have:

i) $|U_{e1}|^2$ and $|U_{e2}|^2$ are constrained by the LSND results [35] and BUGEY neutrino oscillation limits [51] to lie in the interval:

$$2 \times 10^{-4} \leq |U_{e1}|^2 + |U_{e2}|^2 \leq 1 \times 10^{-2}; \tag{45}$$

ii) $|U_{e3}|$ and $|U_{e4}|$ are related to the solar mixing angle $\theta_\odot$:

$$\begin{aligned} |U_{e3}|^2 &= \cos^2\theta_\odot \ (1 - \sum_{i=1}^{2} |U_{ei}|^2), \\ |U_{e4}|^2 &= \sin^2\theta_\odot \ (1 - \sum_{i=1}^{2} |U_{ei}|^2), \end{aligned} \tag{46}$$

In the case of the 2+2A scheme under discussion, the effective neutrino mass m_{ν_e}, which can be determined from the measurement of the end-point part of the β-spectrum of ^{3}H, is given by

$$m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m^2_{\rm SBL}} \tag{52}$$

From the results of the analysis of the LSND data [35] it follows that for $m_1 < 0.1$ eV one has:

$$0.4 \text{ eV} \leq m_{\nu_e} \leq 1.4 \text{ eV}. \tag{53}$$

If $m_1 \gtrsim 0.1$ eV, the lower bound in (53), in particular, will be larger. Consequently, for the 2+2A type of neutrino mass spectrum and any value of m_1, m_{ν_e} is predicted to lie in the range planned to be probed by the future Karlsruhe-Mainz-Troitzk experiment KATRIN [67]. Thus, the realization of the KATRIN project will allow to check directly the possibility of 2+2A type of neutrino mass spectrum. A measurement of $m_{\nu_e} \gtrsim 0.4$ eV and and a more accurate knowledge of $\Delta m^2_{\rm SBL}$ would permit to determine the value of m_1. This would allow to determine also the values of $m_{2,3,4}$ in the case of the 2+2A spectrum.

Neglecting $m_1|U_{e\,1}|^2$ and $m_2|U_{e\,2}|^2$ with respect to the terms $\sim m_3, m_4$, we get:

$$|\!<\!m\!>\!| \; \simeq m_{\nu_e} \sqrt{1 - \sin^2(2\theta_\odot)\; \sin^2 \frac{\alpha_{31} - \alpha_{41}}{2}}. \tag{54}$$

In eq. (54) we have neglected the contribution of the two lightest neutrinos since

$$\left| m_1 |U_{\rm e1}|^2 + m_2 |U_{\rm e2}|^2 e^{i\alpha_{21}} \right| \leq m_1 |U_{\rm e1}|^2 + m_2 |U_{\rm e2}|^2 \ll m_4 \tag{55}$$

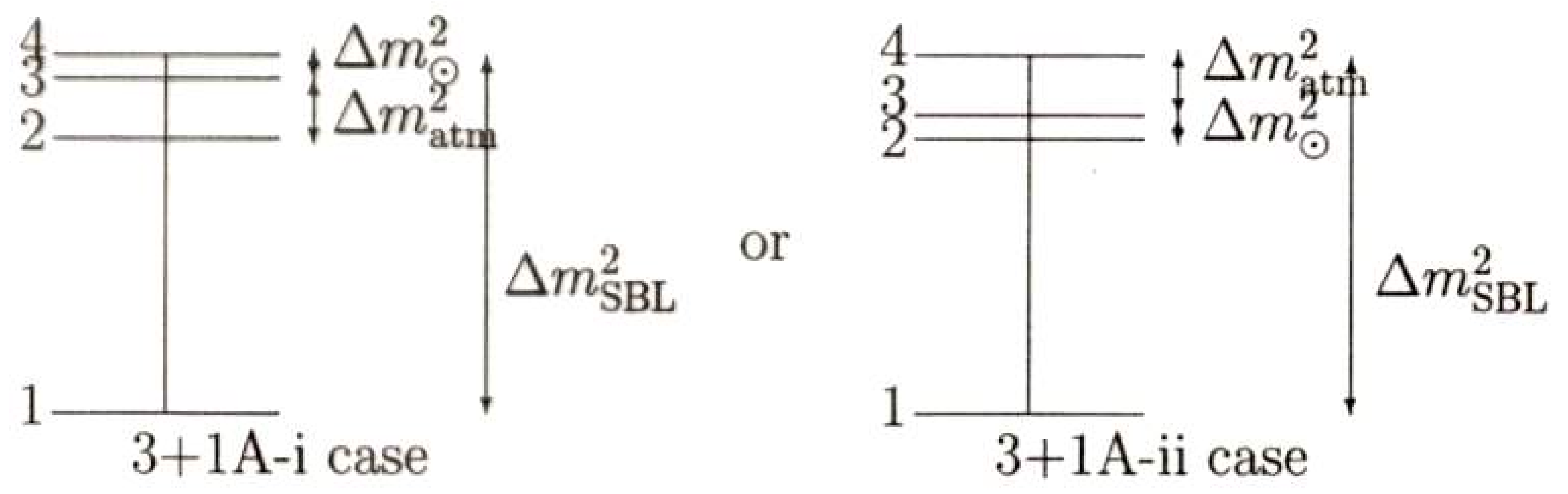
For $m_1 < 0.1$ eV this contribution does not exceed approximately 1.5×10^{-3} eV.

We have

$$\sin^2 \frac{\alpha_{41} - \alpha_{31}}{2} \simeq \frac{1}{\sin^2 2\theta_\odot} \left(1 - \frac{|\!<\!m\!>\!|^{\,2}}{m_1^2 + \Delta m^2_{\rm SBL}} \right) \simeq \frac{1}{\sin^2 2\theta_\odot} \left(1 - \frac{|\!<\!m\!>\!|^{\,2}}{m^2_{\nu_e}} \right). \tag{56}$$

6 The 3+1A mass spectrum

The 3+1A mass spectrum is characterized by three nearly degenerate neutrinos, $\nu_{2,3,4}$, having masses sufficiently larger than the fourth one, ν_1: $m_1 < (\ll) \; m_2, m_3, m_4$. There exist two possibilities:



One has $m_1 \ll m_4$, if $m_1 < 0.1$ eV. These patterns can also be characterized by

$$\Delta m^2_{43} \ll \Delta m^2_{32} \ll \Delta m^2_{41}, \qquad \text{3+1A-i case;} \tag{85}$$

$$\Delta m^2_{32} \ll \Delta m^2_{43} \ll \Delta m^2_{41}, \qquad \text{3+1A-ii case.} \tag{86}$$

One can make the identification:

$$\Delta m^2_{43} \equiv \Delta m^2_\odot, \quad \Delta m^2_{32} \equiv \Delta m^2_{\rm atm}, \quad \Delta m^2_{41} \equiv \Delta m^2_{\rm SBL}. \tag{87}$$

One also finds:

i) $|U_{e1}|$ is limited by the data of the SBL experiments [35, 51]: $2.0 \times 10^{-4} \leq |U_{e1}|^2 < 1.0 \times 10^{-2}$;

ii) $|U_{e2}|$ should satisfy the CHOOZ limit [55, 58]: $|U_{e2}|^2 < 0.08$ (99% C.L.);

iii) $|U_{e3}|$ and $|U_{e4}|$ are related to the solar neutrino mixing angle $\theta_\odot$:

$$|U_{e3}|^2 = \cos^2\theta_\odot \; (1 - \sum_{i=1}^{2} |U_{ei}|^2), \tag{88}$$

$$|U_{e4}|^2 = \sin^2\theta_\odot \; (1 - \sum_{i=1}^{2} |U_{ei}|^2). \tag{89}$$

Neglecting $\Delta m^2_{\rm atm}$ and $\Delta m^2_\odot$ in comparison with $\Delta m^2_{\rm SBL}$, one finds :

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{\rm SBL} \; - \Delta m^2_\odot \; - \Delta m^2_{\rm atm}} \, ,$$

$$m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm SBL} \; - \Delta m^2_\odot} \, ,$$

$$m_4 = \sqrt{m_1^2 + \Delta m^2_{\rm SBL}} \, .$$

Neglecting terms $\sim \Delta m^2_\odot / \Delta m^2_{\rm SBL}$ and $\sim \Delta m^2_{\rm atm} / \Delta m^2_{\rm SBL}$ in $m_{2,3}$, whose contributions in $|{<}m{>}|$ do not exceed respectively 10^{-3} eV and 1.5×10^{-3} eV, we get:

$$m_2 \simeq m_3 \simeq m_4 = \sqrt{m_1^2 + \Delta m^2_{\rm SBL}} \, . \qquad (90)$$

As far as the approximation (90) holds, i.e., up to corrections $\sim 10^{-3}$ eV, the 3+1A-i and the 3+1A-ii patterns lead to the same predictions for $|{<}m{>}|$.

As like in the 2+2A scheme, m_{ν_e} measured in the ^{3}H $\beta-$decay experiments is given, up to $O(\lesssim 10^{-2}$ eV) by:

$$m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m^2_{\rm SBL}} \, . \qquad (91)$$

From the analysis of the LSND data:

$$m_{\nu_e} \gtrsim 0.4 \text{ eV}$$

for any value of m_1 .

Thus, the 3+1A type spectrum can be tested in the KATRIN ^{3}H $\beta-$decay experiment. The observation made for the case of the 2+2A spectrum, that a measurement of $m_{\nu_e} \gtrsim 0.4$ eV and a more precise knowledge of $\Delta m^2_{\rm SBL}$ would permit to determine the value of m_1 and would allow to fix the values of $m_{2,3,4}$ as well, is valid also for the 3+1A spectrum.

Neglecting $O(\lesssim 10^{-2}$ eV) corrections, one finds

$$|{<}m{>}| \; \simeq \sqrt{m_1^2 + \Delta m^2_{\rm SBL}} \; | \, |U_{e2}|^2 + (1 - |U_{e2}|^2)(\cos^2\theta_\odot e^{i(\alpha_3 - \alpha_2)} + \sin^2\theta_\odot e^{i(\alpha_4 - \alpha_2)})|. \qquad (92)$$

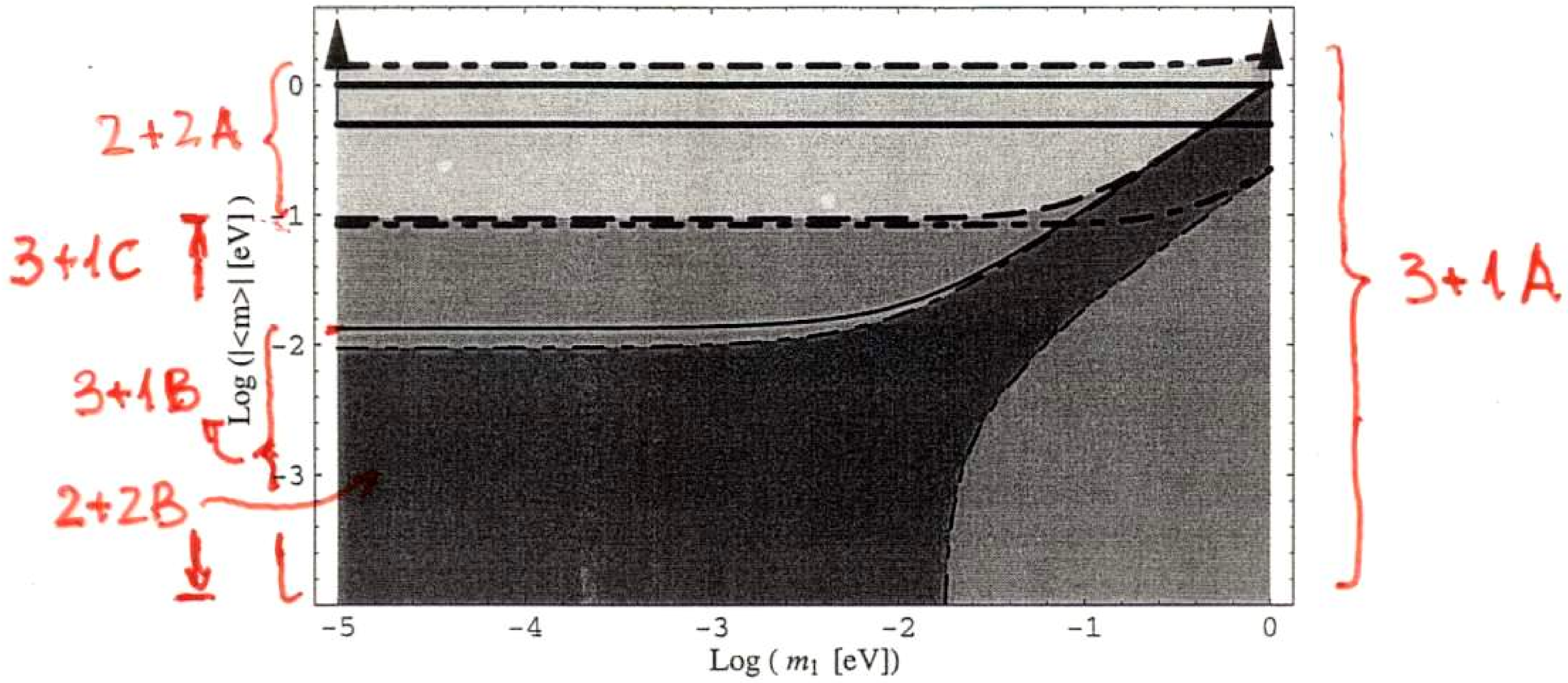


Figure 16: The dependence of $|{<}m{>}|$ on m_1 for the LMA solution of the solar-ν problem. For the 2+2 neutrino mass spectra, the figure is obtained using the allowed values of $\Delta m^2_{\rm SBL}$ and $\theta_{\rm SBL}$ at 95% C.L. derived in ref. [35], of $\Delta m^2_{\rm atm}$ found at 90% C.L. in ref. [41], of $\Delta m^2_\odot$ and $\theta_\odot$ obtained for $\cos^2\beta = 0.3$ at 90% C.L. in ref. [42], and the 90% C.L. limit on $\theta_{\rm CHOOZ}$ from ref. [58]. For the 3+1 neutrino mass spectra, the allowed regions of $|{<}m{>}|$ are derived using the 95% C.L. allowed ranges of $\Delta m^2_{\rm SBL}$ and $\theta_{\rm SBL}$ from ref. [35] and the allowed values of $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$, $\theta_\odot$ and $\theta_{\rm CHOOZ}$ at 90% C.L. from ref. [55]. The allowed regions for $|{<}m{>}|$ correspond *i)* for the 2+2A neutrino mass spectrum, eq. (39) - to the light-grey, medium-grey and dark-grey regions between the two doubly-thick dash-dotted lines; *ii)* for the 2+2B neutrino mass spectrum, eq. (58) - to the dark grey region limited by the two thick dash-dotted lines and the axes; *iii)* for the 3+1A neutrino mass spectrum, eq. (79) - to the grey regions below the upper doubly-thick dash-dotted line; *iv)* for the 3+1B neutrino mass spectrum, eq. (102) - to the medium-grey and dark-grey region below the thick solid line; *v)* for the 3+1C neutrino mass spectrum, eq. (123) - to the medium-grey and dark-grey region below the doubly-thick dashed line. The two horizontal lines show the upper limits [29], quoted in eq. (4).

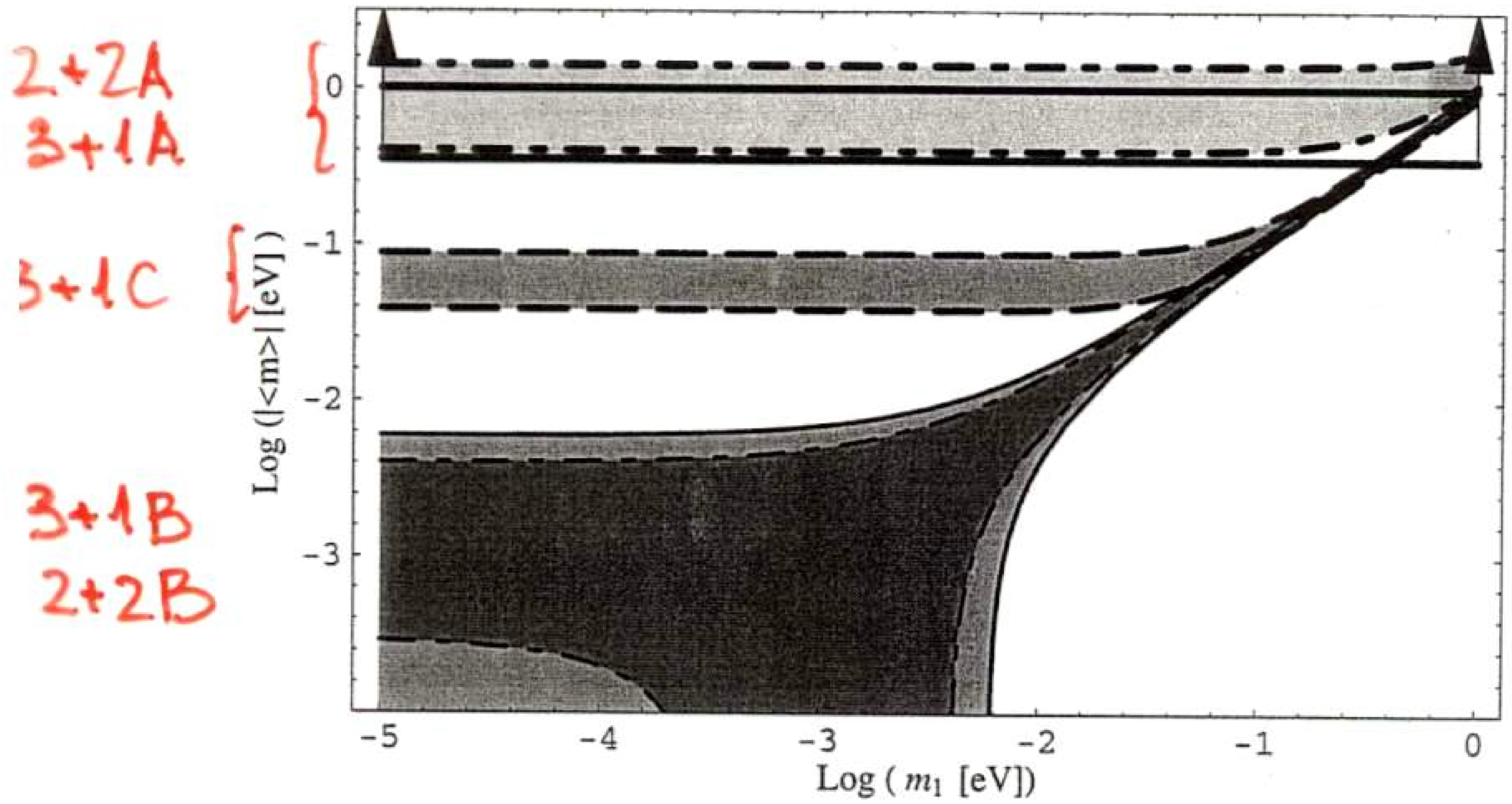


Figure 18: The same as in Fig. 16 but for the SMA MSW solution of the solar neutrino problem. The allowed values of $|<m>|$ correspond *i)* for the 2+2A and 3+1A neutrino mass spectra, eqs. (39) and (79) - to the light-grey region between the two doubly-thick dash-dotted lines; *ii)* for the 2+2B neutrino mass spectrum, eq. (58) - to the dark-grey region limited by the thick dash-dotted lines and the axes; *iii)* for the 3+1B neutrino mass spectrum, eq. (102) - to the medium-grey and dark-grey region limited by the two thick solid lines and the axes; *iv)* for the 3+1C neutrino mass spectrum, eq. (123) - to the medium-grey region between the two doubly-thick dashed lines.

CONCLUSIONS.

FUTURE $(\beta\beta)_{0\nu}$ - DECAY EXPERIMENTS CAN PROVIDE INFORMATION ON

- THE NEUTRINO MASS SPECTRUM
- m_1 (SMA MSW) $\Rightarrow m_1, m_2, m_3$ OR
- $m_1 = [m_1^{min}, m_1^{max}]$ (LMA, LOW-QVO)
- CP - VIOLATION IN THE LEPTON SECTOR (LMA, LOW-QVO, $\Delta m^2_{\odot} = \Delta m^2_{32}, |U_{e1}|^2 \lesssim 5 \cdot 10^{-2}$)

COMBINED WITH DATA FROM THE ^3H β-DECAY EXPERIMENTS (KATRIN), A POSITIVE RESULT FROM $(\beta\beta)_{0\nu}$ - DECAY EXPERIMENTS, i.e.,

$$|<m>| \gtrsim 0.02\ eV, \quad m_{\nu_e} \gtrsim 0.35\ eV$$

WOULD ALLOW TO DETERMINE

m_1, m_2, m_3 AND TO ANSWER THE QUESTION

WHETHER CP - SYMMETRY IS VIOLATED IN THE LEPTON SECTOR OR NOT.

VERY IMORTANT TO KNOW

$$\min(|\cos 2\theta_{\odot}|), \ |U_{e3}|^2 \ (|U_{e1}|^2)$$

$$\Delta m^2_{\odot} = \Delta m^2_{21} \text{ or } \Delta m^2_{\odot} = \Delta m^2_{32}.$$