FOUR–NEUTRINO OSCILLATIONS

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OUTLINE

I. Solar 2–ν Oscillations: Active versus Sterile

II. Atmospheric 2–ν Oscillations: Active versus Sterile

III. Four Neutrino Mixing:

Solar Neutrinos

Atmospheric Neutrinos

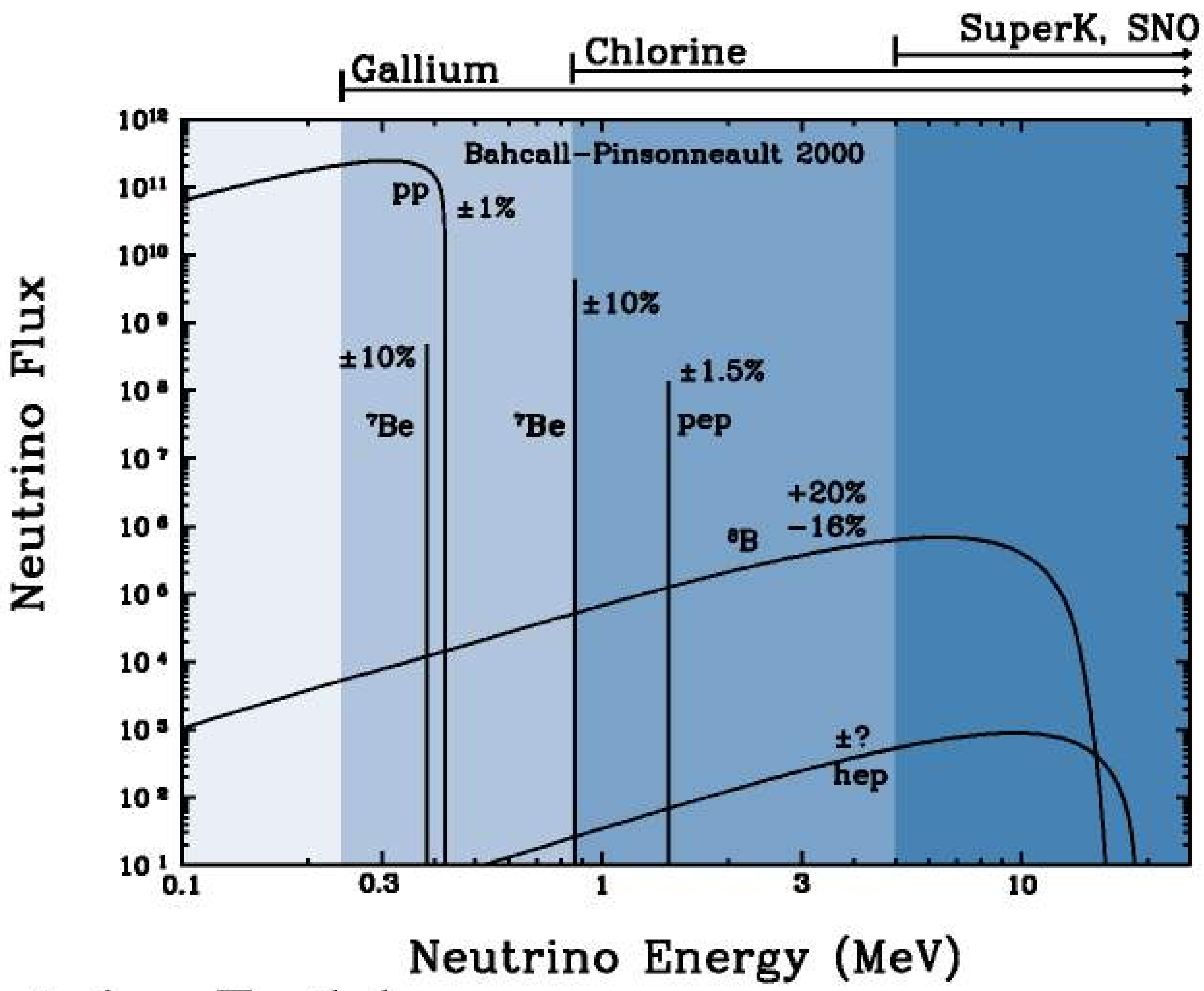
Combined Analysis

IV. Conclusions

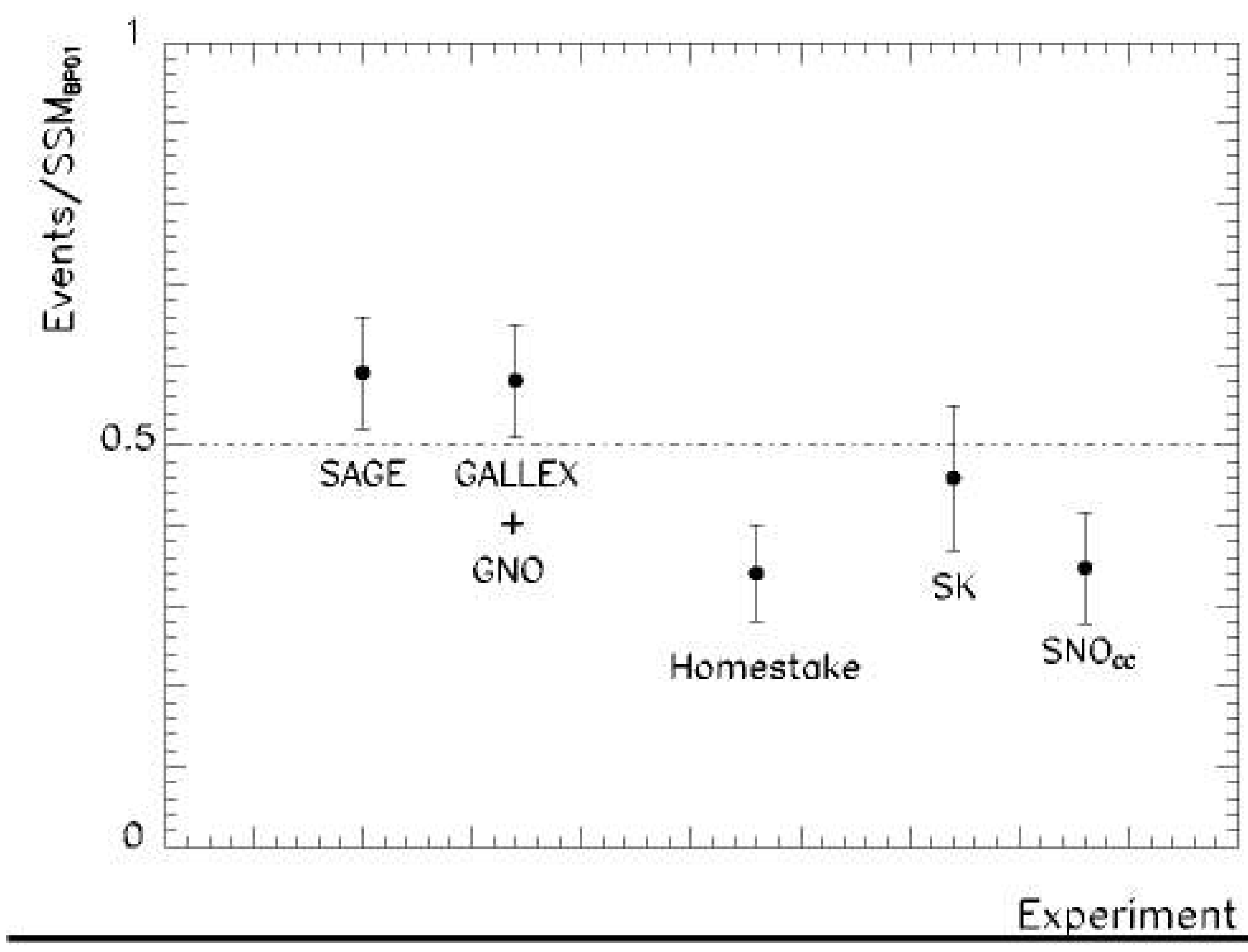
Les Houches, June 2001

Solar 2–ν Oscillations: Active versus Sterile

- The sun emits $\nu_e' s$. The Standard Solar Model fluxes



- $\nu_e' s$ are detected on Earth by
  – Homestake (" Clorine") $\nu_e +^{37} Cl \to^{37} Ar + e^-$
  – SAGE and GALLEX +GNO $\nu_e +^{71} Ga \to^{71} Ge + e^-$
  –Kamiokande and SuperK $\nu_e e$ scattering in H_2O Target
  –SNO(Sudbury,Canada)
  CC $\nu_e + d \to p + p + e^-$
  NC $\nu_x + d \to \nu_x + p + n$
  ES and $\nu_x e^-$ scattering in D_2O Target
- The experiments are sensitive to different E_ν
- They observe a deficit...

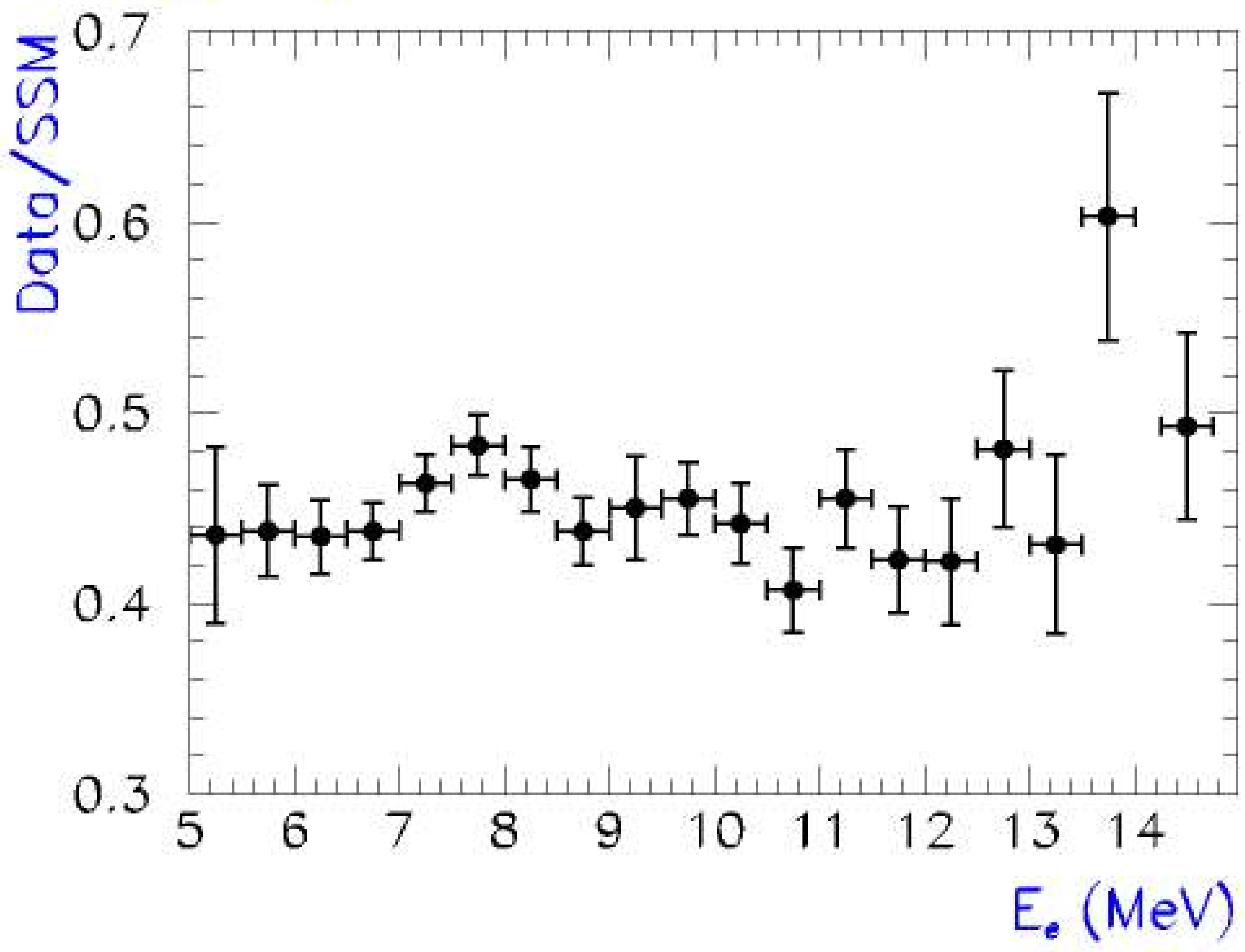


...Of about 30–60 %cd

Other Super–Kamiokande Measurements

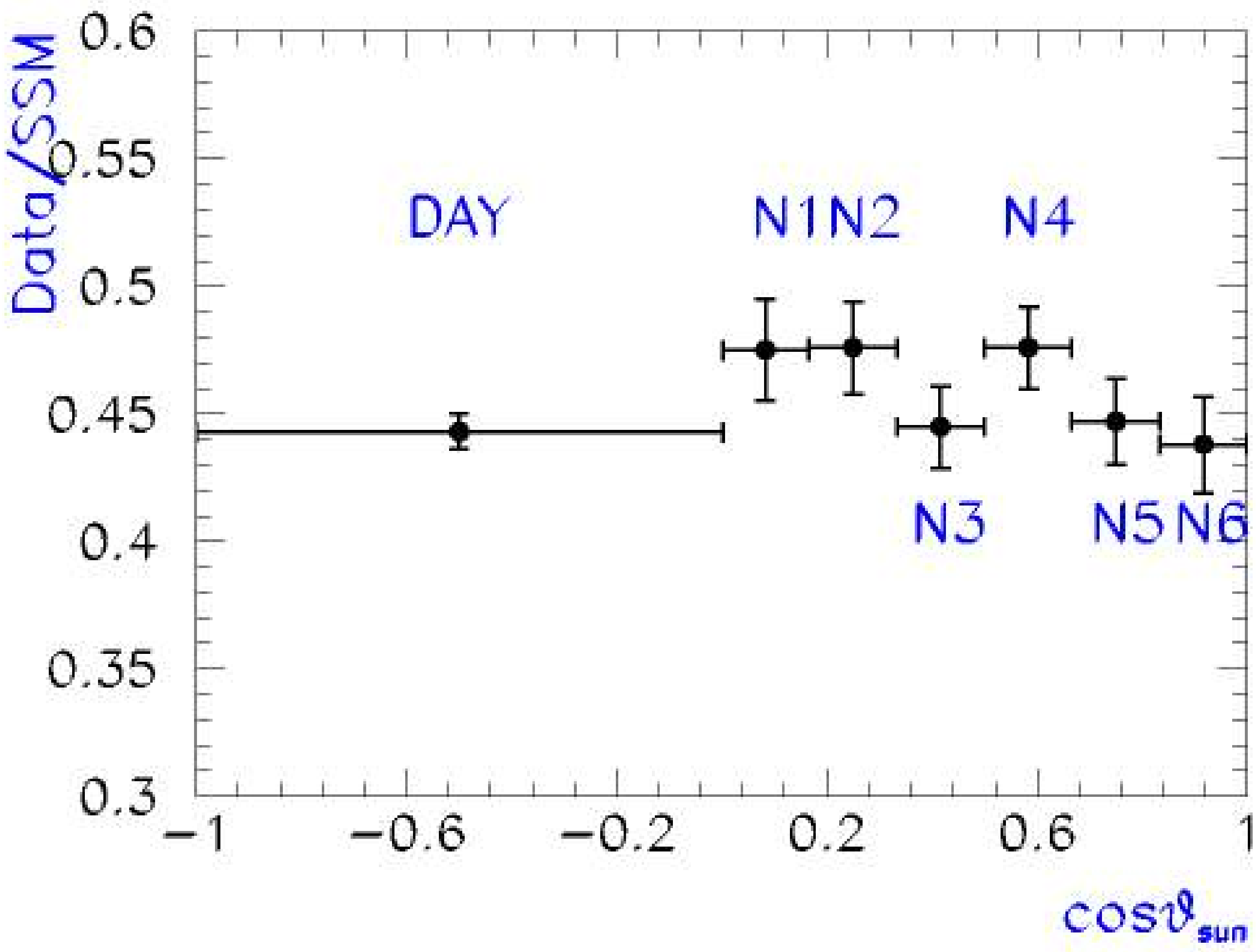
Super–Kamiokande 1258 Days

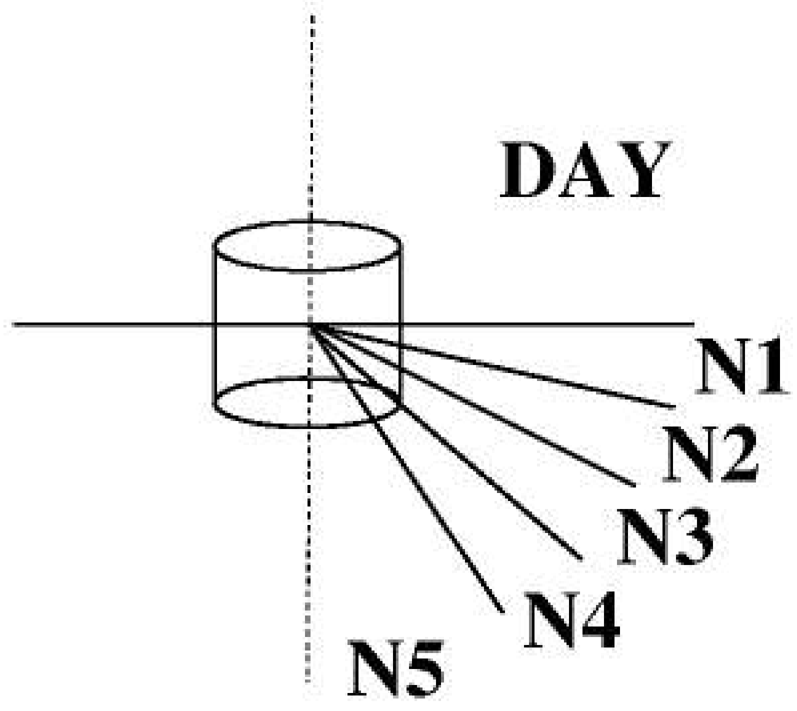
- Recoil Electron Energy Spectrum → no distortion $\chi^2_{flat} = 19/(18dof)$



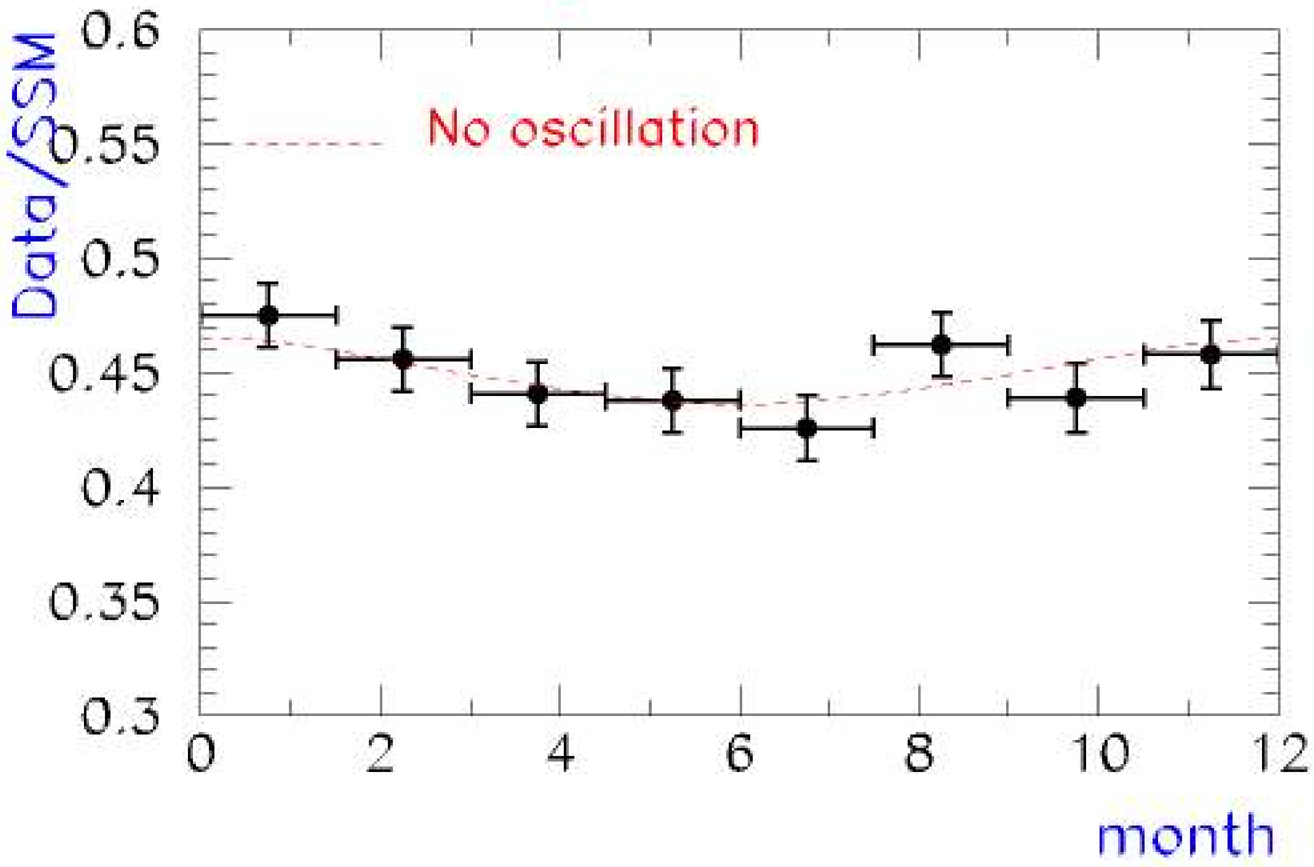
- Zenith Angle Distribution (Day/Night Effect): Effect of Earth Matter

→ Few more events at N than D $2\frac{D-N}{D+N} = -0.033 \pm 0.022 \pm 0.013\,(1.3\sigma)$





- Seasonal Variation → Small Beyond Earth Orbit Eccentricity



• The *Standard* Interpretation:

Oscillation of ν_e into ν_μ, ν_τ (active $\nu's$) , or ν_s (sterile ν)

– The ν_e survival amplitude after propagation from the Sun to the detector at the Earth:

$$A(\nu_e \to \nu_e) = A_{Sun}(\nu_e \to \nu_1) \times A_{\rm vac}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e) + A_{Sun}(\nu_e \to \nu_2) \times A_{\rm vac}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e)$$

– Where

$$|A_{Sun}(\nu_e \to \nu_1)|^2 = P^{Sun}_{e1} = 1 - |A_{Sun}(\nu_e \to \nu_2)|^2$$

$$|A_{Earth}(\nu_1 \to \nu_e)|^2 = P^{Earth}_{1e} = 1 - |A_{Earth}(\nu_2 \to \nu_e)|^2$$

$$A_{vac}(\nu_i \to \nu_i) = \exp\left(-i\, m_i^2\, (L - R_{Sun})/2E\right)$$

– So in general:

$$P_{ee} = P^{Sun}_{e1} P^{Earth}_{1e} + (1 - P^{Sun}_{e1})(1 - P^{Earth}_{1e}) + 2\sqrt{P^{Sun}_{e1}(1 - P^{Sun}_{e1}) P^{Earth}_{1e}(1 - P^{Earth}_{1e})} \cos\left(\frac{\Delta m^2 L}{2E} + \delta\right)$$

δ small phase $\sim 10^{-3}$.

– For $\Delta m^2 \sim E/L$ matter effects can be neglected

$P^{Sun}_{e1} = P^{Earth}_{1e} = \cos^2\theta \Rightarrow$ Vacuum Oscillations

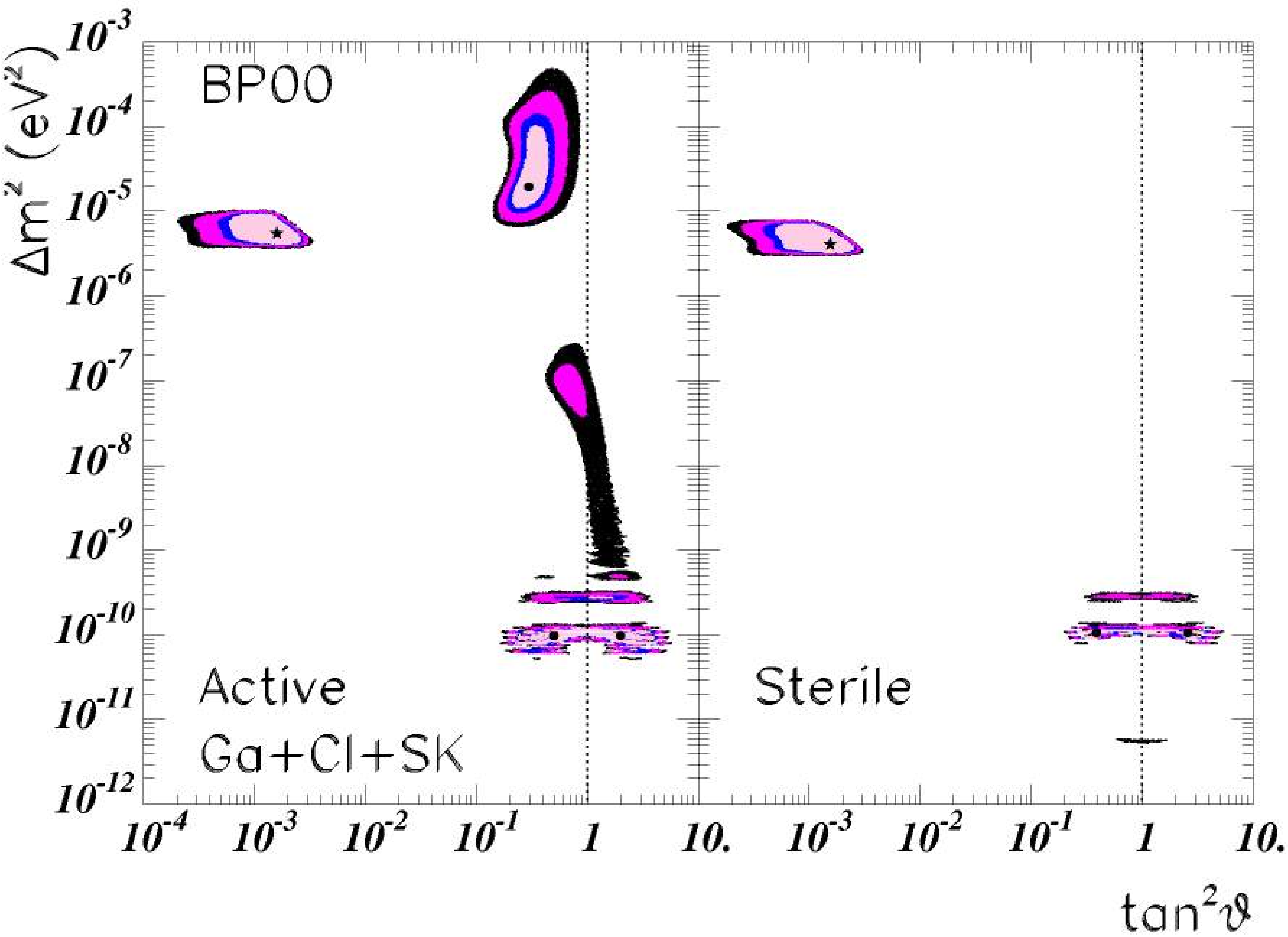
– For $\Delta m^2 \gtrsim 10^{-8}$ eV$^2 \Rightarrow \langle \cos\left(\frac{\Delta m^2 L}{2E}\right)\rangle = 0 \Rightarrow$ MSW Transitions

– In the intermediate range Quasi-vacuum Oscillations

Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$

Including new GNO, Sage and Super–Kamiokande 1258 Days

Allowed regions from Rates: M.C.G-G, C. Peña-Garay



– Why differences for oscillations into active or sterile?

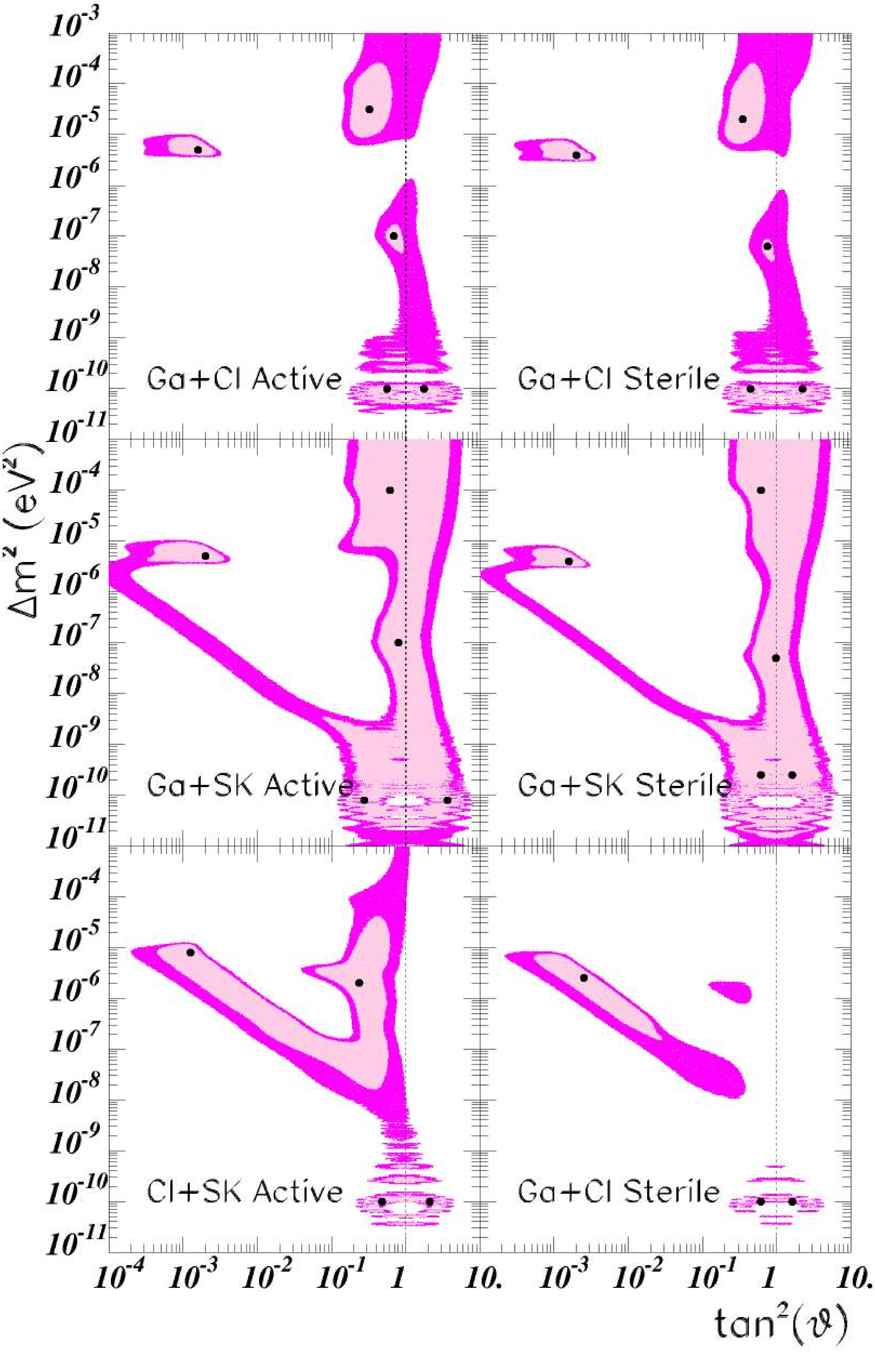
Main effect is different contribution to event rates in SuperK

$$\nu_{\mu(\tau)} + e \to \nu_{\mu(\tau)} + e \to \text{NC events in SuperK}$$
$$\nu_s + e \not\to \nu_s + e \to \text{no NC events in SuperK}$$

Also slightly different survival probabilities in the sun

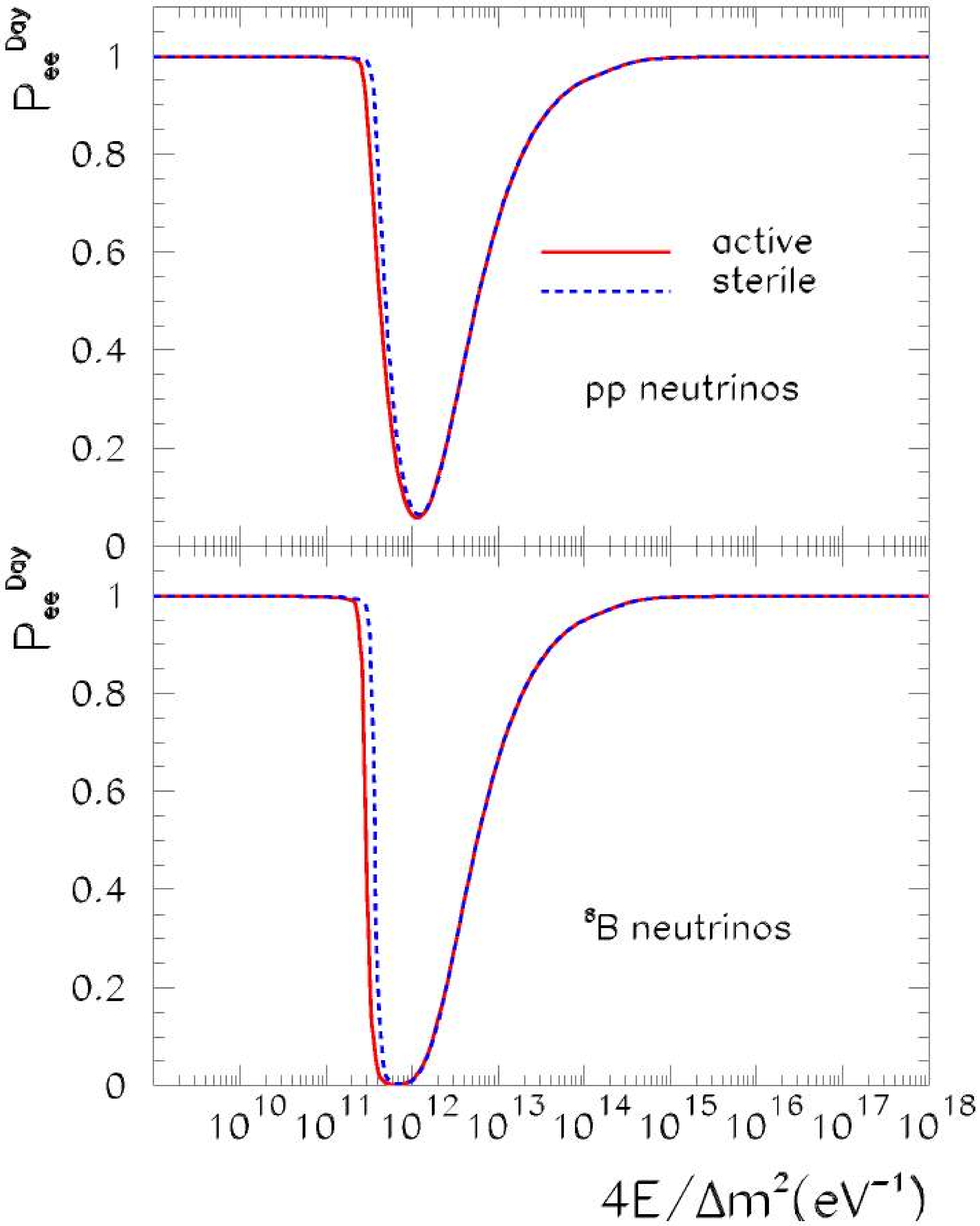
		Active				Sterile
		SMA	LMA	LOW	VAC	SMA
Rates	Δm^2/eV^2	7.1×10^{-6}	2.1×10^{-5}	1.0×10^{-7}	7.9×10^{-11}	4.9×10^{-6}
	$\tan^2\theta$	1.6×10^{-3}	0.34	0.63	0.29 (3.4)	5.5×10^{-4}
	Prob (%)	10 %	15 %	1.3 %	17.5 %	0.01%

Regions with Only Two Experiments



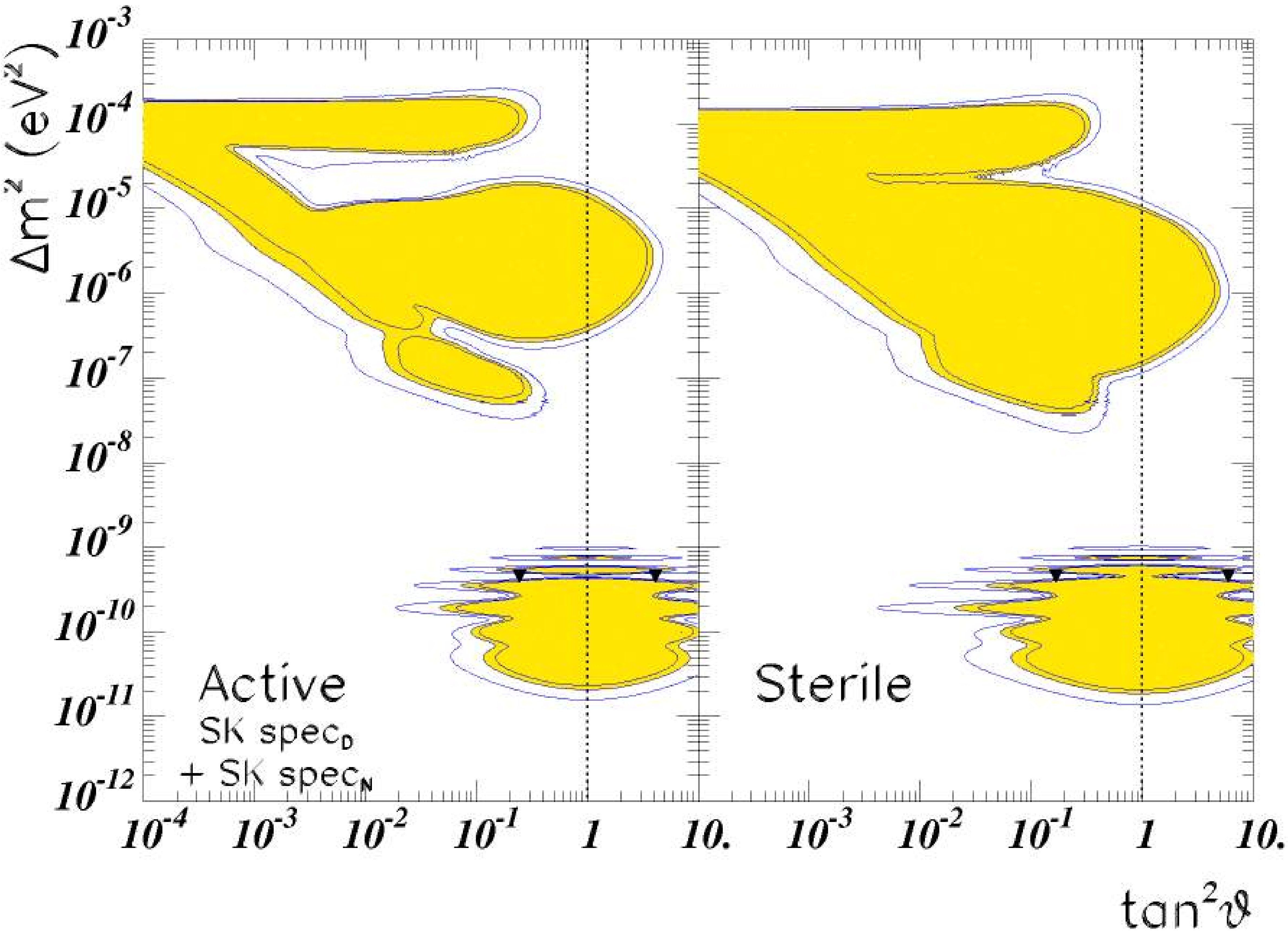
Main difference between Active and Sterile due to SK+Cl

Survival Probabilities For Active and Sterile Neutrinos



Due to smaller matter potential the SMA region for sterile is slightly shifted down

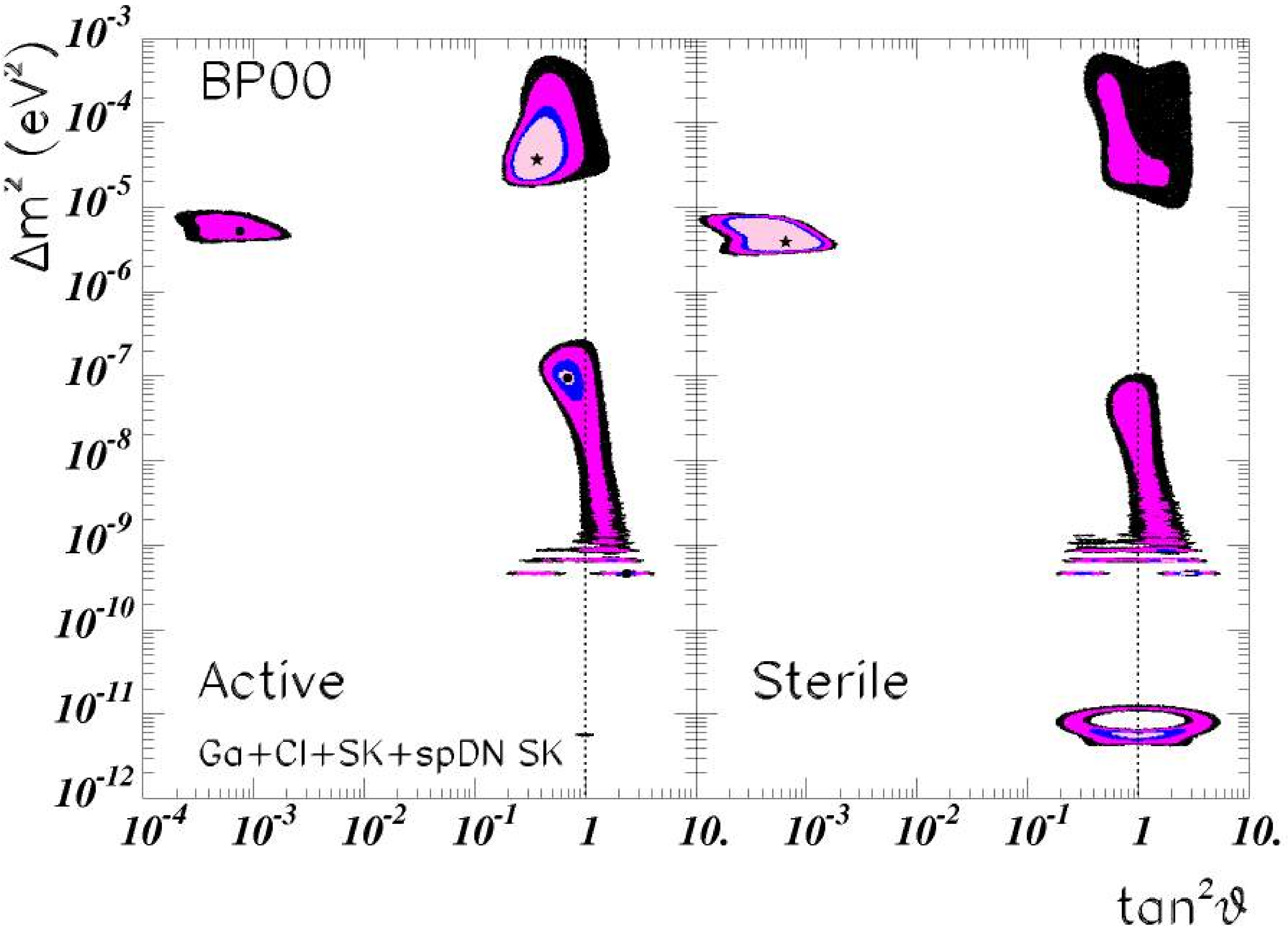
Excluded regions from Day–Night Spectrum:



Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$

Including new GNO, Sage and Super–Kamiokande 1258 Days

Allowed regions from Global Analysis: M.C.G-G, C.Peña-Garay



		Active				Sterile
		SMA	LMA	LOW	VAC	SMA
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	$\tan^2 \theta$	1.6×10^{-3}	0.34	0.63	0.29 (3.4)	5.5×10^{-4}
	Prob (%)	10 %	15 %	1.3 %	17.5 %	0.01%
Global	$\Delta m^2/\text{eV}^2$	4.7×10^{-6}	4.5×10^{-5}	1.0×10^{-7}	4.6×10^{-10}	4.5×10^{-6}
	$\tan^2 \theta$	3.9×10^{-4}	0.41	0.70	2.4	3.1×10^{-4}
	Prob (%)	22 %	64 %	50 %	47 %	19%

– *LMA cut for $\Delta m^2 \gtrsim 8 \times 10^{-4}$ eV^2 due to Chooz bound*

– SMA sterile allowed but with smaller mixing (worse than any active)

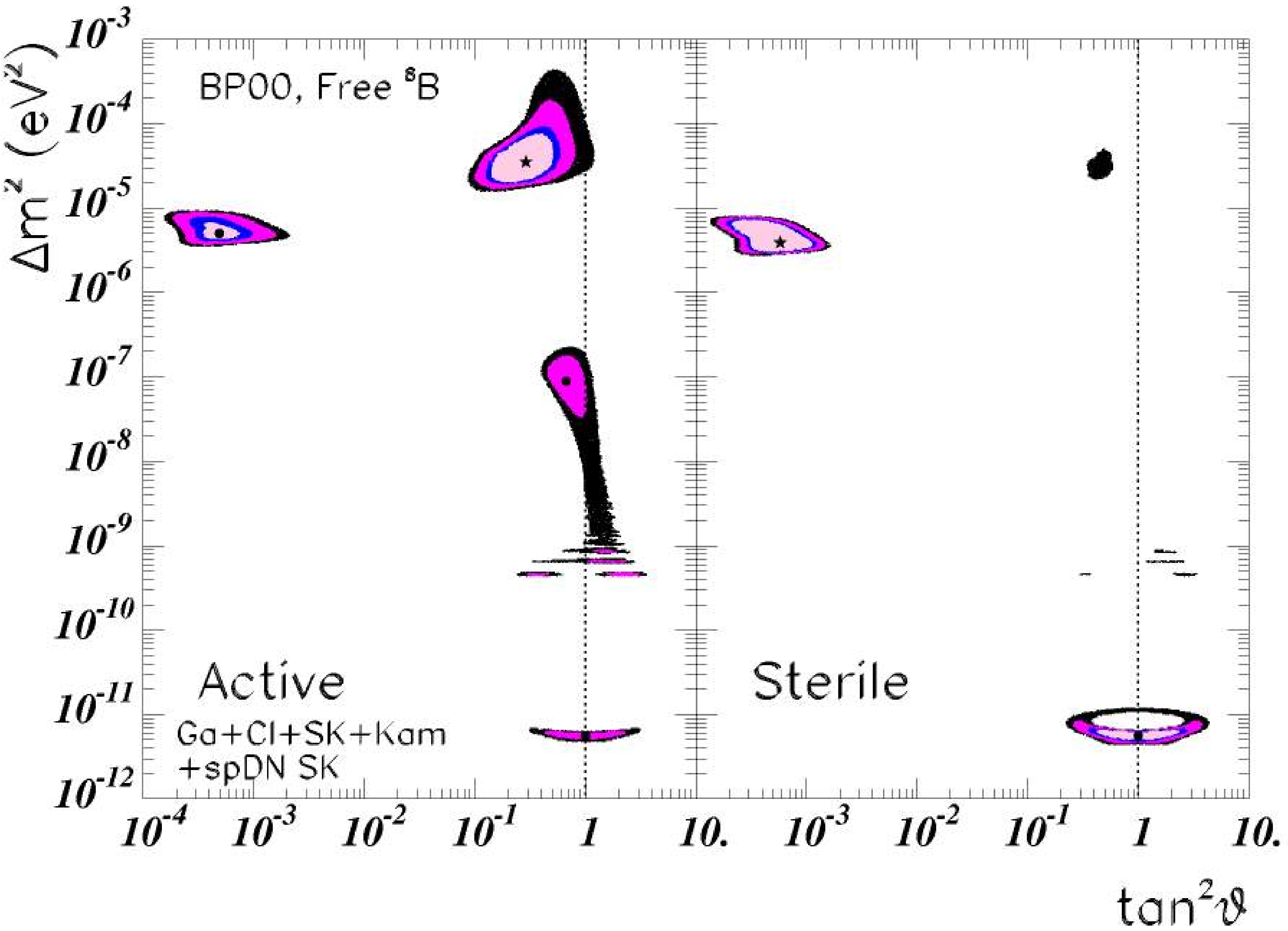
– SMA active allowed at $\sim$ 95 % CL but with smaller mixing

– LOW solution better than SMA

Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$

Allowed regions from Global Analysis with free ^{8}B:

M.C.G-G, C. Peña-Garay hep-ph/0009041



		Active			Sterile
		SMA	LMA	LOW	SMA
Rates	Δm^2/eV^2	4.0×10^{-6}	3.5×10^{-5}	1.0×10^{-7}	3.9×10^{-6}
+Sp$_D$	$\tan^2\theta$	0.00049	0.29	0.65	0.00058
+Sp$_N$	Prob (%)	47 %	62 %	34 %	37 %

– Improvement of SMA solutions Active and Sterile:

Flatter spectrum ⇒ SMA shifted to smaller mixing⇒ Larger ^8B flux

– Small improvement of the active LMA:

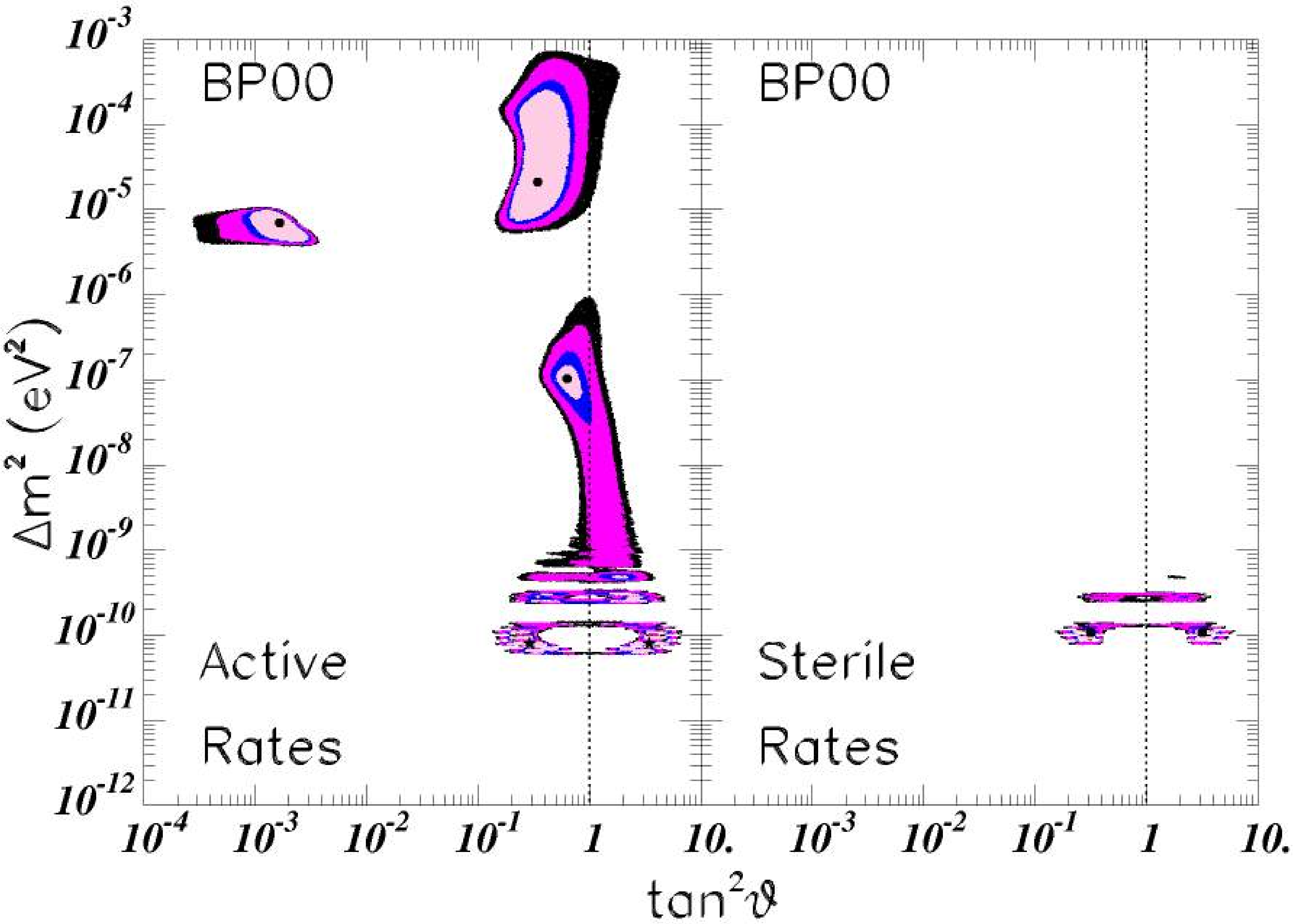
Easier simultaneous agreement of Cl and SK rates

– Just So^2 improves

Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$

Allowed regions from Rates (Including SNO):

J. Bahcall, M.C.G-G, C. Peña-Garay



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$$\nu_{\mu(\tau)} + e \to \nu_{\mu(\tau)} + e \to \text{NC events in SuperK}$$

$$\nu_s + e \not\to \nu_s + e \to \text{no NC events in SuperK}$$

Difficult to fit SuperK and SNO CC

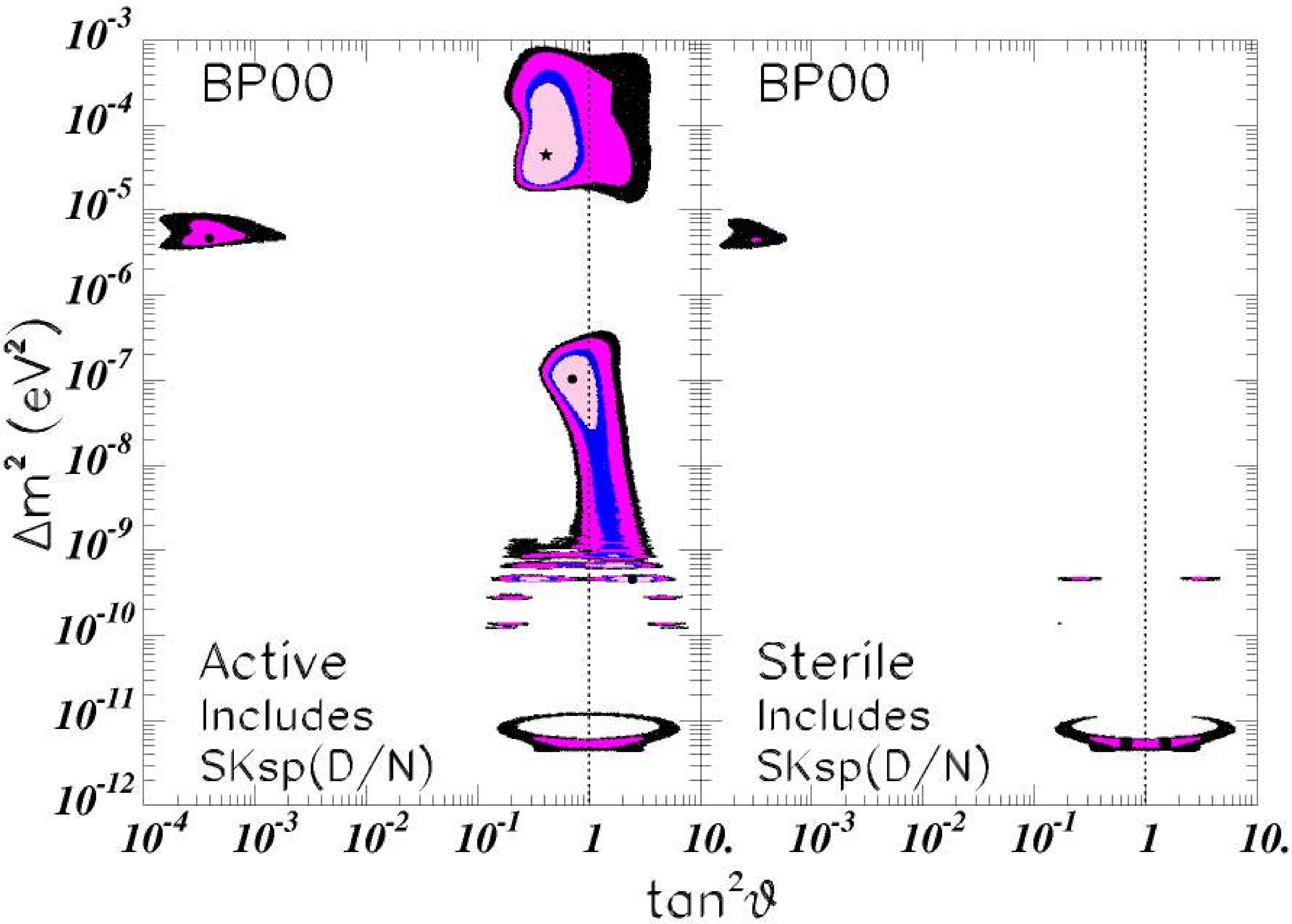
	Active				Sterile	
RATES	SMA	LMA	LOW	VAC	SMA	VAC
Δm^2/eV^2	6.9×10^{-6}	2.1×10^{-5}	1.0×10^{-7}	7.9×10^{-11}	4.9×10^{-6}	1.1×10^{-10}
$\tan^2\theta$	1.6×10^{-3}	0.34	0.63	0.29 (3.4)	5.5×10^{-4}	0.29 (3.4)
Prob (%)	7 %	15 %	1.3 %	17 %	0.01%	0.63%

Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$

* Including SNO

Allowed regions from Global Analysis (Including SNO):

J. Bahcall, M.C.G-G, C. Peña-Garay



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	$\tan^2\theta$	3.9×10^{-4}	0.41	0.70	2.4	3.1×10^{-4}
	Prob (%)	19 %	59 %	45 %	42 %	16%

– *LMA cut for $\Delta m^2 \gtrsim 8 \times 10^{-4}$ eV^2 due to Chooz bound*

– SMA sterile marginally allowed (worse than any active)

– SMA active allowed at $\sim$ 99 % CL but with smaller mixing

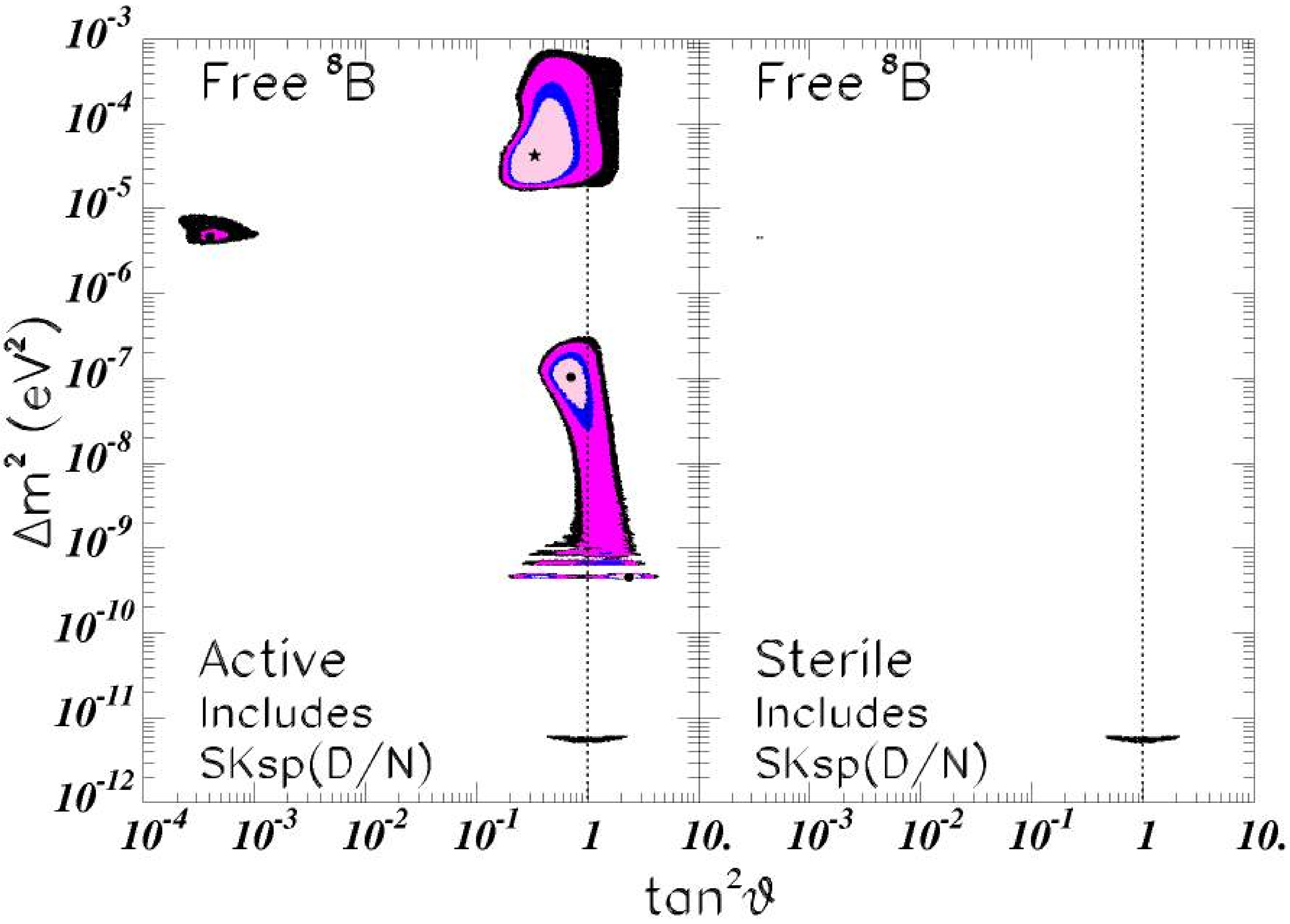
– LOW solution better than SMA

– Just so^2 appears

Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$

Allowed regions from Global Analysis with free ^{8}B (Including SNO):

J. Bahcall, M.C.G-G, C. Peña-Garay



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– Improvement of SMA solutions Active and Sterile:

Flatter spectrum $\Rightarrow$ SMA shifted to smaller mixing$\Rightarrow$ Larger ^8B flux

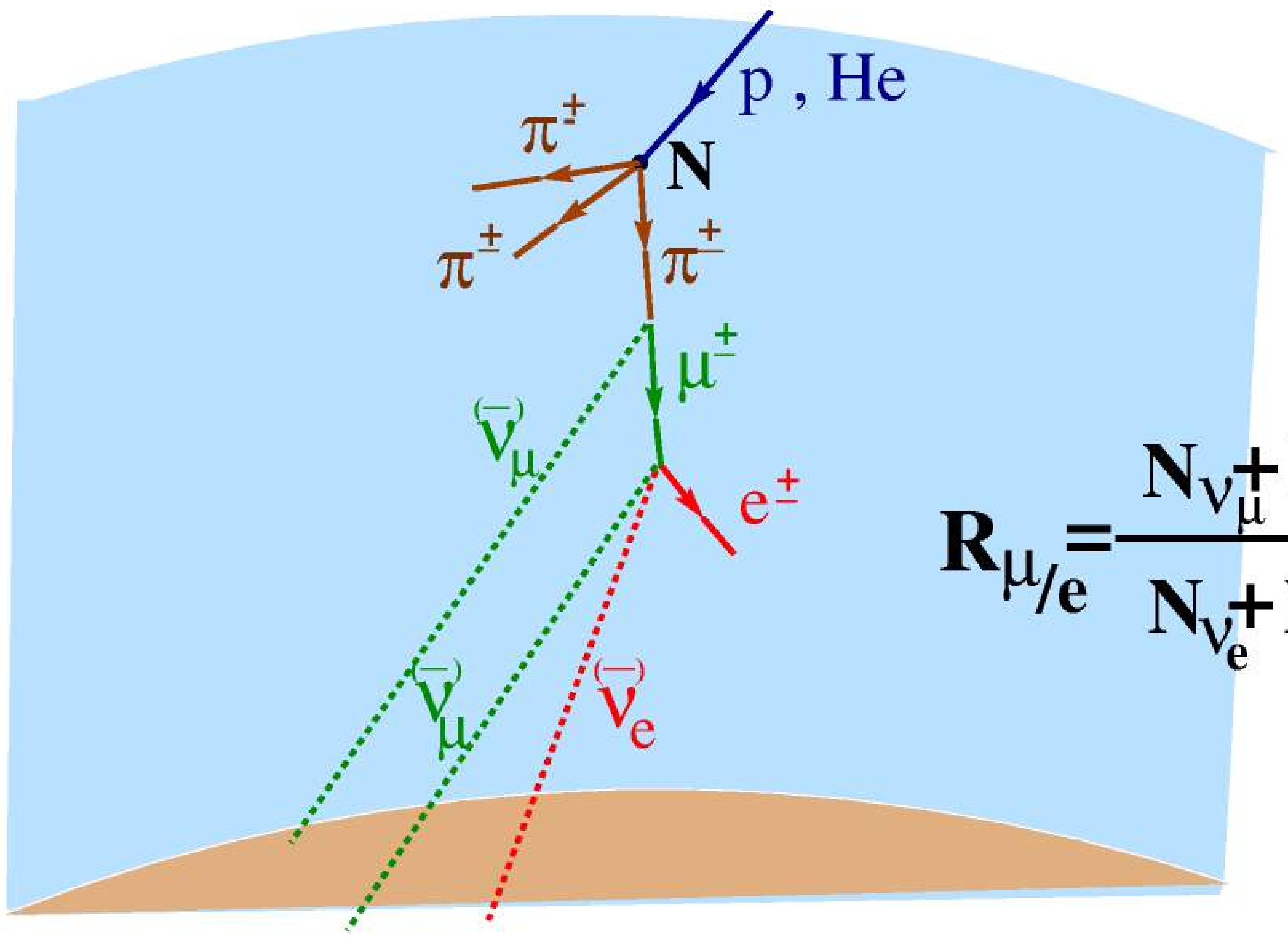
– Small improvement of the active LMA:

Easier simultaneous agreement of Cl and SK rates

– Just So^2 at 3 σ

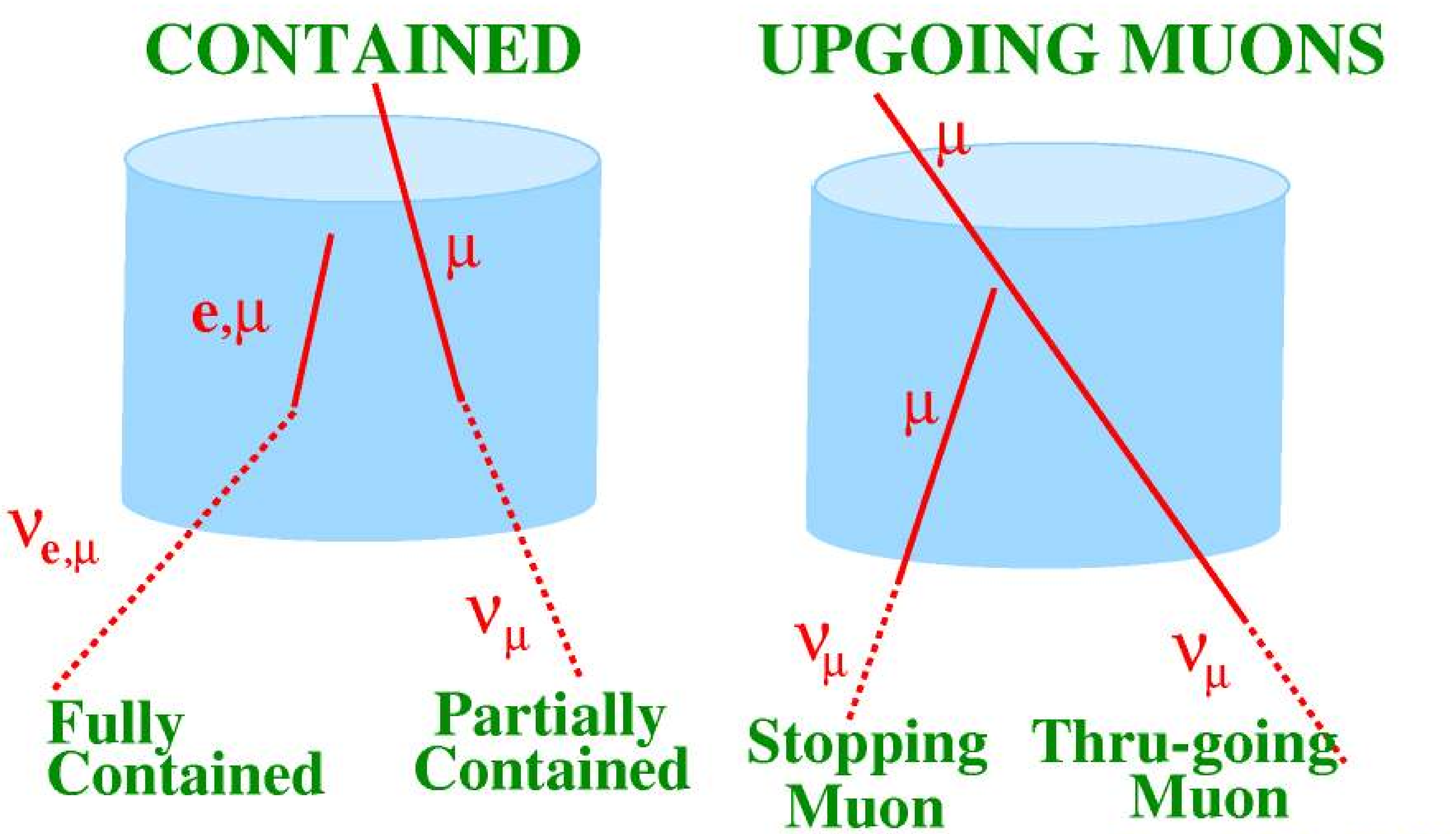
Atmospheric 2–ν Oscillations: Active vs Sterile

Atmospheric $\nu_{e,\mu}$ are produced by the interaction of cosmic rays (p, He . . .) with the atmosphere



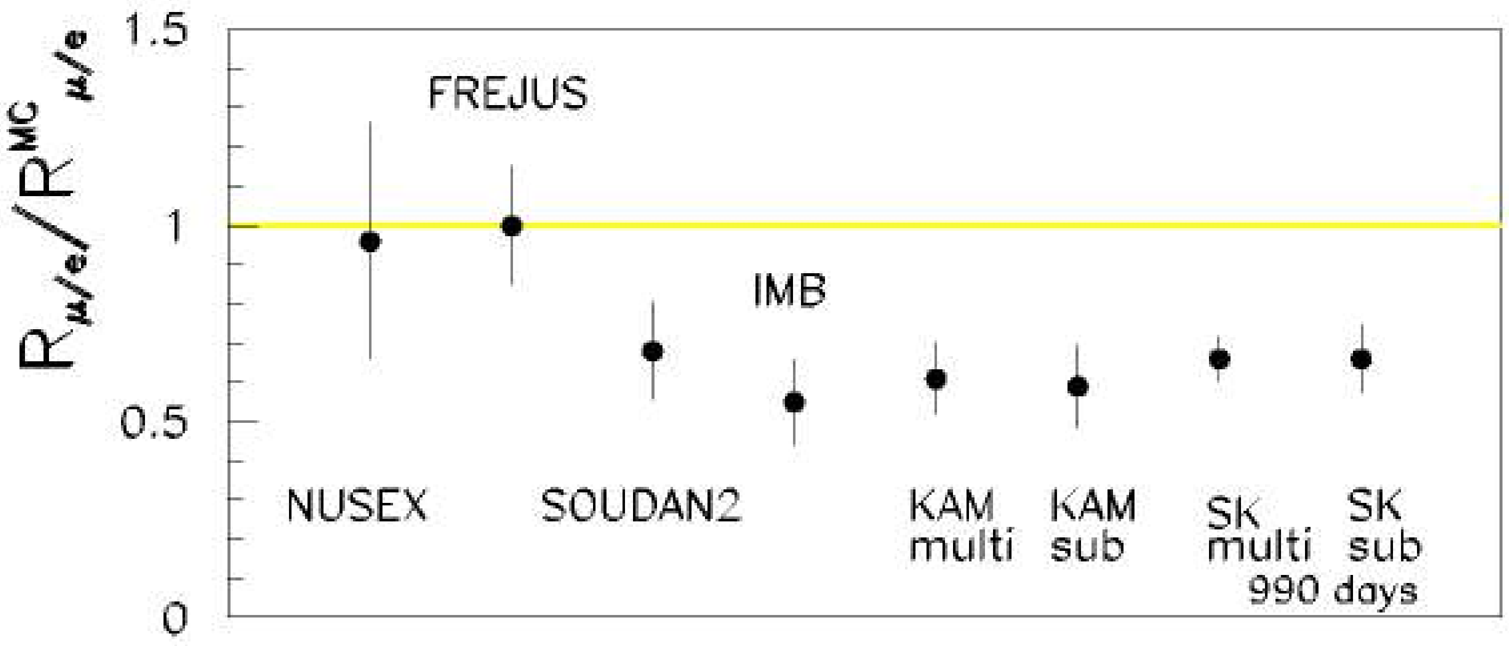
$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

EVENT CLASSIFICATION

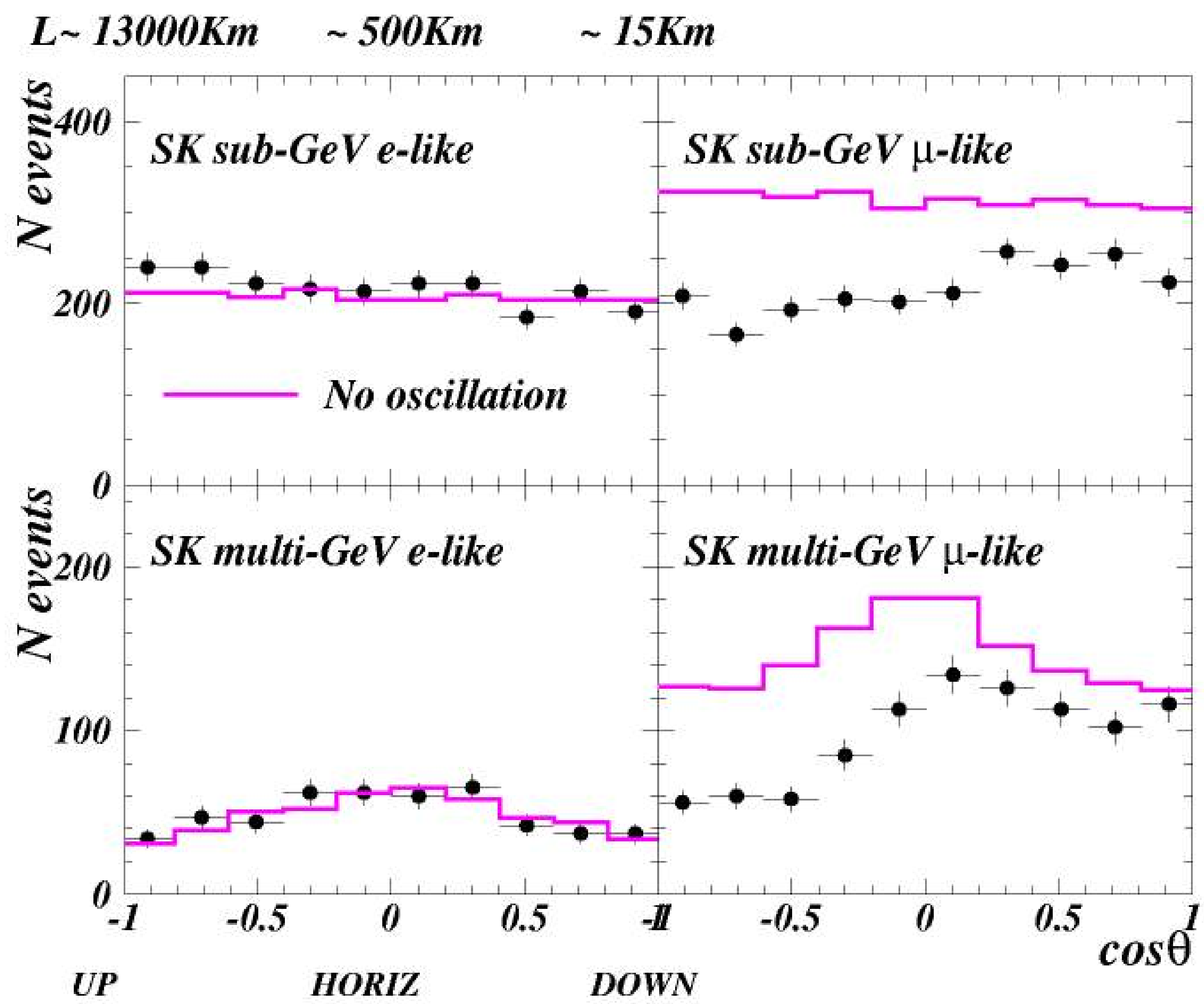


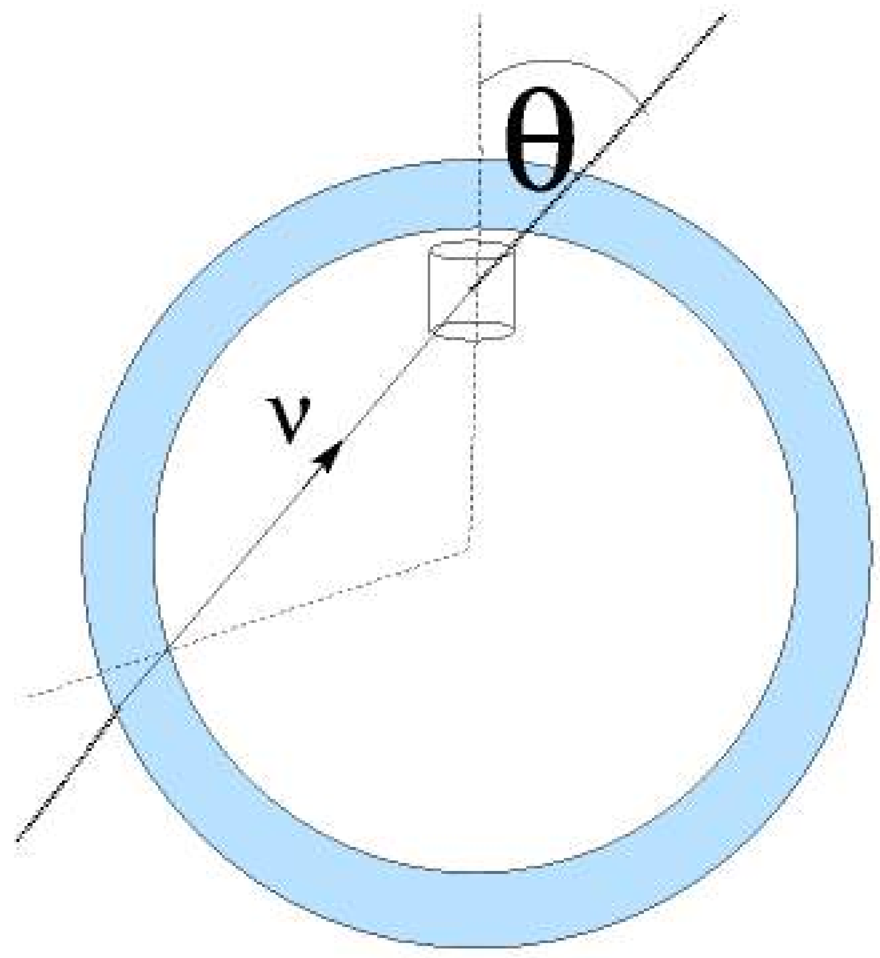
E_ν 0.1 - few GeV | few GeV | few 10 GeV | few 100 GeV

• Total Rates for Contained Events

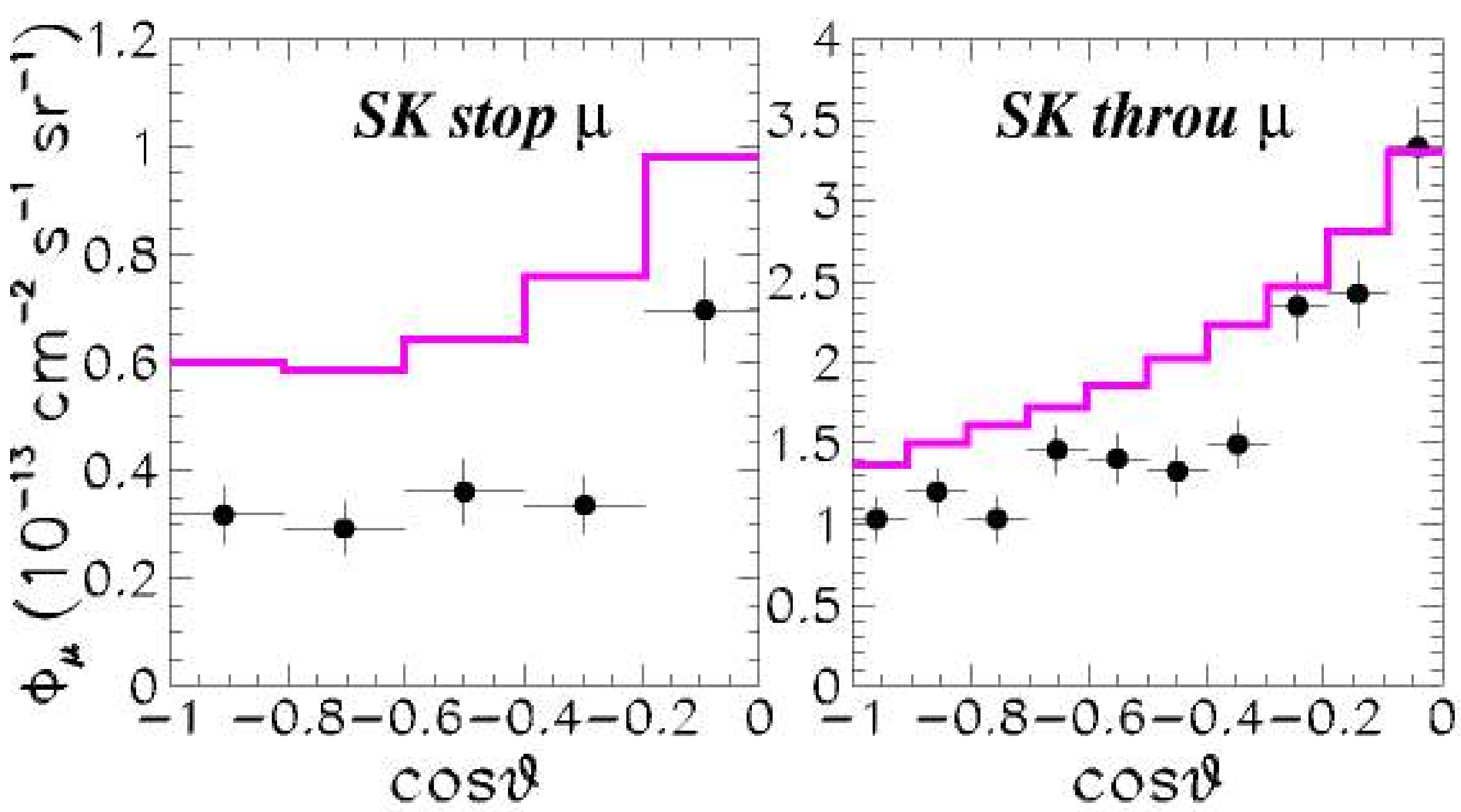


• Angular Distribution of Contained Events → Deficit grows with L





• Upward Going Muon Fluxes → Deficit decreases with E



$$\frac{\left.\frac{STOP}{THROU}\right|_{DATA}}{\left.\frac{STOP}{THROU}\right|_{MC}} = 0.61 \pm 0.1$$

Atmospheric Neutrino Data Oscillation Analysis

- In presence of neutrino oscillations:

– The expected number of contained events

$$N_\mu = N_{\mu\mu} + N_{e\mu} \qquad N_e = N_{ee} + N_{\mu e}$$

$$N_{\alpha\beta} = n_t T \int \frac{d^2\Phi_\alpha}{dE_\nu d\cos\theta_\nu} \kappa_\alpha(h) P_{\alpha\beta} \frac{d\sigma}{dE_\beta} \varepsilon(E_\beta) dE_\nu dE_\beta d\cos\theta_\nu dh$$

– The expected upgoing-μ fluxes:

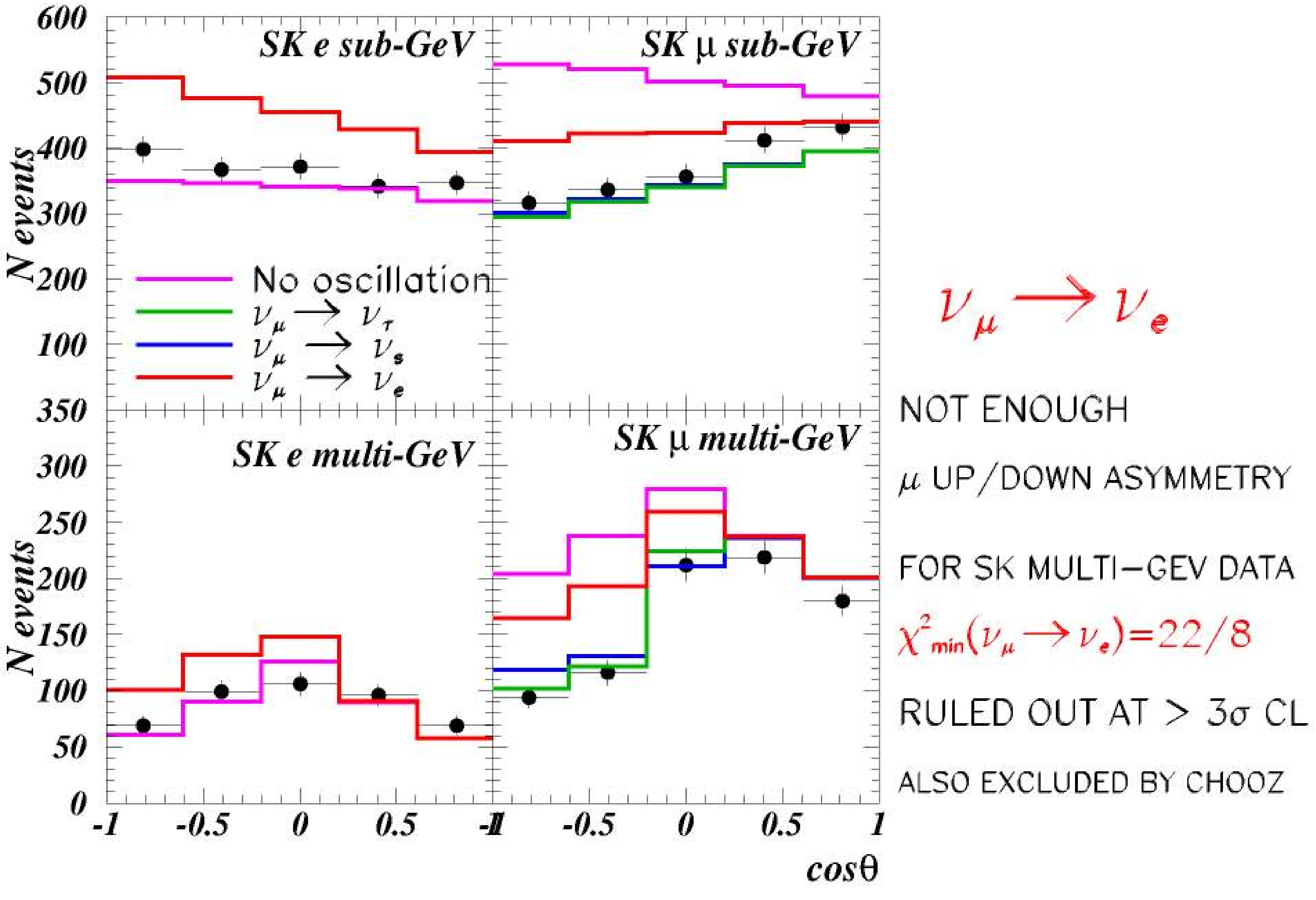
$$\Phi_\mu(\theta)_{S,T} = \frac{1}{A(L_{min},\theta)} \int \frac{d\Phi_\mu(E_\mu,\cos\theta)}{dE_\mu d\cos\theta} A_{S,T}(E_\mu,\theta) dE_\mu$$

$$\frac{d\Phi_\mu}{dE_\mu d\cos\theta} = \int_0^\infty \frac{d\Phi_{\nu_\mu}}{dE_\nu d\cos\theta} P_{\mu\mu} \frac{d\sigma}{dE_{\mu 0}} F_{rock}(E_{\mu 0}, E_\mu, X) N_A dE_\nu dE_{\nu 0} dX$$

- Three possible oscillation channels:

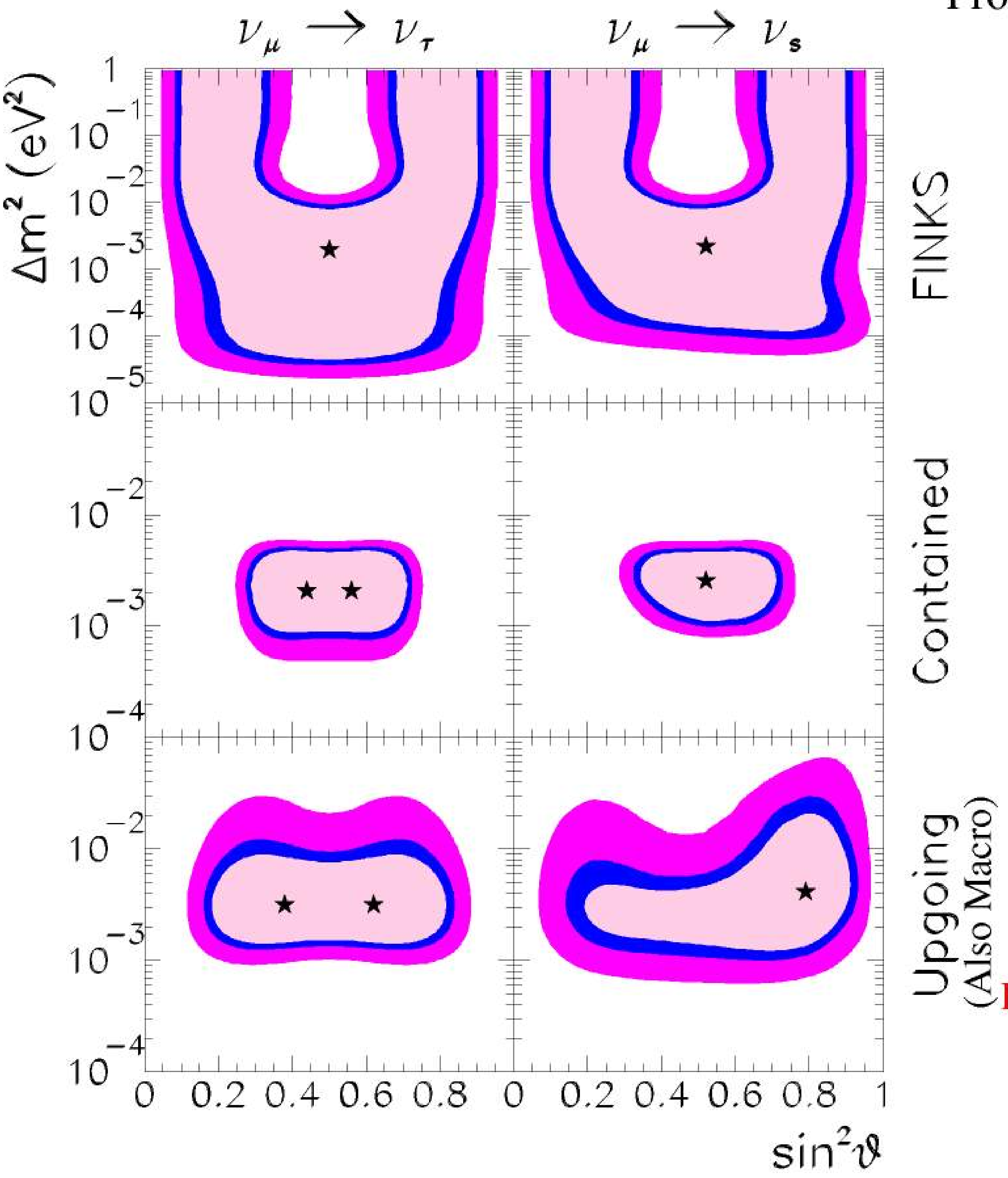
$$\nu_\mu \to \nu_X \; X = e, \tau, \text{sterile}$$

- For $X = e$ and $X = sterile$ Earth matter effects



Atmospheric Neutrino Oscillation Parameters

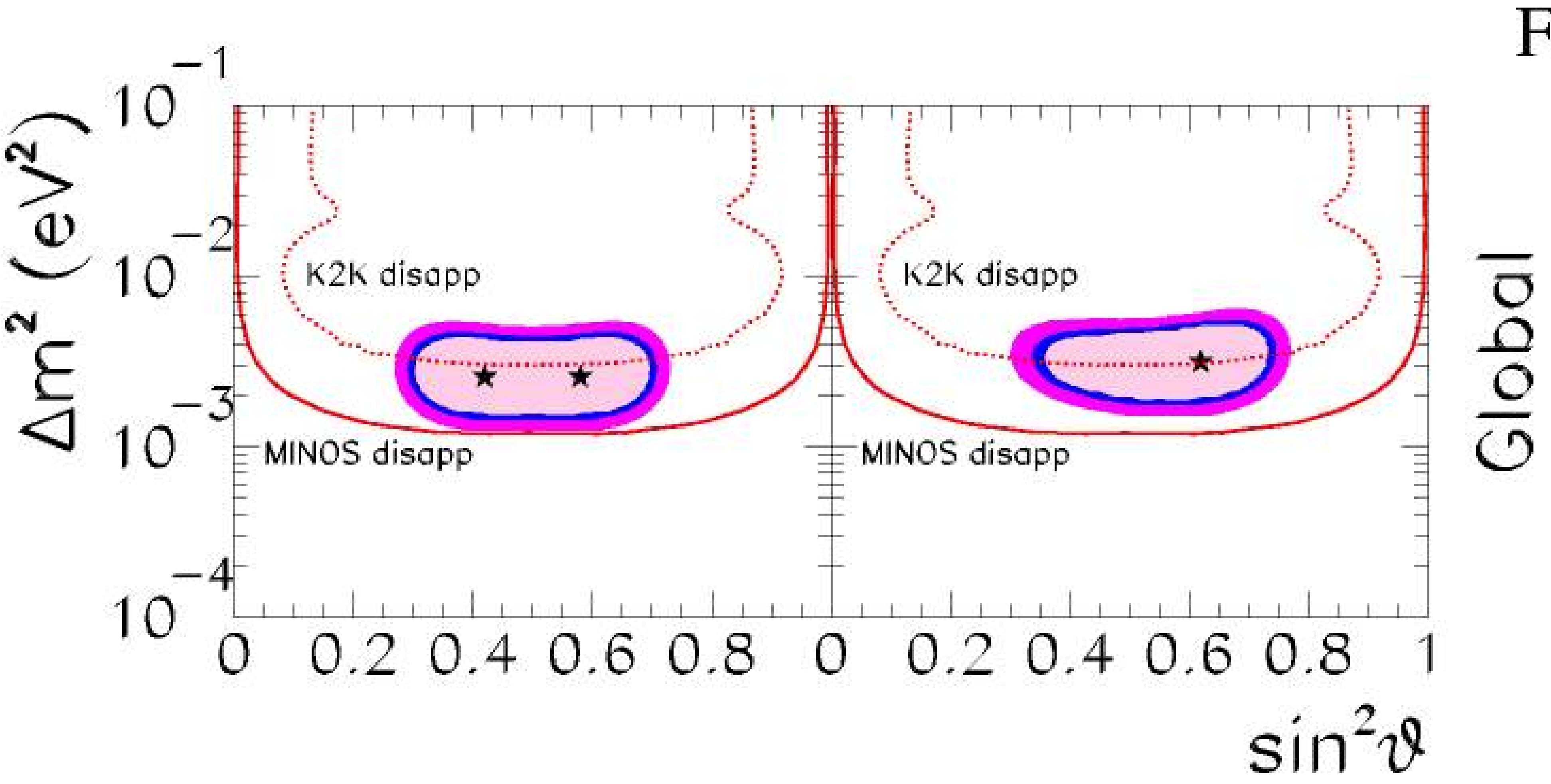
M.C.G-G, M. Maltoni, C.Peña-Garay



From Partial Analysis

	χ^2_{min}	
	ν_τ	ν_s
FINKS	$16/8$	$16/8$
Contained	$36/38$	$38/38$
Upgoing (Also Macro)	$19/23$	$29/23$

Difference from matter effects in upgoig-μ



From Global Analysis

	χ^2_{min}	
	ν_τ	ν_s
Global	$56/63$	$72/63$

$\nu_\mu \rightarrow \nu_\tau$ Favoured

Super–Kamiokande from the analysis of the zenith angle dependence of high-E PC + thrugoing + NC enriched events:

$\nu_\mu \rightarrow \nu_s$ disfavoured with $\sim 3\sigma$.

Four Neutrino Mixing

- To fit solar, atmospheric and LSND

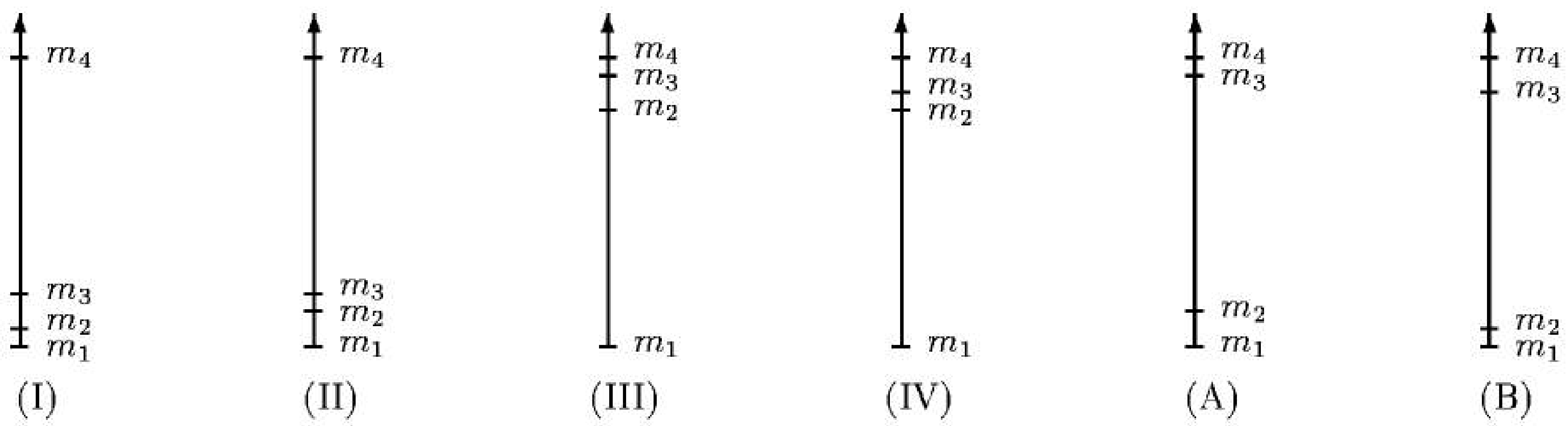
  $\rightarrow 3\ \Delta m^2 : \Delta m^2_{LSND} \ggg \Delta m^2_{atm} \gg \Delta m^2_{sun}$

  $\rightarrow 4\ \nu' s$

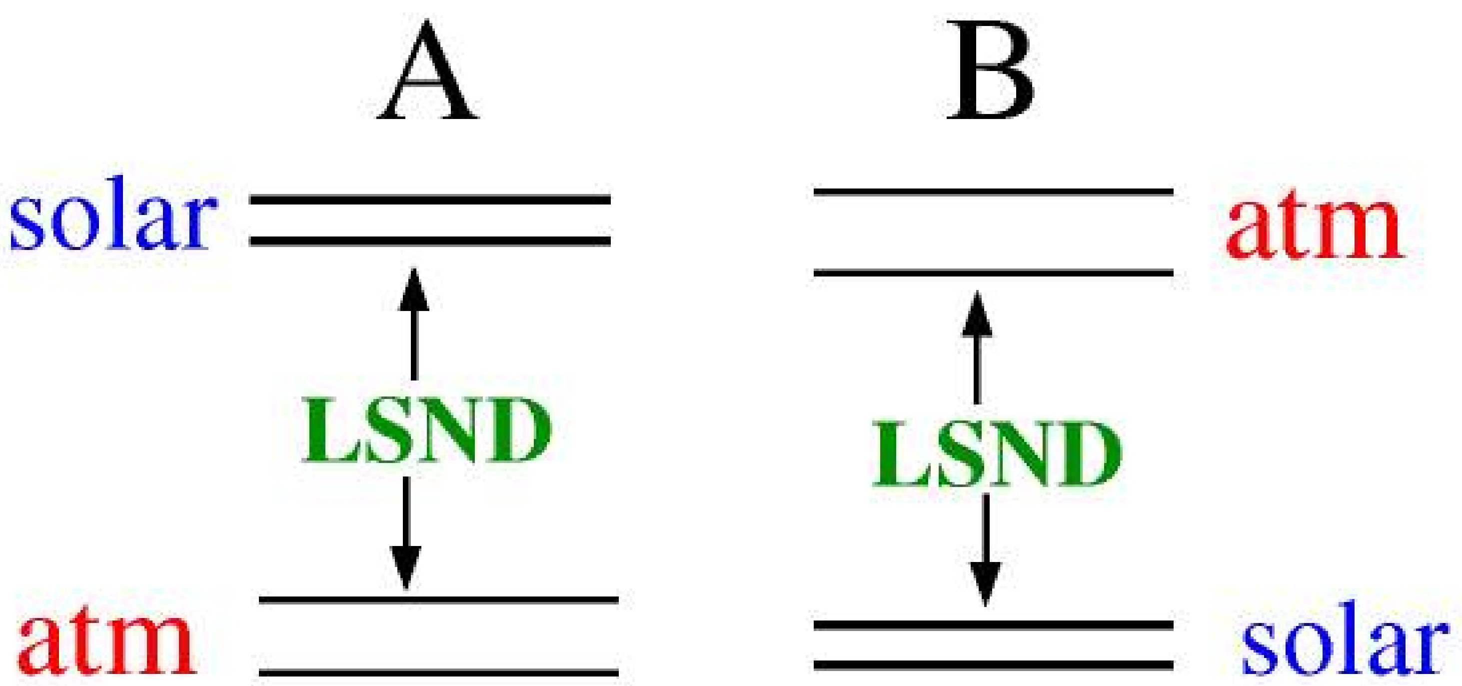
- But LEP data implies only 3 neutrino flavours

  $\rightarrow$ 4th ν must be sterile

- U: 6 mixing angles and 3 CP phases
- Possible mass spectra:



- Limits from Accelerator and Reactor: $\rightarrow$ A and B favoured



In B for example, naively:

Atm: $\nu_\mu \rightarrow \nu_+$

Solar: $\nu_e \rightarrow \nu_-$

$\nu_- \simeq c_\chi \nu_s + s_\chi \nu_\tau$

$\nu_+ \simeq -s_\chi \nu_s + c_\chi \nu_\tau$

χ fuction of angles in U

- Open Question: is ν_s part of Atm or Solar Oscillation?

  * In 2-ν oscillations we have seen the limiting cases:

$$c_\chi = 0 \rightarrow \begin{cases} \text{Atm: } \nu_\mu \rightarrow \nu_s \\ \text{Solar: } \nu_e \rightarrow \nu_\tau \end{cases}$$

$$c_\chi = 1 \rightarrow \begin{cases} \text{Atm: } \nu_\mu \rightarrow \nu_\tau \\ \text{Solar: } \nu_e \rightarrow \nu_s \end{cases}$$

  *But intermediate cases $0 < s_\chi < 1$ also possible

• More precisely for B:

$$\Delta m^2_\odot = \Delta m^2_{21} \ll \Delta m^2_{\rm atm} = \Delta m^2_{43}$$
$$\ll \Delta m^2_{\rm LSND} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{31} \simeq \Delta m^2_{32}$$

• The mixing matrix U

$$U = U_{24}\, U_{23}\, U_{14}\, U_{13}\, U_{34}\, U_{12}$$

• Limits from Reactor Experiments

$$P_{ee}^{Bugey} = 1 - 2\left(|U_{e3}|^2 + |U_{e4}|^2\right)\left(1 - |U_{e3}|^2 - |U_{e4}|^2\right)\langle \sin^2 \frac{\Delta m^2_{\rm LSND} L}{4E}\rangle \gtrsim 0.99$$

$\Rightarrow |U_{e3}|^2 + |U_{e4}|^2 = c^2_{14} s^2_{13} + s^2_{14} \lesssim 10^{-2}$

$\Rightarrow \vartheta_{13}, \vartheta_{14}$ negligeable in solar and atmospheric oscillations

$\Rightarrow$ And the simplifed mixing matrix U

$$\begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12}c_{23}c_{24} & c_{12}c_{23}c_{24} & s_{23}c_{24}c_{34} - s_{24}s_{34} & s_{23}c_{24}s_{34} + s_{24}c_{34} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23}c_{34} & c_{23}s_{34} \\ s_{12}c_{23}s_{24} & -c_{12}c_{23}s_{24} & -s_{23}s_{24}c_{34} - c_{24}s_{34} & -s_{23}s_{24}s_{34} + c_{24}c_{34} \end{pmatrix}$$

$\Rightarrow$ Four relevant mixing angles:

– $\vartheta_{12} \rightarrow$ amplitude of oscillations with $\Delta m^2_\odot$

– $\vartheta_{34} \rightarrow$ amplitude of oscillations with Δm^2_{atm}

– ϑ_{23} and $\vartheta_{34} \rightarrow$ active–sterile admixture.

Solar Neutrinos

• $\Delta m^2_\odot$ oscillations occur between the states

$$\nu_e \to \nu_\alpha \quad \text{with} \quad \nu_\alpha = c_{23}c_{24}\nu_s + \sqrt{1 - c_{23}^2 c_{24}^2}\,\nu_a$$

(ν_a linear combination of ν_μ and ν_τ)

with mixing angle ϑ_{12}

• $c_{23}^2 c_{24}^2 = 1 - |U_{a1}|^2 - |U_{a2}|^2 = |U_{s1}|^2 + |U_{s2}|^2$
gives the active–sterile admixture

• Cases

* $c_{23}c_{24} = 0 \to$ pure $\nu_e \to \nu_{active}$ oscillations with mixing θ_{12}
* $c_{23}c_{24} = 1 \to$ pure $\nu_e \to \nu_{sterile}$ oscillations with mixing θ_{12}
* $0 < c_{23}c_{24} < 1 \to$ simultaneous $\nu_e \to \nu_s$ and $\nu_e \to \nu_a$

– In this general case:

$$P_{ee}^{4\nu,Sun} = P_{ee}^{2\nu,Sun}$$
$$P_{es}^{4\nu,Sun} = c_{23}^2 c_{24}^2 \left(1 - P_{ee}^{Sun}\right)$$

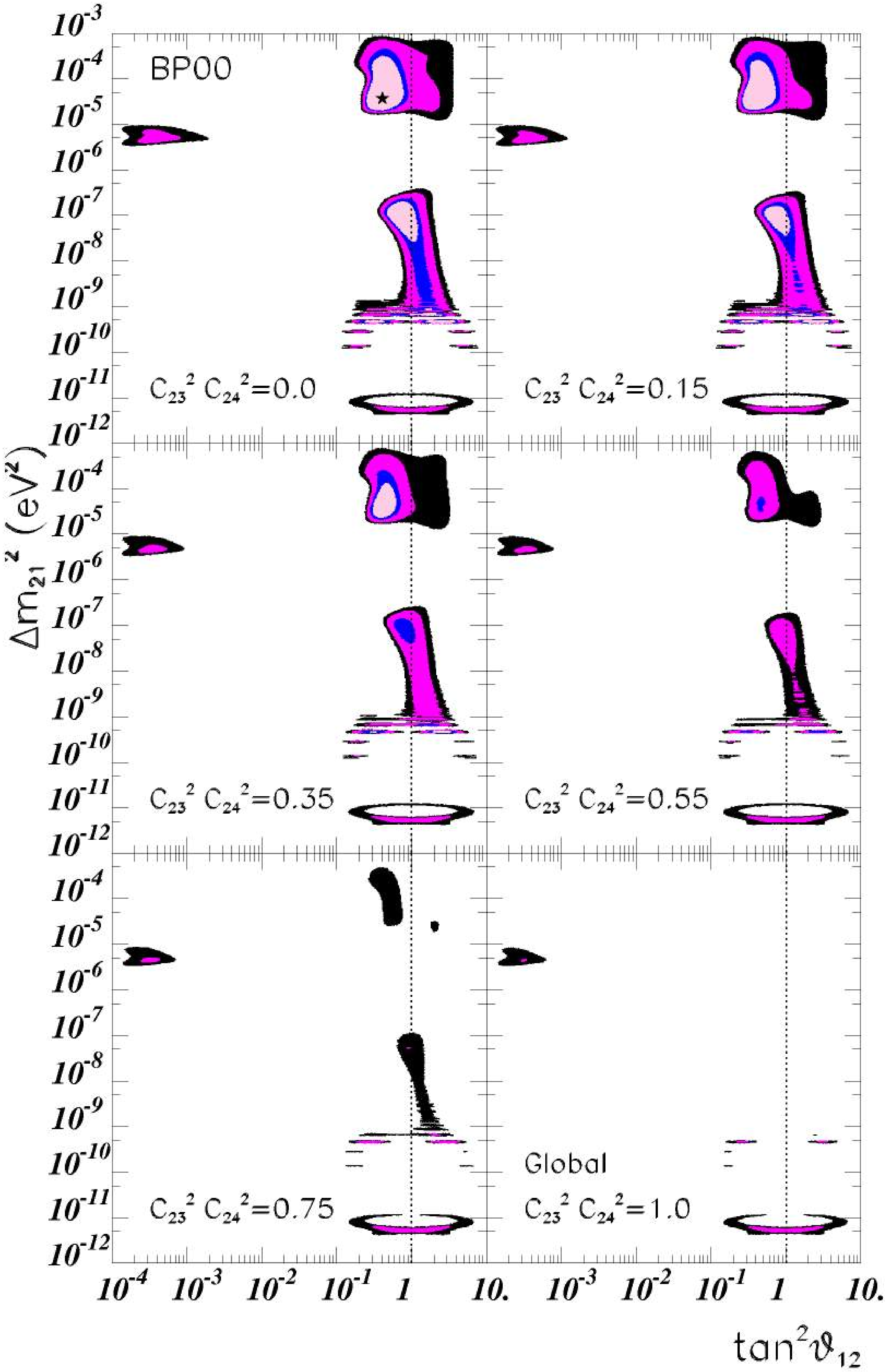
but $P_{ee}^{2\nu,Sun}$ is computed for a matter potential

$$A \equiv A_{CC} + c_{23}^2 c_{24}^2\, A_{NC}$$

– Same analysis but with 3 parameters: Δm^2_{12}, θ_{12}, and $c_{23}^2 c_{24}^2$

Solutions for Solar 4–ν Oscillations

M.C.G-G, M. Maltoni, C.Peña-Garay hep-ph/0105269 (Updated with SNO)



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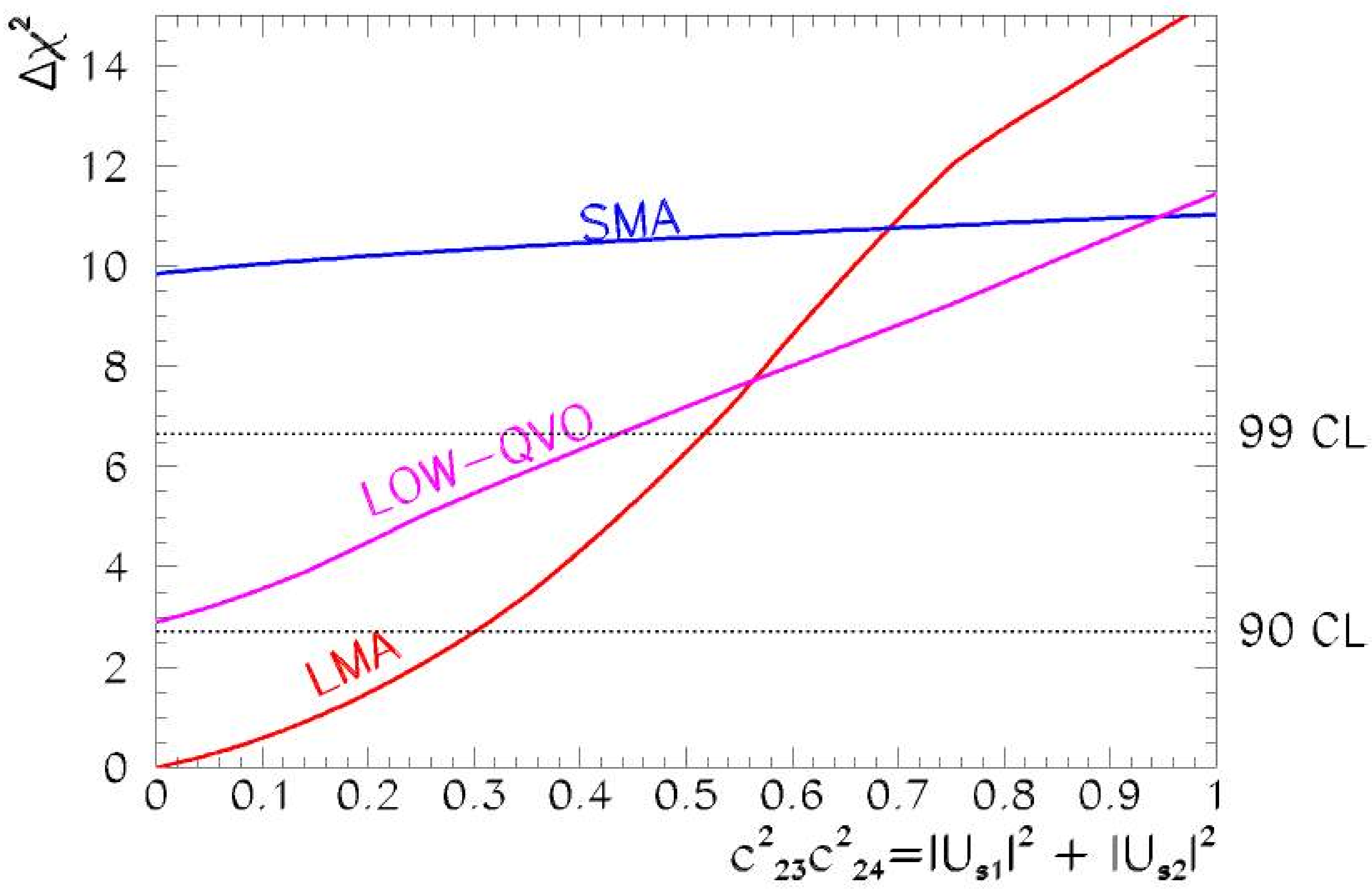

– Best fit point for LMA pure $\nu_e \to \nu_a$

– LMA, LOW–QVO can have a sizeable subdominant $\nu_e \to \nu_s$ component

– For large ν_s projection SMA favoured

– Regions disappear for:

$$|U_{s1}|^2 + |U_{s2}|^2 = c_{23}^2 c_{24}^2$$

CL	SMA	LMA	LOW–QVO
90	$\geq$0.0	$\geq$0.45	$\geq$0.4
99	never	$\geq$0.72	$\geq$0.99

Atmospheric Neutrinos

• Δm^2_{atm} oscillations occur between the states $\nu_\beta \to \nu_\gamma$

$$\nu_\beta = s_{23} c_{24} \nu_s + c_{23} \nu_\mu - s_{23} s_{24} \nu_\tau \quad \text{and} \quad \nu_\gamma = s_{24} \nu_s + c_{24} \nu_\tau$$

with mixing angle ϑ_{34}

• Cases

- * $c_{23} = 1$ $(U_{\mu 1} = U_{\mu 2} = 0) \to \nu_\beta = \nu_\mu$
  - $c_{23} = c_{24} = 1 \to$ pure $\nu_\mu \to \nu_\tau$ oscillations
  - $c_{23} = 1, c_{24} = 0 \to$ pure $\nu_\mu \to \nu_s$ oscillations
  - $0 < c_{24} < 1 \to$ simultaneous $\nu_\mu \to \nu_s$ and $\nu_\mu \to \nu_\tau$
- * $c_{23} = 0 \to$ no atmospheric ν_μ oscillations

• $c^2_{24} \equiv \tau$–sterile admixture

• $c^2_{23} = |U_{\mu 3}|^2 + |U_{\mu 4}|^2 \equiv \nu_\mu$ projection on Δm^2_{atm} oscillations

$\Rightarrow$ s^2_{23} limited by the atmospheric analysis.

$\Rightarrow$ Also ν_μ disappearance searches at CDHSW $\Rightarrow$ $s^2_{23} \lesssim 0.2$

– In the general case survival probabilities obtained numerically with Earth matter potential

$$A \equiv (s^2_{24} - s^2_{23} c^2_{24})\, A_{NC}$$

– Same analysis but with 4 parameters: Δm^2_{12}, θ_{12}, and c^2_{23}, c^2_{24}

– In the general case $-\frac{\pi}{2} \leq \vartheta_{34} \leq \frac{\pi}{2}$

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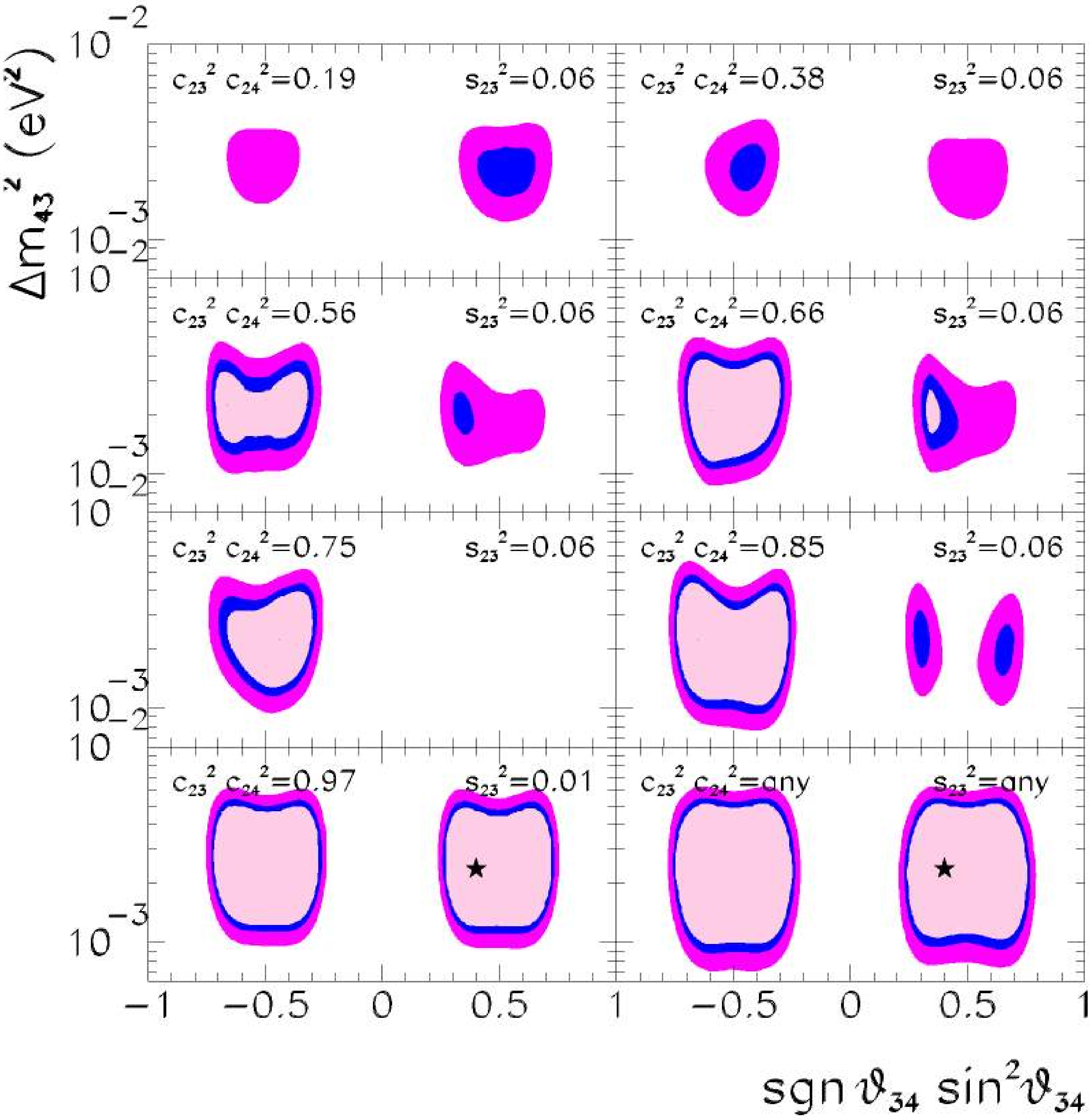
Varying both ϑ_{24} and ϑ_{23}



– Best fit almost pure ν_μ–ν_τ

$$|U_{s1}|^2 + |U_{s2}|^2 = c^2_{23}c^2_{24} = 0.97$$

$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 = s^2_{23} = 0.01$$

$$\Delta m^2_{43} = 2.4 \times 10^{-3}\ \text{eV}^2$$

$$\vartheta_{34} = 39^\circ$$

Solutions for Atmospheric 4–ν Oscillations

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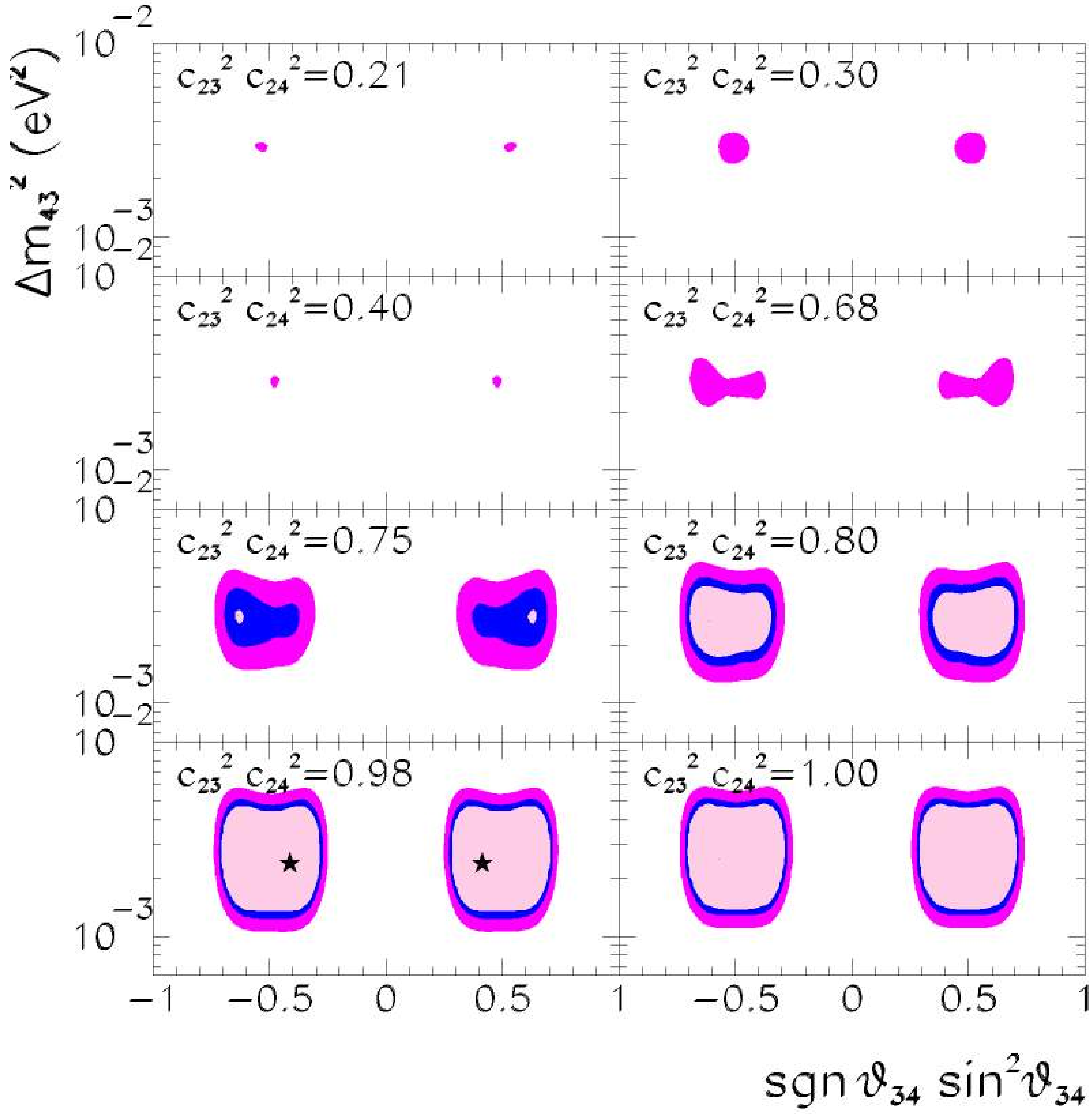
Fixing $\vartheta_{23} = 0$ (“restricted” case)



Solutions for Atmospheric 4–ν Oscillations

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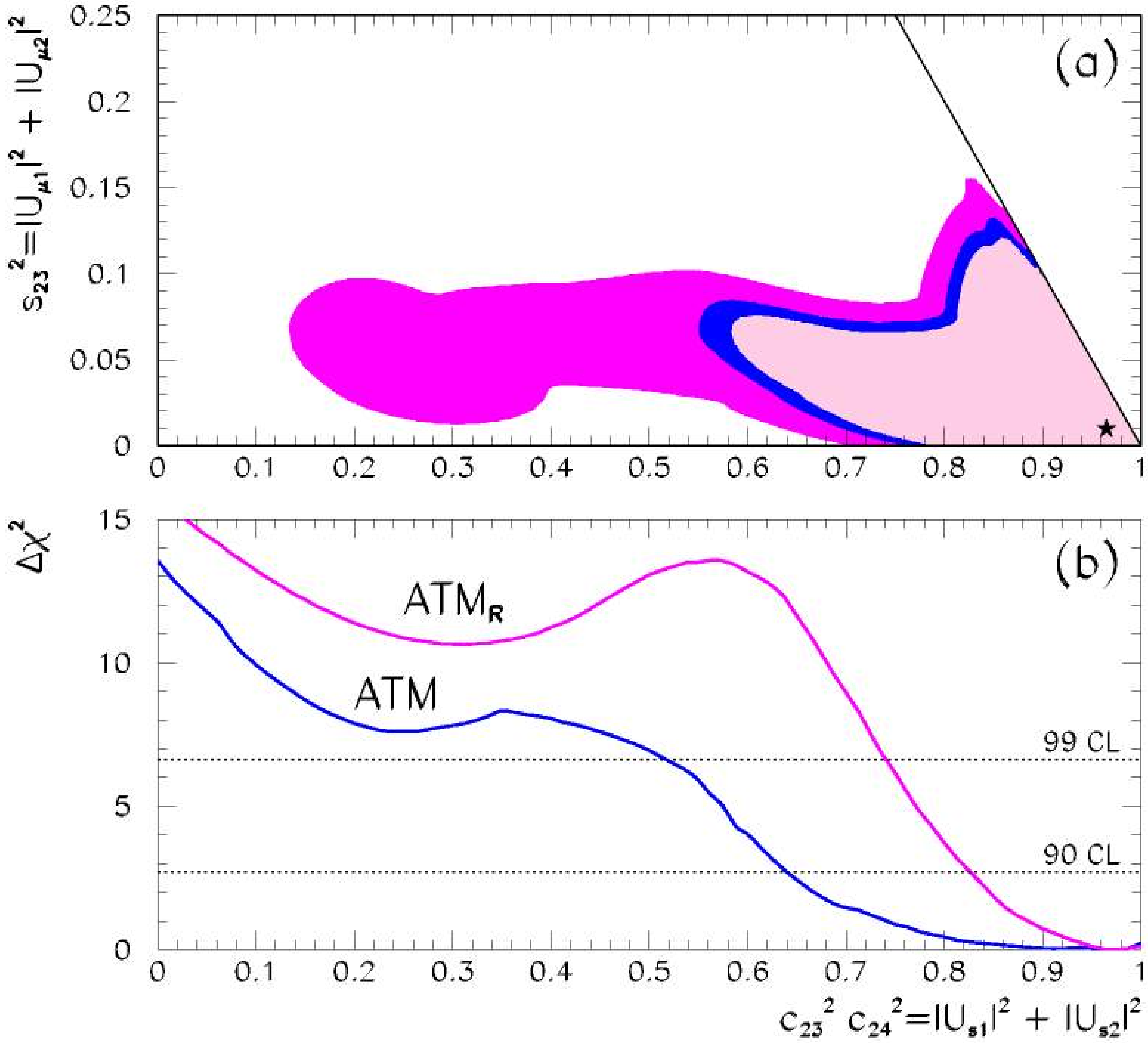

- ν_β =close-to-pure ν_μ ($s_{23}^2 \lesssim 0.12(0.16)$ at 90 (99%))
- this oscillates into a state which is preferably composed by ν_τ, but a 48% admixture of sterile still allowed at 99% CL
- "restricted" case $s_{23}=0 \Rightarrow$ sterile admixture tightened by ~ 2
- flatter SK upgoing muon data at $-0.7 \leq \cos\theta \leq -0.4 \Rightarrow$ lost of sensitivity to large active–sterile admixture

Solutions for Solar and Atmospheric 4–ν Oscillations

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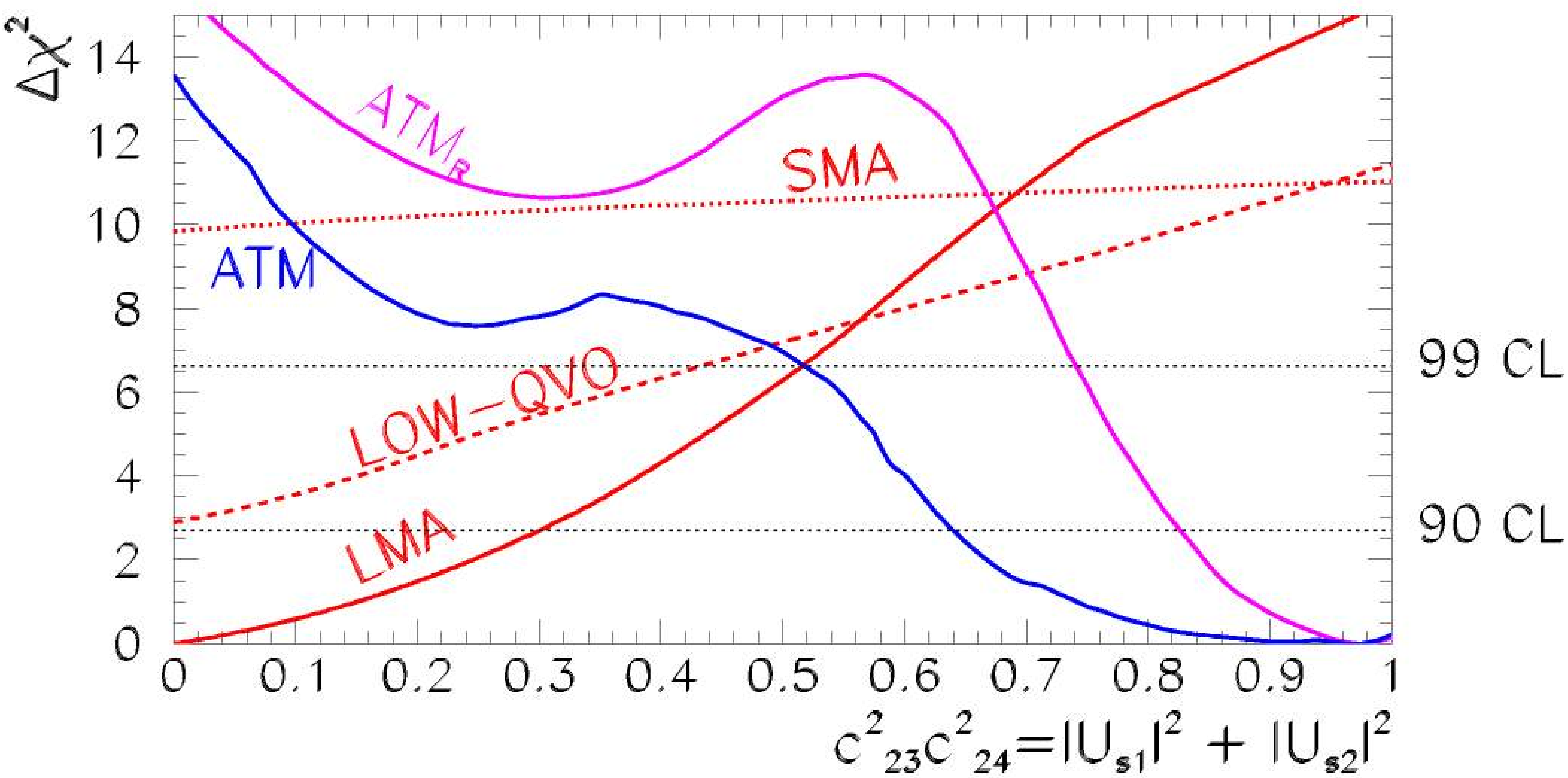
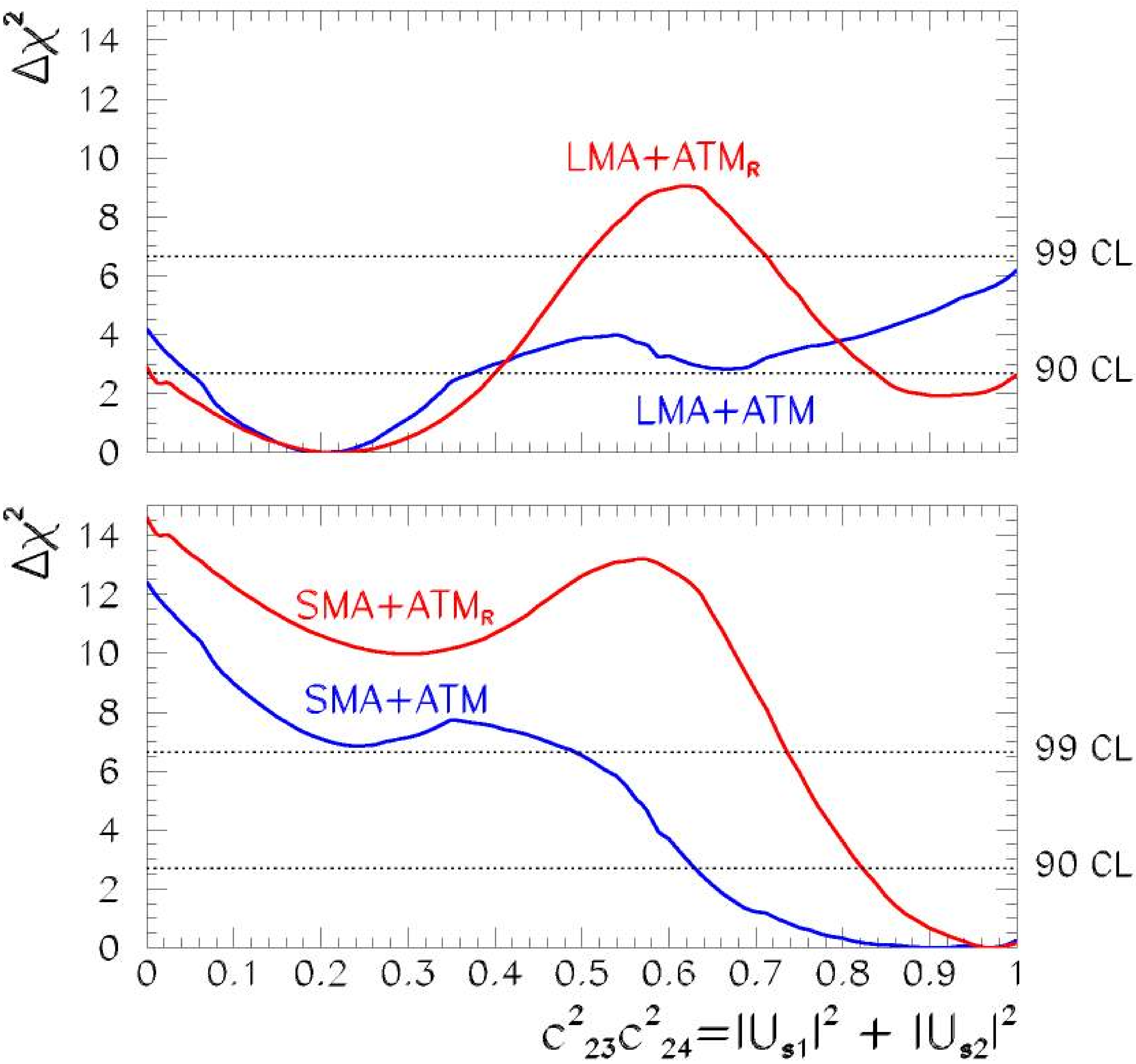


- Naively:
  - Solar Analysis $\Rightarrow c^2_{23}c^2_{24} < 0.52$ at 99% C
  - Atmospheric Analysis $\Rightarrow c^2_{23}c^2_{24} > 0.52$ at 99% CL

$\Rightarrow$ 2+2 scenarios are valid at 99% CL

But if combined properly...

Combined Analysis



Scenario	Best fit	$(s^2_{23})_{\rm min}$	$(c^2_{23}c^2_{24})_{\rm min}$	$\chi^2_{\rm min}$ (GOF)
ATM+SOL$_{\rm LMA}$	CAS+CSA	0.065	0.21	73.8 (68%)
ATM$_{\rm R}$+SOL$_{\rm LMA}$	CAS+CSA	0.	0.22	77.4 (56%)
ATM+SOL$_{\rm SMA}$	NPSS+NPAA	0.030	0.91	75.4 (62%)
ATM$_{\rm R}$+SOL$_{\rm SMA}$	NPSS+NPAA	0.	0.98	75.5 (62%)

Conclusions

- Maximal active–sterile (MAS) admixture

$$|U_{s1}|^2 + |U_{s2}|^2 = c_{23}^2 c_{24}^2 = 0.5$$

always disfavoured

- Two favourite configurations (but not very deep in minima)

  * CAS+CSA: close-to-active solar neutrino oscillations plus close-to-sterile atmospheric neutrino oscillations

$$|U_{s1}|^2 + |U_{s2}|^2 \sim 0.18\text{–}0.2$$

    – active-sterile admixture controlled by solar data
    – *favoured* when solar is LMA
    – *favoured* quality of global fit
    – MAS ruled out at $\sim$ 95 % CL
    – secondary minimum at complementary CSS+CAA scenario

  * NPSS+NPAA: near-pure-sterile solar neutrino oscillations + near-pure-active atmospheric neutrino oscillations

$$|U_{s1}|^2 + |U_{s2}|^2 \sim 0.91\text{–}0.97$$

    – active-sterile admixture controlled by atmospheric data
    – *favoured* when solar is SMA
    – *worse* quality of global fit
    – MAS ruled out at more than 99 % CL