QED CORRECTIONS TO THE SCATTERING OF SOLAR ν_s AND ELECTRONS

M. PASSERA

ITP - U. BERN

LES HOUCHES, June 19 2001

R.C. to $\nu - e^-$ SCATTERING

A LONG STORY!

1964 : T.D. LEE - A. SIRLIN

- M. RAM (1966)
- 't HOOFT (1971)
- SALOMONSON - UEDA (75)
  ZHIZHIN - KONOPLICH - NIKITIN - RODIONOV (75)
  GREEN - VELTMAN (80)
  MARCIANO - SIRLIN (80)
  AOKI - HIOKI - KAWABE - KONUMA - MUTA (81)
  SARANTAKOS - SIRLIN - MARCIANO (83)
  BARDIN - DOKUCHAEVA (84)
  BERNABEU - BILENKY - BOTELLA - SEGURA (94)
  \- - - - -
  \- - - - -

SOLAR NEUTRINOS

- J. N. BAHCALL (1987): R.M. PHYS. 59 505

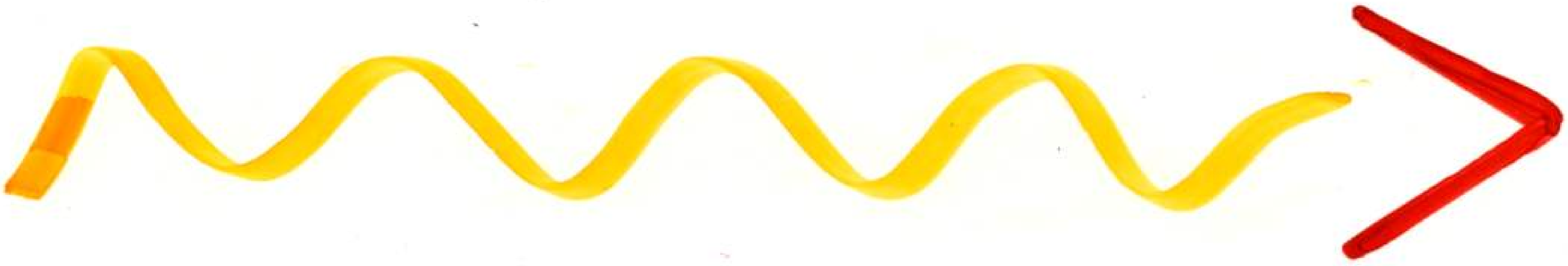
  TOTAL CROSS SECTIONS,
  ENERGY SPECTRA,
  ANGULAR DISTRIBUTIONS.

- BAHCALL - KAMIONKOWSKI - SIRLIN (1995): PRD 51 6146

  R.C. ARE IMPORTANT FOR THE ANALYSIS of PRECISE SOLAR ν-e^- SCATTERING EXPERIMENTS...

• BACK TO QED...

DO PRECISE EXPS.
REALLY MEASURE
THE ELECRON RECOIL
ENERGY??



γ

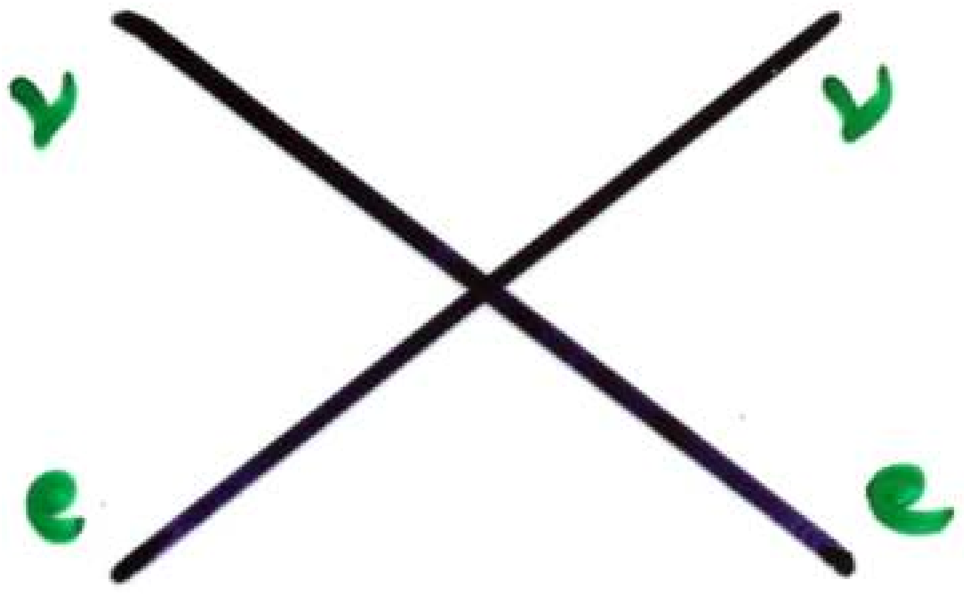
"QED" CORRECTIONS

$\nu_{\mu,\tau} + e^- \rightarrow \nu_{\mu,\tau} + e^-$ NC

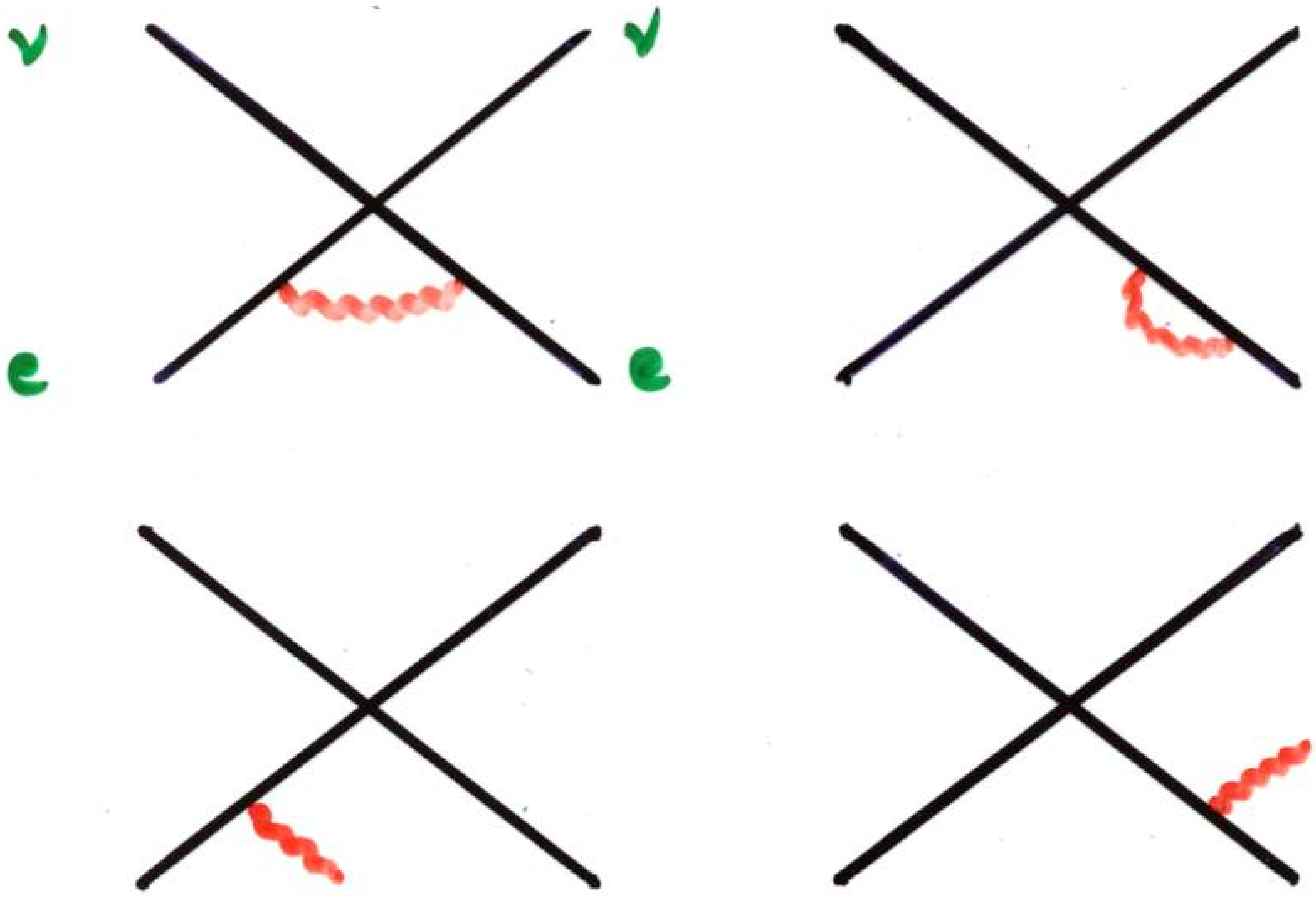
$\nu_e + e^- \rightarrow \nu_e + e^-$ NC+CC

L.O.

$\frac{|q^2|}{M_W^2} \ll 1$



QED



V

B—ε—
H
S

L.O. SM PREDICTION

$$\frac{d\sigma}{dE} = \frac{2 m G_\mu^2}{\pi} \Big\{ \underbrace{g_L^2}_{\text{"L"}} + \underbrace{g_R^2 (1-z)^2}_{\text{"R"}} - \underbrace{g_L g_R \frac{m z}{\nu}}_{\text{"LR"}} \Big\}$$

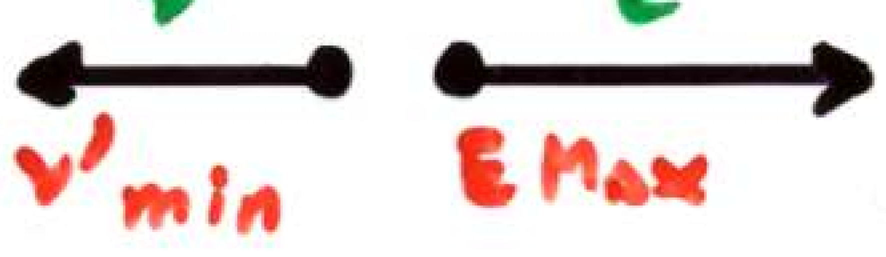
$$G_\mu = 1.16637(1) \times 10^{-5} \ \mathrm{GeV}^{-2}$$

$$g_L = \sin^2\theta_W \pm \tfrac{1}{2} \begin{cases} \nu_e \\ \nu_{\mu,\tau} \end{cases}$$

$$g_R = \sin^2\theta_W$$

$$z = \frac{E-m}{\nu}$$

$$E \in \left[m, \frac{m^2 + (2\nu+m)^2}{2(2\nu+m)} \right]$$



VIRTUAL CORRECTIONS

$$\left.\frac{d\sigma}{dE}\right|_V = \frac{2mG_\mu^2}{\pi}\,\frac{\alpha}{\pi}\,\delta(E,\nu)$$

- $$\begin{aligned}\delta(E,\nu) = \; & g_L^2\left\{V_1(E) + V_2(E)\left[z-1-\frac{mz}{2\nu}\right]\right\} \\ & + g_R^2\left\{V_1(E)(1-z)^2 + V_2(E)\left[z-1-\frac{mz}{2\nu}\right]\right\} \\ & - g_L g_R\left\{\left[V_1(E)-V_2(E)\right]\frac{mz}{\nu} \right. \\ & \left. \qquad + 2V_2(E)\left[z-1-z^2\right]\right\}\end{aligned}$$

- $$\begin{aligned}V_1(E) = \; & \left(2\ln\frac{m}{\lambda}\right)\left[1-\frac{E}{2\ell}\ln\frac{E+\ell}{E-\ell}\right] - 2 \\ & - \frac{E}{\ell}\left[\mathrm{Li}_2\left(\frac{\ell-E+m}{2\ell}\right) - \mathrm{Li}_2\left(\frac{\ell+E-m}{2\ell}\right)\right] \\ & + \frac{1}{4\ell}\left[3E+m-E\ln\frac{2E+2m}{m}\right]\ln\frac{E+\ell}{E-\ell}\end{aligned}$$

- $$V_2(E) = \frac{m}{4\ell}\ln\frac{E+\ell}{E-\ell}$$

$$\ell = (E^2-m^2)^{1/2} \qquad\qquad \mathrm{Li}_2(x) = -\int_0^x dt\,\frac{\ln(1-t)}{t}$$

L and R parts agree with LS'64

SOFT BREMSS. CORRECTIONS

$$\left.\frac{d\sigma}{dE}\right|_{SB} = \frac{\alpha}{\pi}\, I_\gamma(E,\epsilon)\, \left.\frac{d\sigma}{dE}\right|_0$$

LEE, SIRLIN, REV. MOD. PHYS. 36 (1964) 666

$$I_\gamma(E,\epsilon) = 2\ln\frac{\lambda}{\epsilon}\left[1 - \frac{E}{2\ell}\ln\frac{E+\ell}{E-\ell}\right] + \frac{E}{2\ell}\left\{L\left(\frac{E+\ell}{E-\ell}\right) - L\left(\frac{E-\ell}{E+\ell}\right) + \ln\frac{E+\ell}{E-\ell}\left[1-2\ln\frac{\ell}{m}\right]\right\} + 1 - 2\ln 2$$

$$L(x) = -\mathrm{Re}\left[Li_2(x)\right], \quad x \in \mathbb{R}$$

- VALID for $\epsilon \ll m$
- IR DIVERGENCE CANCELS

HB and TOTAL QED CORRECTIONS

$$\nu_\ell + e^- \to \nu_\ell + e^- \ (+\gamma)$$

$$\left.\frac{d\sigma}{dE}\right|_{SM} = \frac{2 m G_\mu^2}{\pi} \Big[$$

$$g_L^2(E)\left[1 + \frac{\alpha}{\pi} f_L(E,\nu)\right] +$$

$$g_R^2(E)\,(1-z)^2\left[1 + \frac{\alpha}{\pi} f_R(E,\nu)\right] -$$

$$g_L(E)\,g_R(E)\,\frac{mz}{\nu}\left[1 + \frac{\alpha}{\pi} f_{LR}(E,\nu)\right] \Big]$$

$$f_X(E) = \underbrace{f_X^{V}(E) + f_X^{SB}(E,\epsilon)}_{f_X^{VS}(E,\epsilon)} + f_X^{HB}(E,\epsilon)$$

L and R : H. RAM (SMALL ϵ LIMIT)

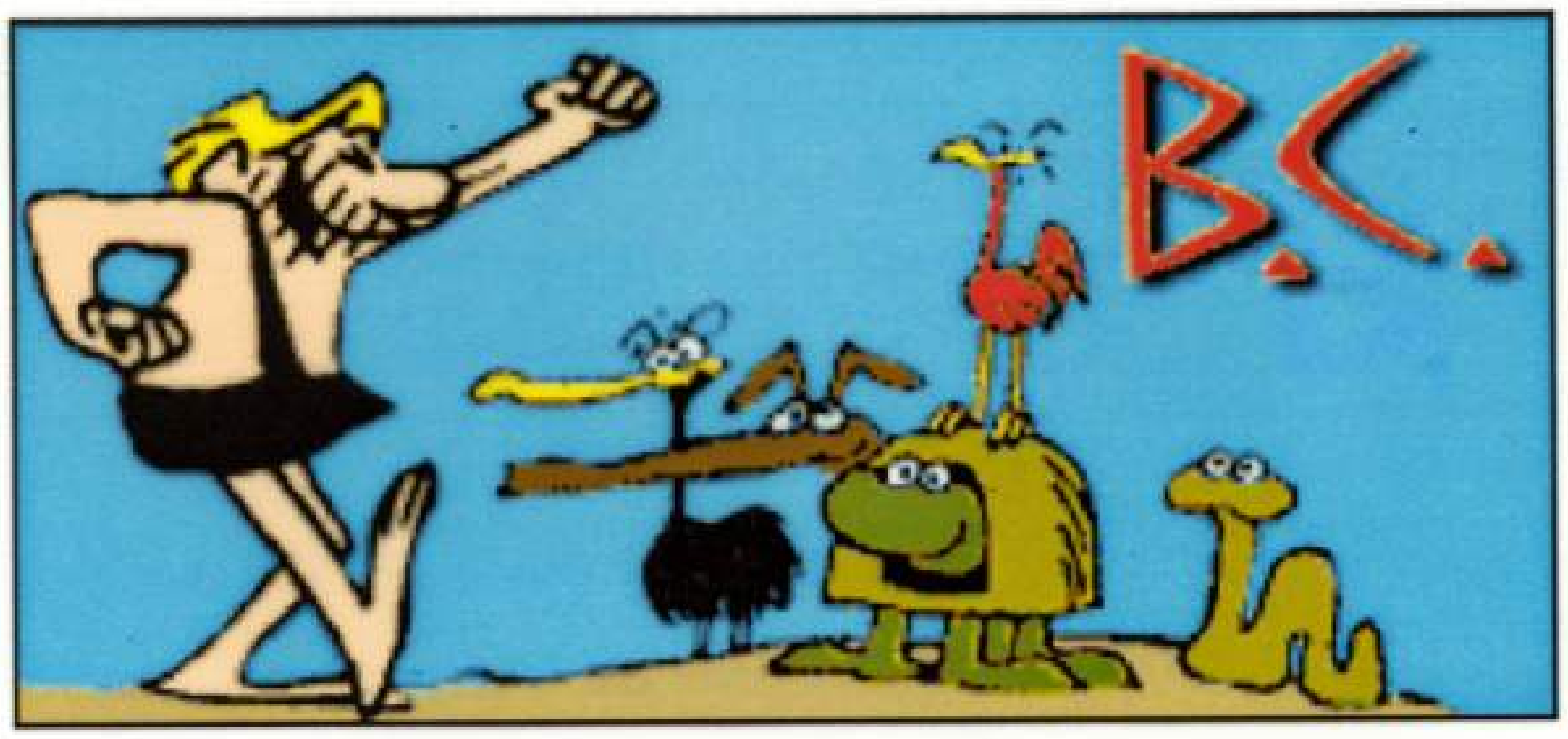
LR : Not PREVIOUSLY COMPUTED

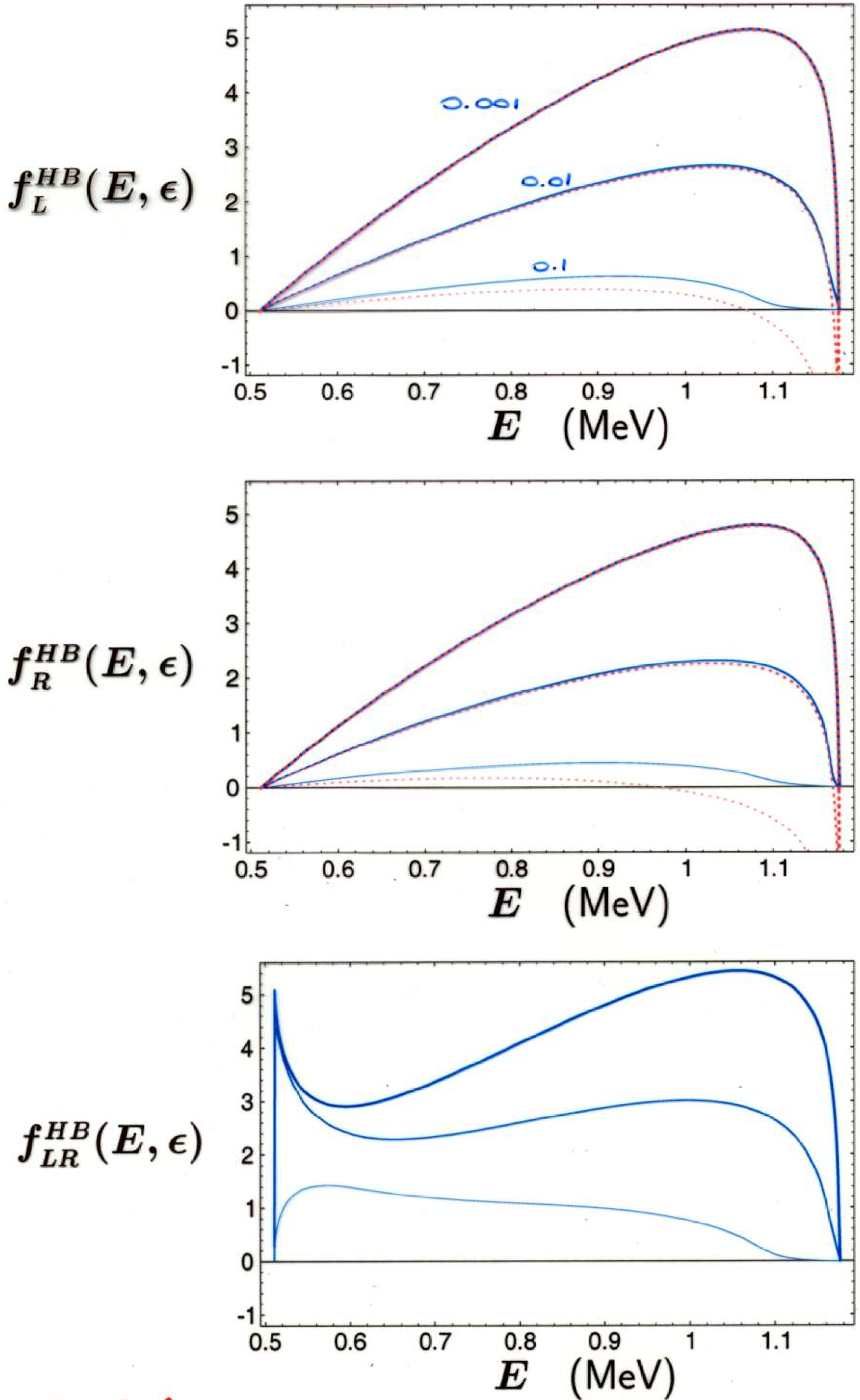
BC

MATHEMATICA – FORTRAN CODE
(USES FEYNCALC AND VEGAS)
TO COMPUTE:

- HB (L, R, LR) ANY ϵ
- TOTAL QED
- "EXACT" VS

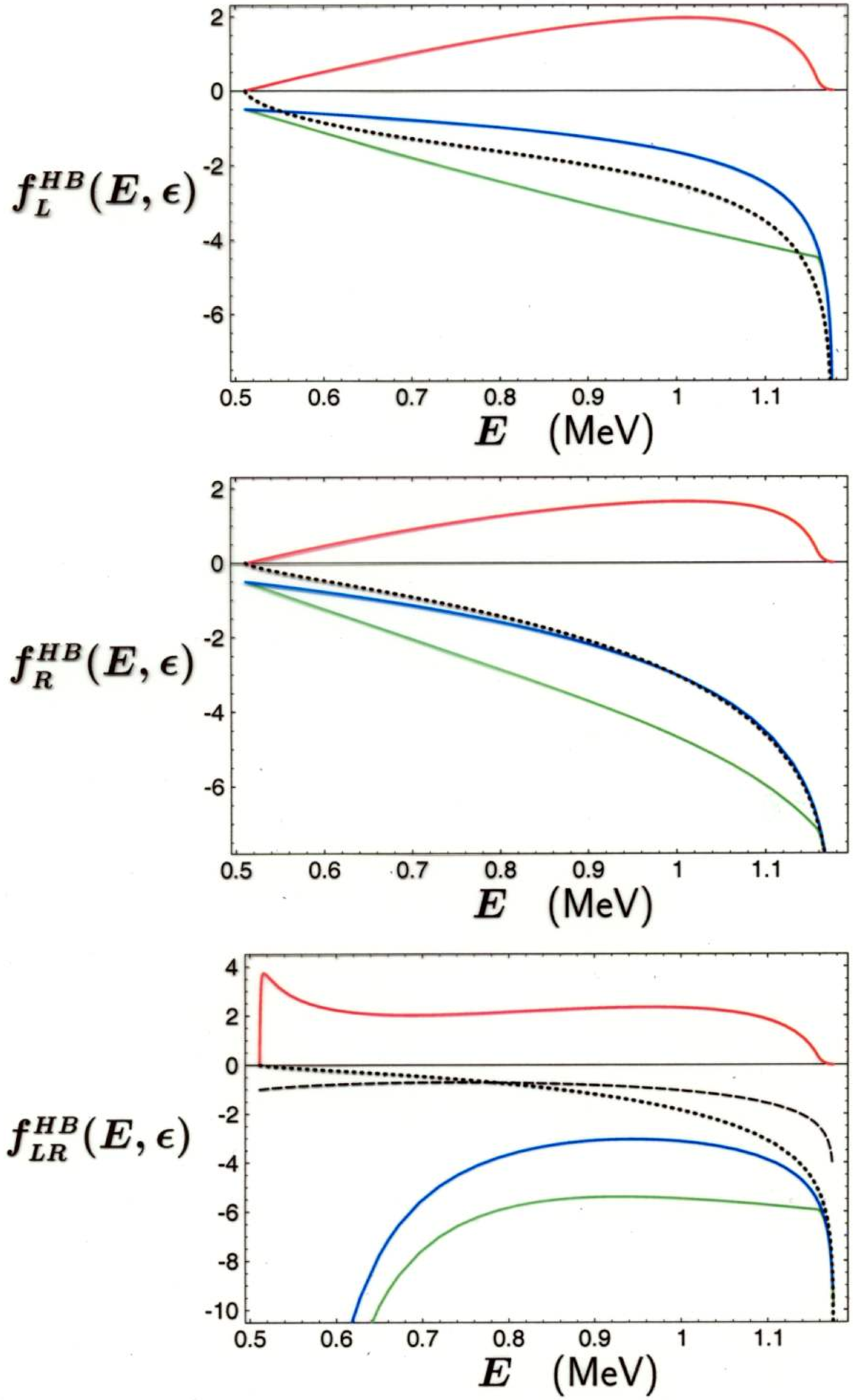
$\longrightarrow$ VS = QED − HB




$f_L^{HB}(E,\epsilon)$
0.001
0.01
0.1
E (MeV)
$f_R^{HB}(E,\epsilon)$
E (MeV)
$f_{LR}^{HB}(E,\epsilon)$
E (MeV)

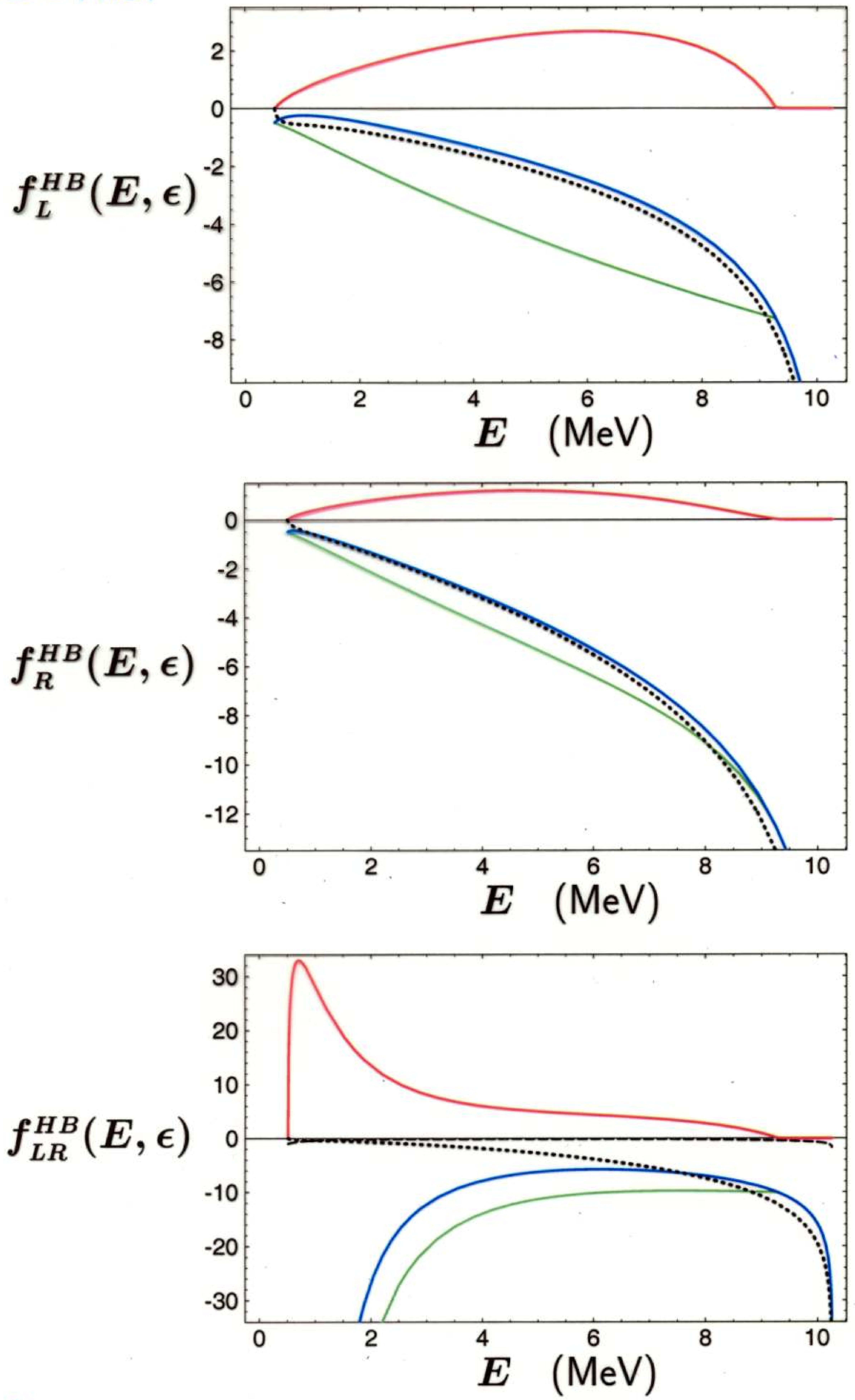
ν = 862 KeV

$\epsilon = 0.02$ MeV



$\nu = 862$ KeV

$\epsilon = 1$ MeV



$\nu = 10$ MeV

SPECTRUM of the COMBINED ENERGY of e^- and γ.

HB STUDY THE ENERGY SPECTRUM of FINAL ν

Define:

$$h(\nu', \epsilon) \equiv \left.\frac{d\sigma}{d\nu'}\right|_{HB} \qquad \bar{h}(E+\omega, \epsilon) \equiv \left.\frac{d\sigma}{d(E+\omega)}\right|_{HB}$$

Such that

$$\int_0^{\nu-\epsilon} h(\nu', \epsilon)\, d\nu' = \int_{m+\epsilon}^{m+\nu} \bar{h}(E+\omega, \epsilon)\, d(E+\omega).$$

$m + \nu = E + \omega + \nu'$ implies

$$\bar{h}(E+\omega, \epsilon) = h(\nu + m - (E+m), \epsilon)$$

COMPUTED BY **BC**

CHECK:

$$\sigma_{HB}(\nu,\epsilon) = \int_{m}^{E_{max}} \frac{d\sigma}{dE}\, dE$$

$$= \int_{m+\epsilon}^{\nu+m} \frac{d\sigma}{d(E+\omega)}\, d(E+\omega)$$

✓

HOW TO COMBINE V, SB and HB?

Define

$$E_\omega \equiv \begin{cases} E+\omega & \text{if } \omega \geq \epsilon \\ E & \text{if } \omega < \epsilon \end{cases}$$

$E \in [m, E_{max}]$

$\omega \in [0, \nu] \longrightarrow$

$E_\omega \in [m, m+\nu]$

... IN SUMMARY

$$\left.\frac{d\sigma}{dE_\omega}\right|_{SM} = \frac{2mG_\mu^2}{\pi}\Big[$$

$$g_L^2(E_\omega)\left(\vartheta + \frac{\alpha}{\pi}\bar{f}_L(E_\omega,\nu)\right) +$$

$$g_R^2(E_\omega)(1-z_\omega)^2\left(\vartheta + \frac{\alpha}{\pi}\bar{f}_R(E_\omega,\nu)\right) -$$

$$g_L(E_\omega)g_R(E_\omega)\frac{mz_\omega}{\nu}\left(\vartheta + \frac{\alpha}{\pi}\bar{f}_{LR}(E_\omega,\nu)\right)\Big]$$

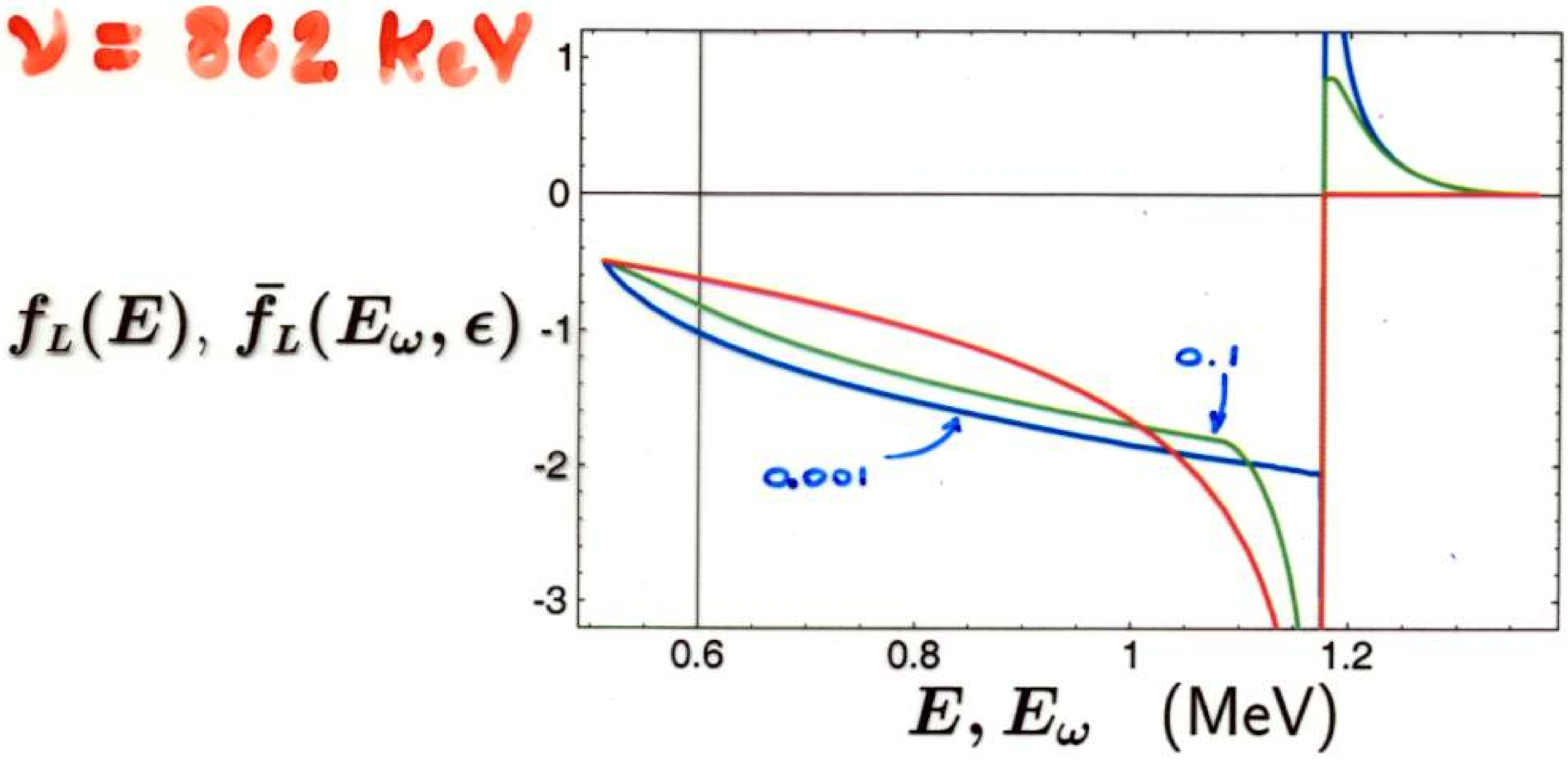
$$z_\omega = \frac{E_\omega - m}{\nu} \qquad \vartheta = \vartheta(E_{max} - E_\omega)$$

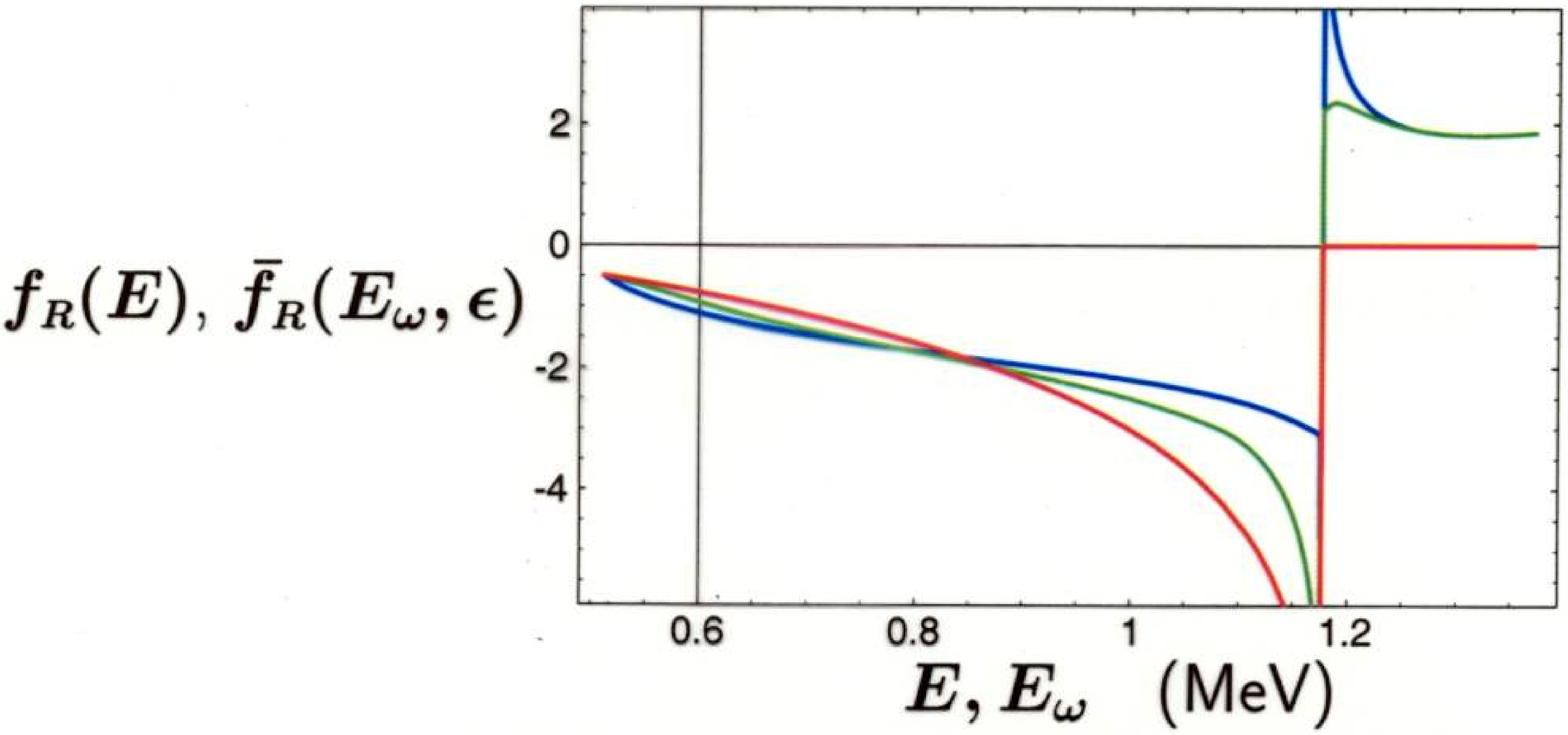
$$\bar{f}_x(E_\omega,\epsilon) = f_x^{VS}(E_\omega,\epsilon) + \bar{f}_x^{HB}(E_\omega,\epsilon)$$

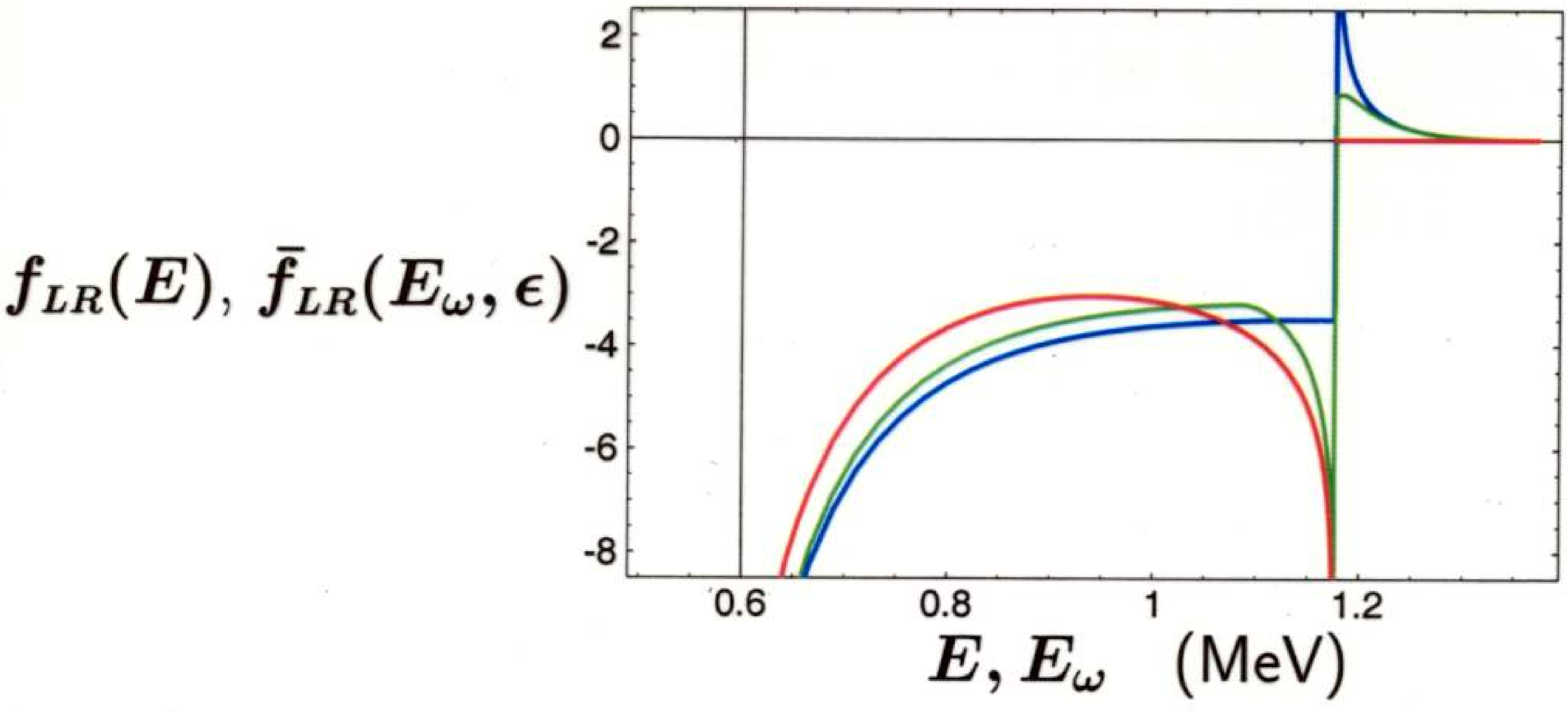
from $\left.\frac{d\sigma}{dE}\right|_{VS}^{\text{"EXACT"}}$

from $\left.\frac{d\sigma}{d\nu'}\right|_{HB}^{\text{"EXACT"}}$

$\nu = 862$ KeV

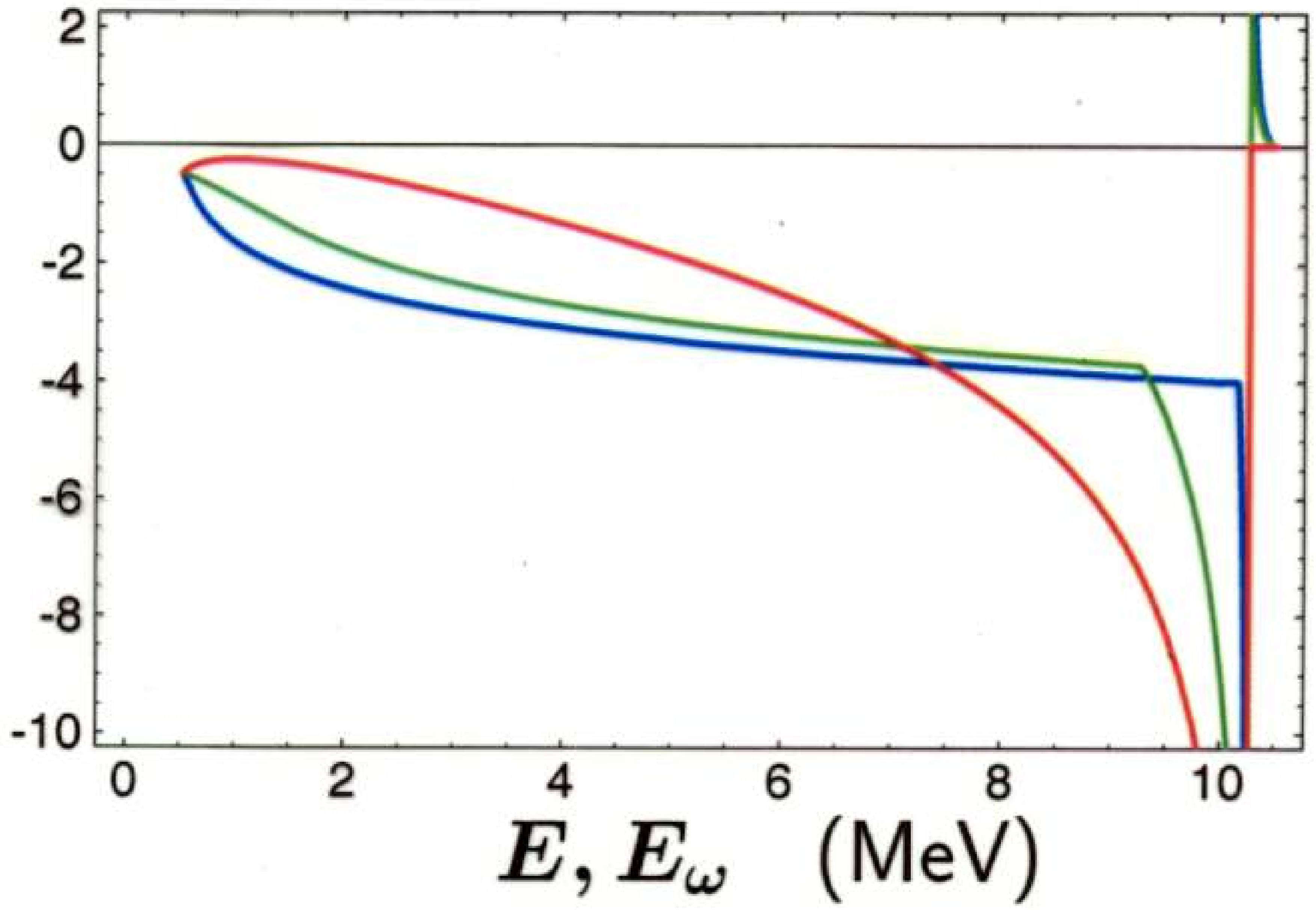

$f_L(E),\ \bar{f}_L(E_\omega, \epsilon)$
1
0
-1
-2
-3
0.1
0.001
0.6
0.8
1
1.2
E, E_ω (MeV)


$f_R(E),\ \bar{f}_R(E_\omega, \epsilon)$
2
0
-2
-4
0.6
0.8
1
1.2
E, E_ω (MeV)

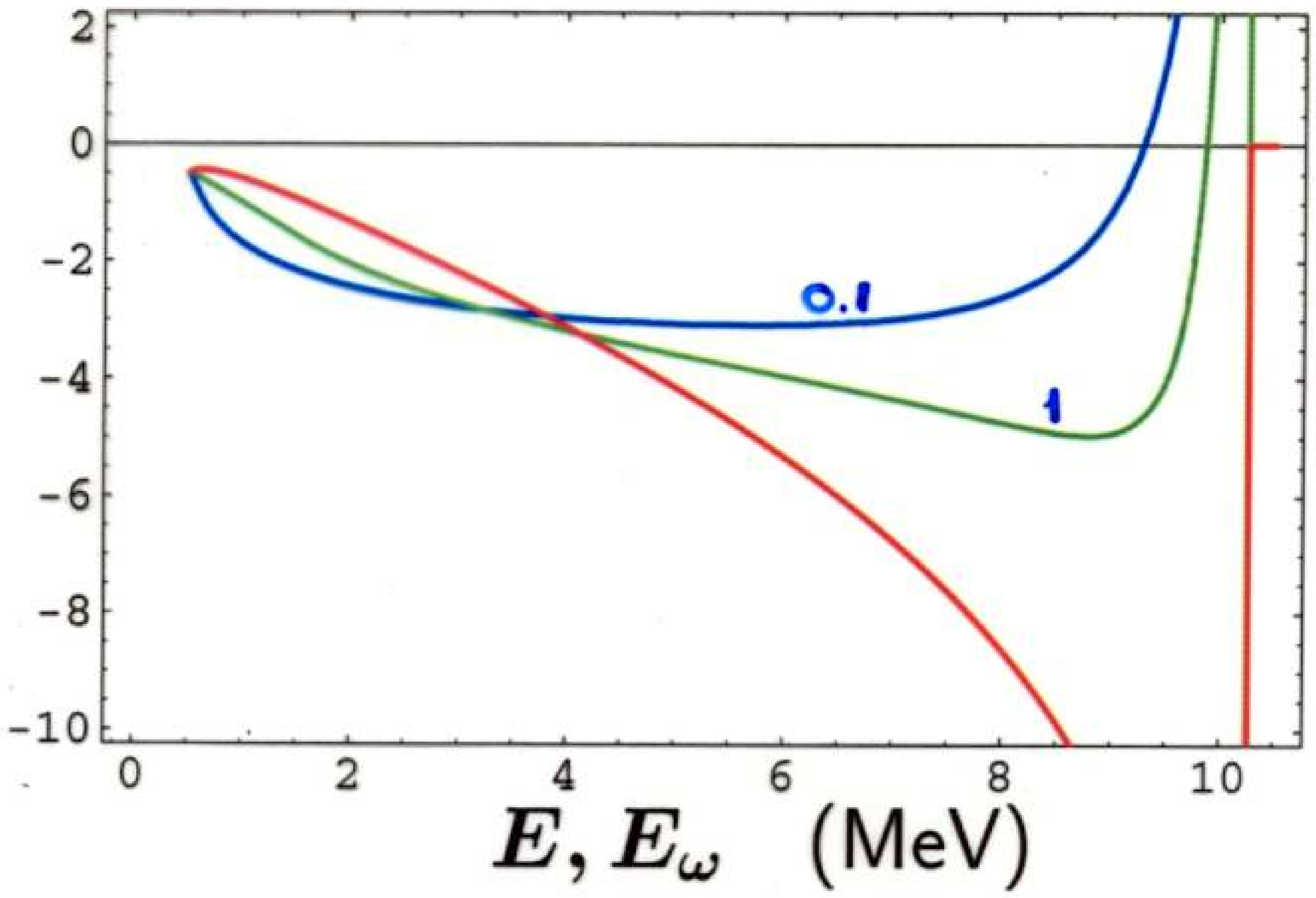

$f_{LR}(E),\ \bar{f}_{LR}(E_\omega, \epsilon)$
2
0
-2
-4
-6
-8
0.6
0.8
1
1.2
E, E_ω (MeV)

$\nu = 10$ MeV

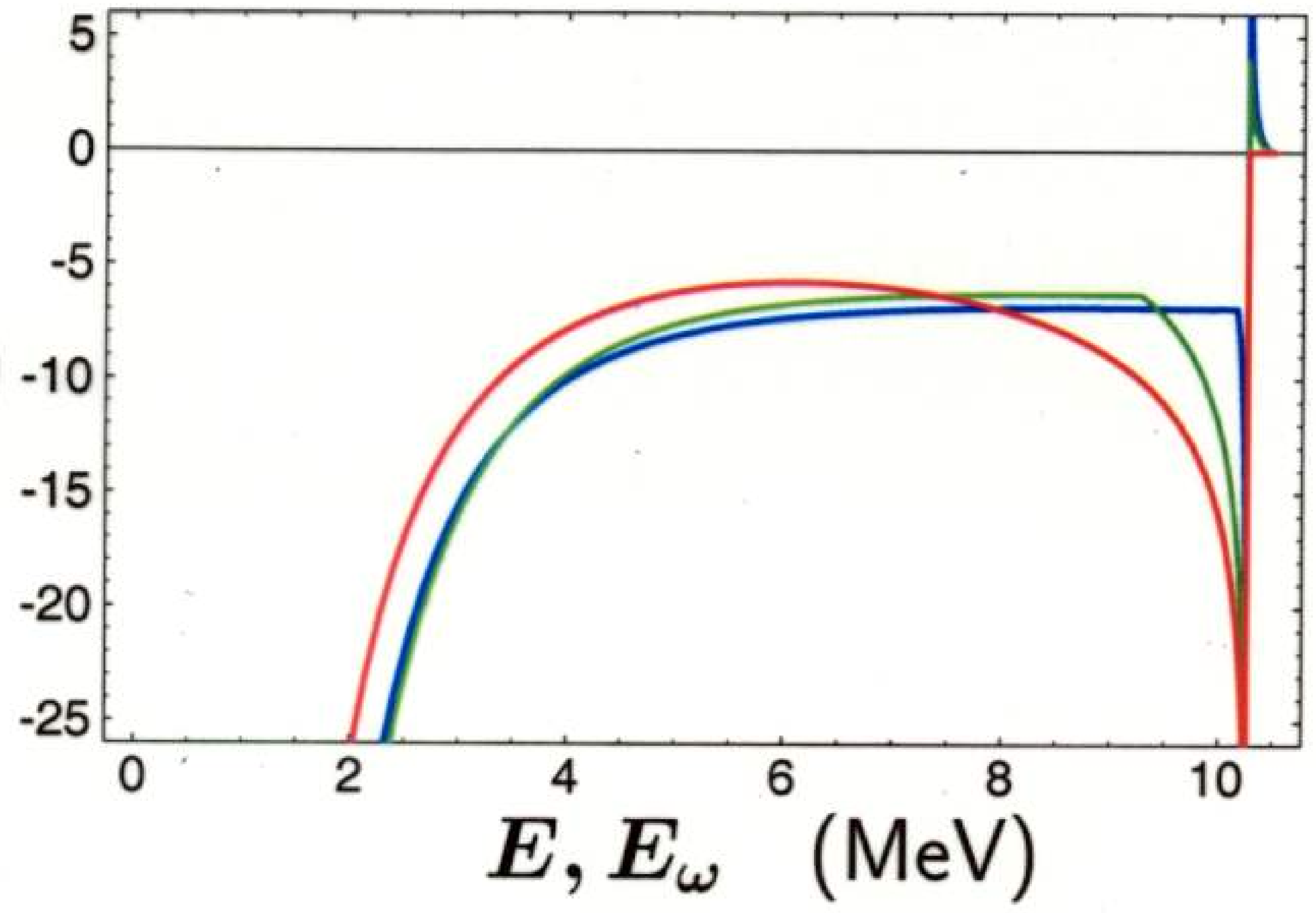
$f_L(E),\ \bar{f}_L(E_\omega, \epsilon)$



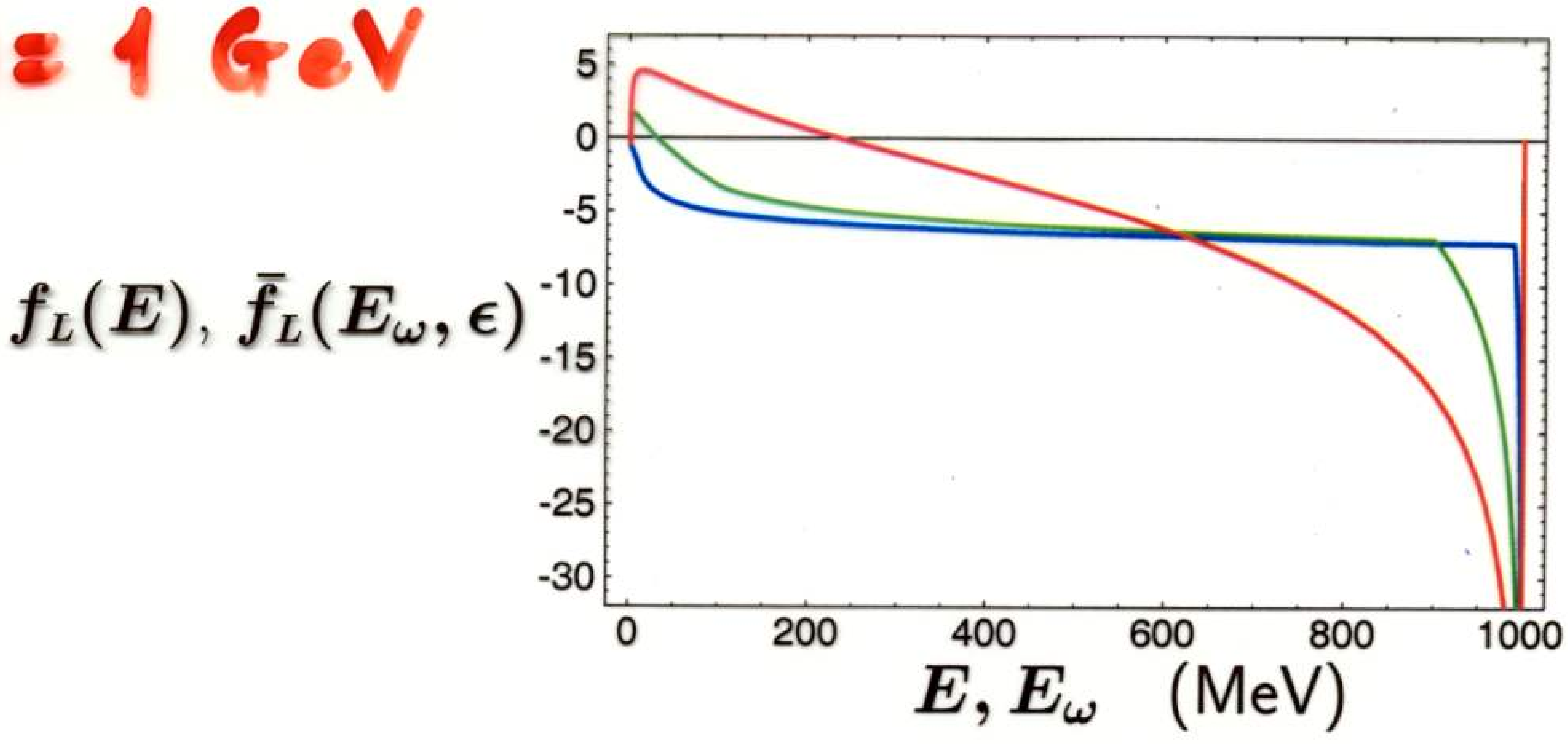
$f_R(E),\ \bar{f}_R(E_\omega, \epsilon)$



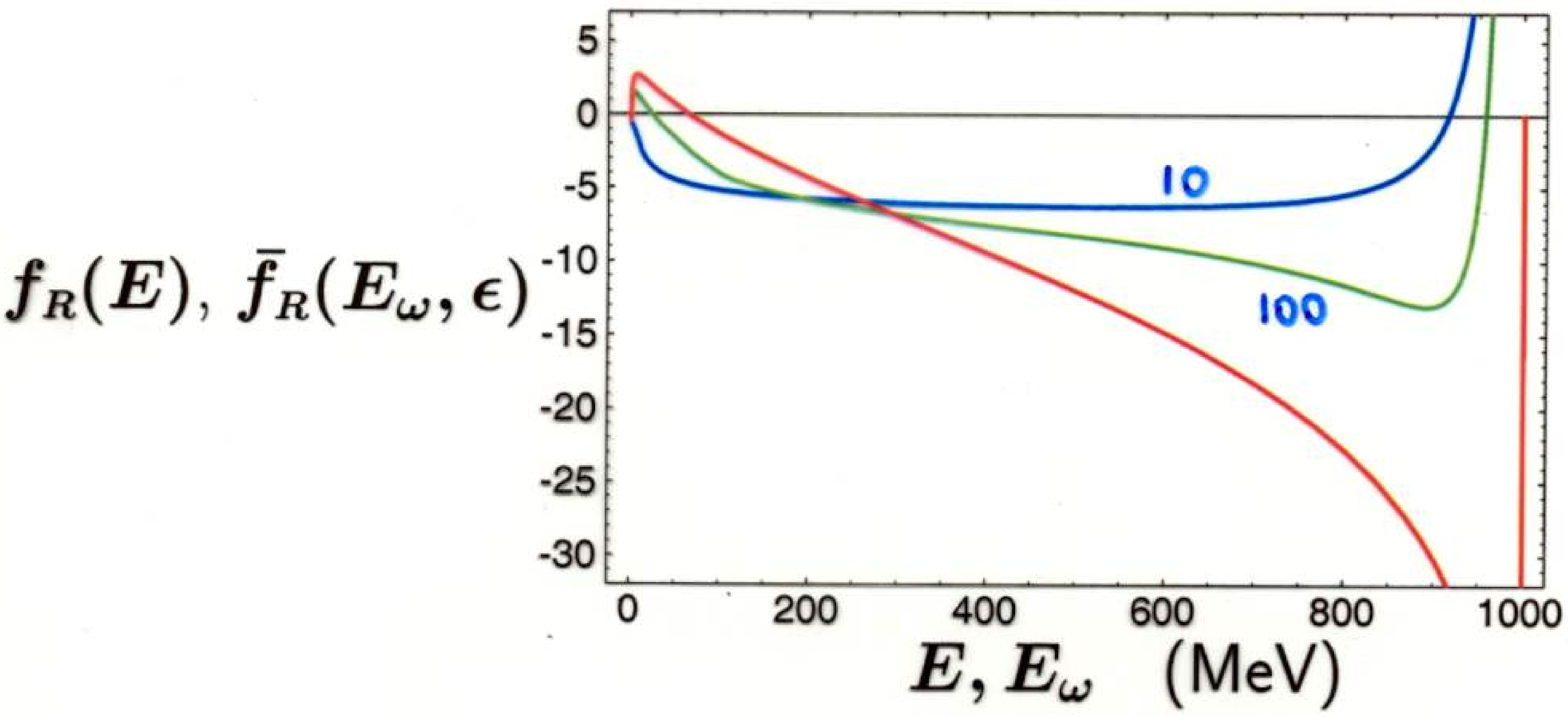
$f_{LR}(E),\ \bar{f}_{LR}(E_\omega, \epsilon)$



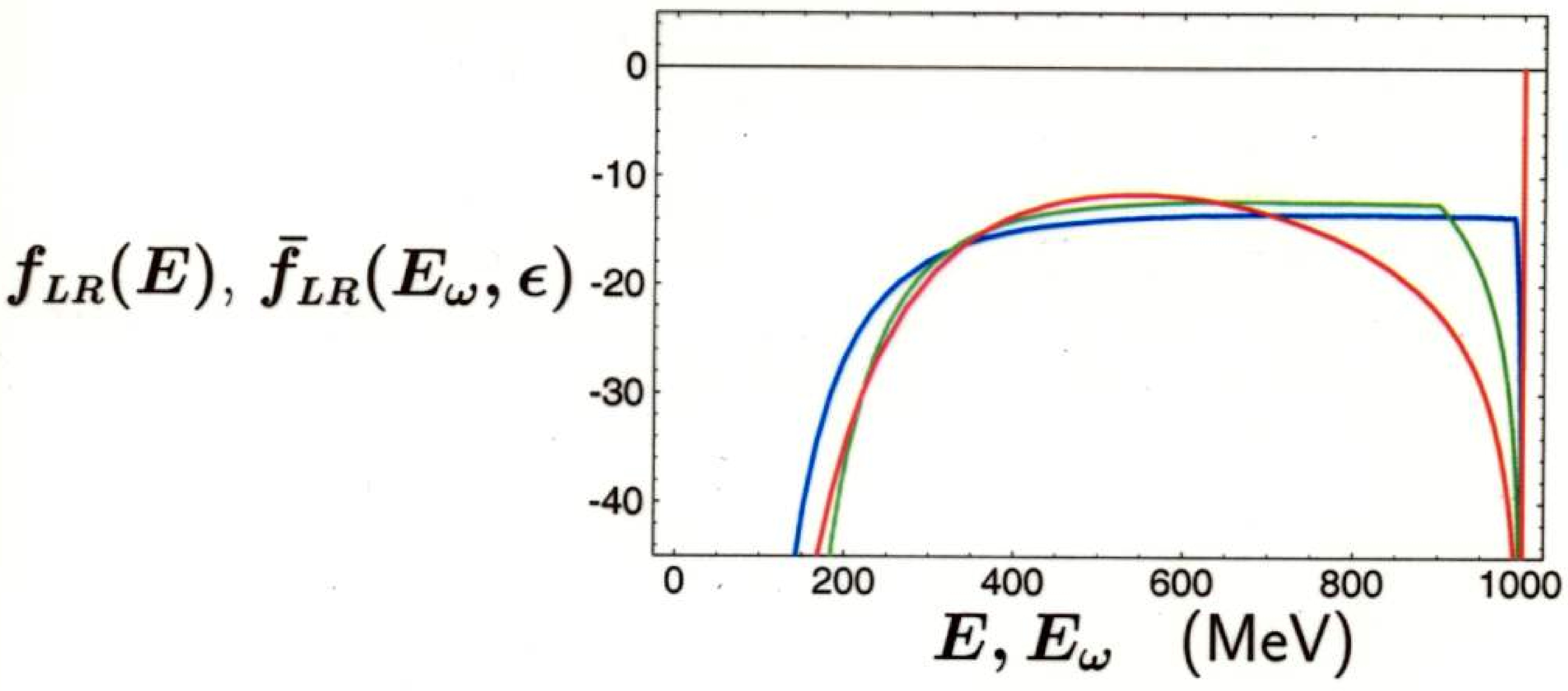
ν = 1 GeV



$f_L(E)$, $\bar{f}_L(E_\omega, \epsilon)$
5
0
-5
-10
-15
-20
-25
-30
0
200
400
600
800
1000
E, E_ω (MeV)



$f_R(E)$, $\bar{f}_R(E_\omega, \epsilon)$
5
0
-5
-10
-15
-20
-25
-30
10
100
0
200
400
600
800
1000
E, E_ω (MeV)



$f_{LR}(E)$, $\bar{f}_{LR}(E_\omega, \epsilon)$
0
-10
-20
-30
-40
0
200
400
600
800
1000
E, E_ω (MeV)

WHEN ARE THESE RESULTS APPLICABLE?

(I) $\dfrac{d\sigma}{dE}$ vs. $\dfrac{d\sigma}{dE_\omega}$ (II)

- (I) IS APPROPRIATE FOR DETECTORS BLIND TO γ OF ALL ENERGIES, BUT ABLE TO MEASURE E PRECISELY.
- $(I)_{HB}$ IS SEPARATELY MEASURABLE IF THE ABOVE DETECTOR CAN ALSO MEASURE γ WITH $\omega > \epsilon$.
- BOTH (I) AND (II) ARE MEASURED BY A DETECTOR ABLE TO DETERMINE PRECISELY E AND HARD γ $(\omega > \epsilon)$, BUT BLIND TO SOFT γ $(\omega < \epsilon)$.

- (II) IS USEFUL WHEN ω CAN'T BE SEPARATELY MEASURED BUT IT FULLY CONTRIBUTES TO THE TOTAL ENERGY MEASUREMENT IF $\omega > \epsilon$.

- THE APPROPRIATE THEORETICAL PREDICTION FOR BOREXINO AND KAMLAND IS (II) WITH SMALL ϵ, BUT THESE CORRECTIONS FOR ^7Be NEUTRINOS ARE $O(\lesssim 1\%)$ ONLY.

- SUPERKAMIOKANDE NEEDS (II) IF WE CAN ASSUME SIMILAR EFFICIENCY IN DETECTING e^- AND γ WITH $\omega > \epsilon$, AND IF ALSO LOW E e^- CONTRIBUTE TO THE TOTAL ENERGY MEASUREMENT. (I) SHOULD NOT BE (BUT IS) USED. FOR ^8B, CORRECTIONS ARE $O(1\%)$.

- $\dfrac{d^2\sigma}{dE\,d\omega}$, $\dfrac{d^3\sigma}{dE\,d\omega\,d\phi}$ MAY BE USEFUL.

CONCLUSIONS

- I HAVE COMPUTED THE $O(\alpha)$ QED CORRECTIONS to $d\sigma/dE$ IN $\nu_\ell + e^- \to \nu_\ell + e^- \; (+\gamma)$ COMPARING the PARTS which HAD BEEN PREVIOUSLY EVALUED

- SB AND HB COMPUTED FOR ARBITRARY PHOTON EN. THR. ε

- STUDIED $d\sigma/dE_\omega$ TO $O(\alpha)$. ANY ε.

- ANALYZED $d\sigma/dE$ VS $d\sigma/dE_\omega$ AND THEIR ROLE IN PRECISE SOLAR ν-e^- SCATTERING EXPs. (BOREXINO, KAMLAND, SUPERK...)