A NuFact Study including all Parameter Correlations

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hep-ph/0105071 M. Freund, P. Huber and M. Lindner

- Analytical formulas for oscillation in matter
- Leading Parameters: Δm^2_{31} and θ_{23}
- Sub-Leading Parameters: θ_{13} and $\mathrm{sgn}\Delta m^2_{31}$
- Small-Δm^2 Effects: Δm^2_{21}, θ_{21} and δ_{CP}
- Conclusions

Mixing Parameters in Matter

Expansion in the mass hierarchy $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$
and assuming small θ_{13}:
(M. Freund, hep-ph/0103300.)

$$\sin^2 2\theta'_{13} = \frac{\sin^2 2\theta_{13}}{\hat{C}^2} + \alpha\,\frac{2\hat{A}(-\hat{A}+\cos 2\theta_{13})\sin^2\theta_{12}\sin^2 2\theta_{13}}{\hat{C}^4}$$

$$\sin 2\theta'_{12} = \alpha\,\frac{2\hat{C}\sin 2\theta_{12}}{|\hat{A}|\cos\theta_{13}\sqrt{2\hat{C}(\mp\hat{A}+\hat{C}\pm\cos 2\theta_{13})}}$$

$$\sin 2\theta'_{23} = \sin 2\theta_{23} + \alpha\cos\delta\,\frac{2\hat{A}\sin 2\theta_{12}\sin\theta_{13}\cos 2\theta_{23}}{\pm 1+\hat{C}\mp\hat{A}\cos 2\theta_{13}}$$

$$\sin\delta' = \sin\delta\left(1-\alpha\,\frac{\cos\delta}{\tan 2\theta_{23}}\,\frac{2\hat{A}\sin 2\theta_{12}\sin\theta_{13}}{\pm 1+\hat{C}\mp\hat{A}\cos 2\theta_{13}}\right)$$

with

$$\hat{C} = \sqrt{(\hat{A}-\cos 2\theta_{13})^2+\sin^2 2\theta_{13}}$$

$$\hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31}$$

$$V = \sqrt{2}G_F n_e.$$

$$\Rightarrow\quad \begin{aligned} \sin^2 2\theta'_{13} &\approx \frac{\sin^2 2\theta_{13}}{\hat{C}^2} \\ \sin^2 2\theta'_{23} &\approx \sin^2 2\theta_{23} \\ \sin\delta' &\approx \sin\delta \\ J'_{CP} &= \frac{\alpha}{|\hat{A}|\hat{C}\cos^2\theta_{13}}J_{CP} \end{aligned}$$

3ν-Formulas in Matter

Probabilities in matter expanded in the mass hierarchy $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ and assuming small θ_{13}:

$$
\begin{aligned}
& P(\nu_\mu \to \nu_\mu) \approx \\
& 1 - \cos^2\theta_{13}\sin^2 2\theta_{23}\sin^2\Delta \\
+ \quad & 2\alpha\cos^2\theta_{13}\cos^2\theta_{12}\sin^2 2\theta_{23}\Delta\cos\Delta
\end{aligned}
$$

$$
\begin{aligned}
& P(\nu_e \to \nu_\mu) \approx \\
& \sin^2\theta_{23}\frac{\sin^2 2\theta_{13}}{(\hat{A}-1)^2}\sin^2((\hat{A}-1)\Delta) \\
\pm \quad & \alpha\,\frac{\sin\delta_{\rm CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})}\sin(\Delta)\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta) \\
+ \quad & \alpha\,\frac{\cos\delta_{\rm CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})}\cos(\Delta)\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta) \\
+ \quad & \alpha^2\frac{\cos^2\theta_{23}\sin^2 2\theta_{12}}{\hat{A}^2}\sin^2(\hat{A}\Delta)
\end{aligned}
$$

$$\Delta = \Delta m^2_{31}L/4E \qquad \hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31} \qquad V = \sqrt{2}G_F n_e.$$

M. Freund, hep-ph/0103300.

A. Cervera *et al.*, Nucl. Phys. **B579** (2000) 17.

Framework

Simulation of event rate spectra

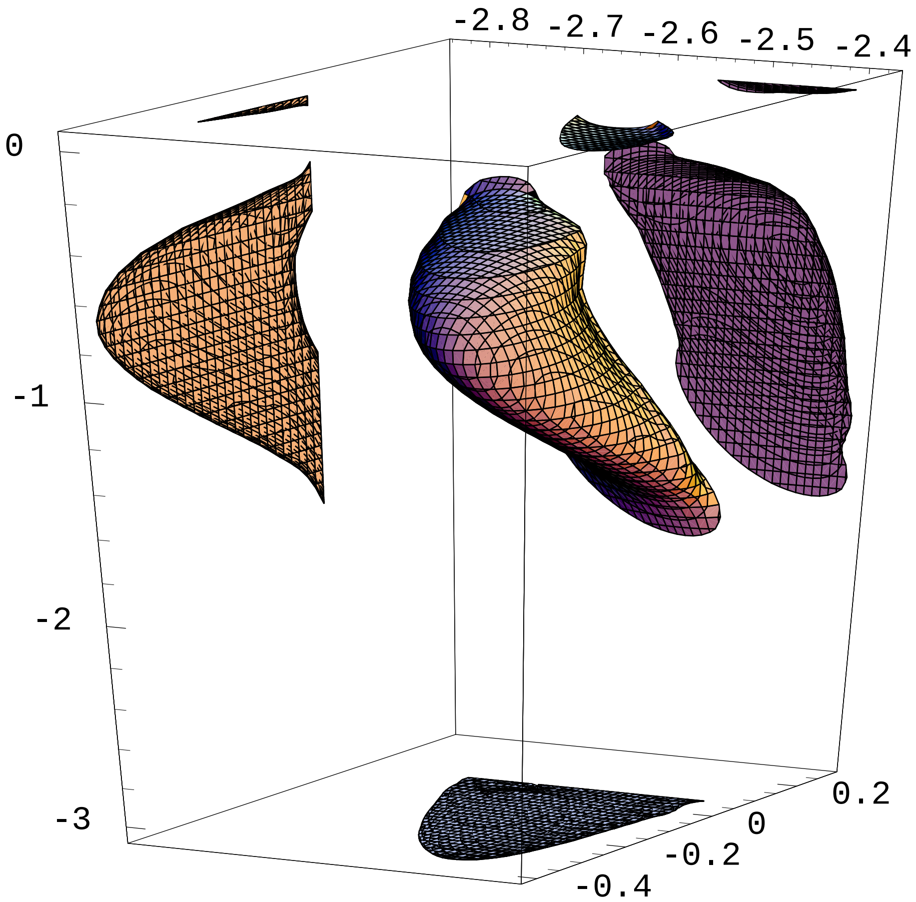
- **channels:** ν_μ-appearance and disappearance
- **flux:** $2 \cdot 10^{20}$ useful muon decays
- **detector:** $10\,\mathrm{kt}$ mass
  perfect charge identification
  Gaussian energy resolution ($\sigma = 0.1E$)
- **systematical errors and backgrounds**: not included

Physical input

- full numerical 3-ν calculation
- matter density profile taken into account
- central values for the simulation: $\theta_{23} = \theta_{12} = \pi/4$
  $\Delta m^2_{31} = 3.5 \cdot 10^{-3}\,\mathrm{eV}^2$

Statistics

full six-parameter fits are too CPU-time consuming for scans of the parameter space!

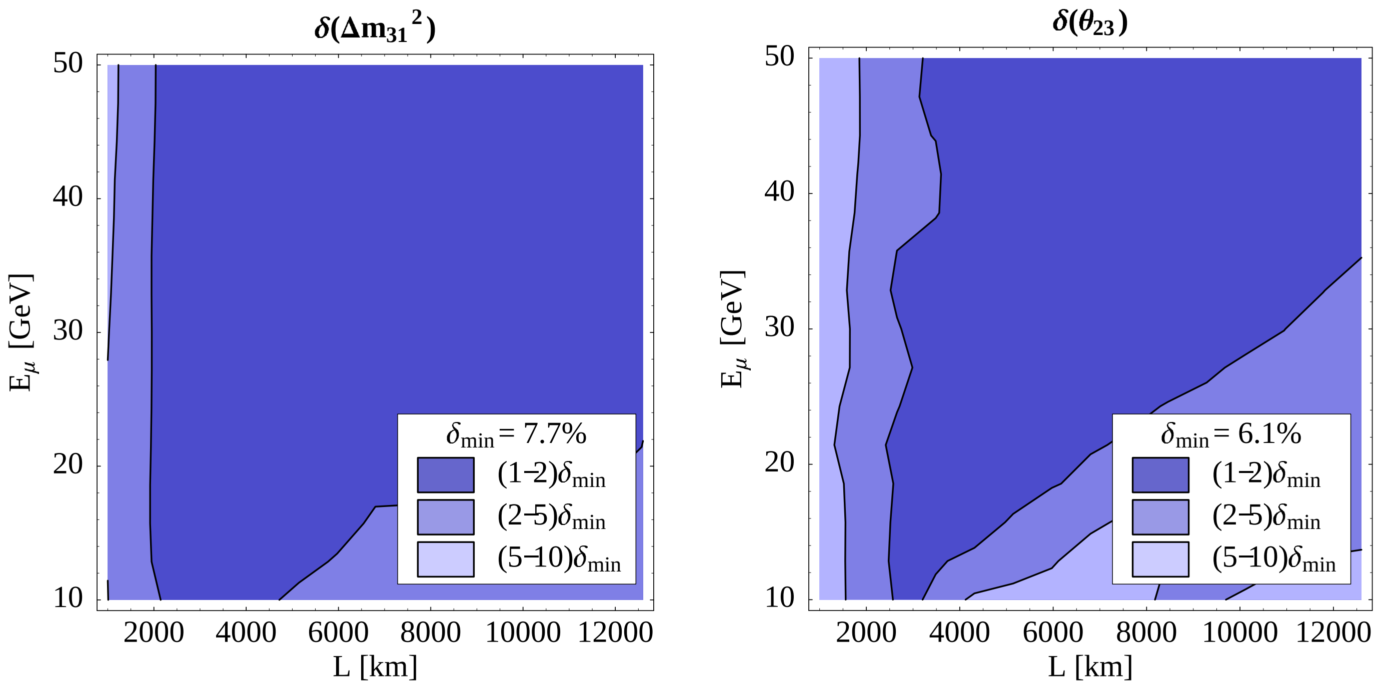


⇒ Calculation of all possible two-parameter fits

Advantage: automatically takes into account all two parameter correlations

Leading Parameters – Δm^2_{31}, θ_{23}

L-E optimization



- Baseline should exceed a minimal value:
  $L > 3000$ for $\Delta m^2_{31} = 3.5 \cdot 10^{-3}\ \mathrm{eV}^2$
  $L > 1000$ for $\Delta m^2_{31} = 5.0 \cdot 10^{-3}\ \mathrm{eV}^2$
  $L > 5000$ for $\Delta m^2_{31} = 1.0 \cdot 10^{-3}\ \mathrm{eV}^2$

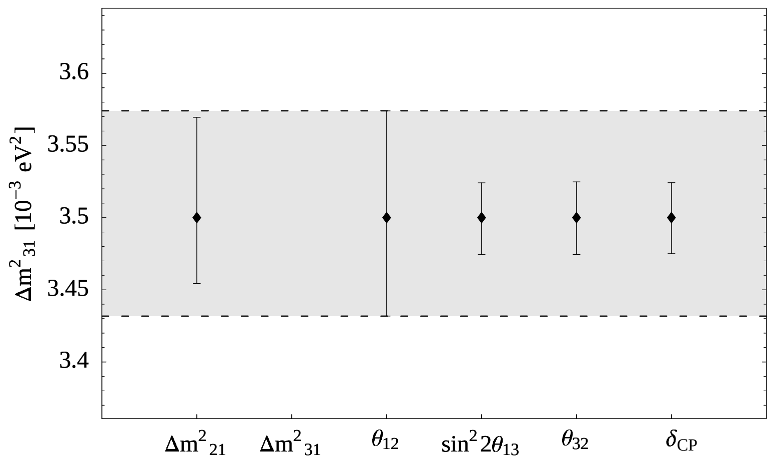
- $E_\mu > 30\,\mathrm{GeV}$ preferable

- achievable precision for Δm^2_{31}: $\delta_{\mathrm{min}} = 8\%$

- achievable precision for θ_{23}: $\delta_{\mathrm{min}} = 6\%$

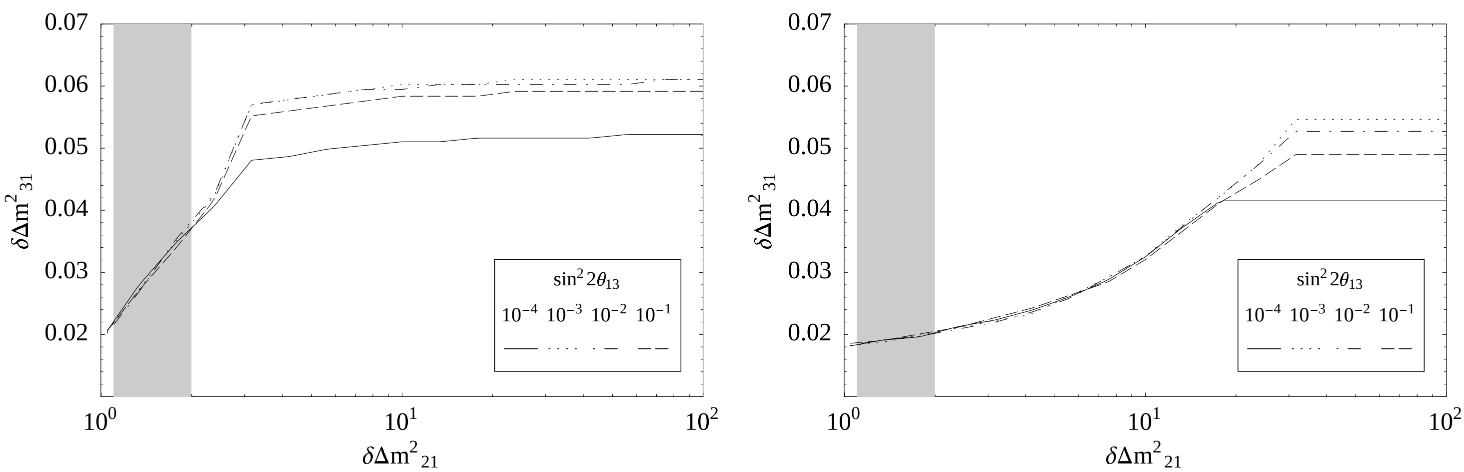
Leading Parameters – Δm^2_{31}

correlation with Δm^2_{21}



$E = 50\,\mathrm{GeV}$, $L = 8\,000\,\mathrm{km}$, $\sin^2 2\theta_{13} = 10^{-3}$, $\Delta m^2_{21} = 10^{-4}\,\mathrm{eV}^2$

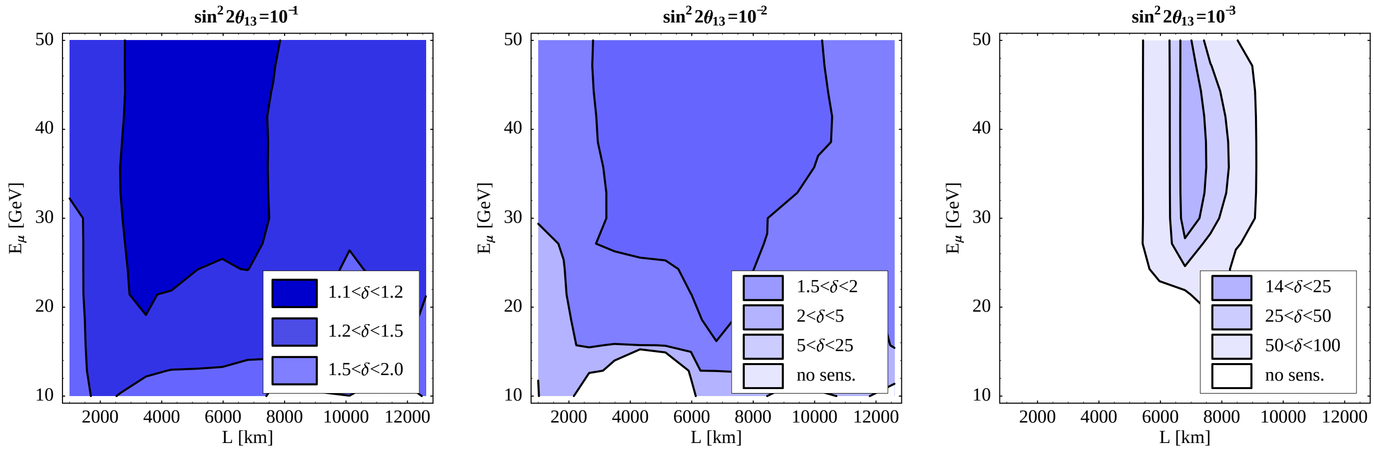
KamLand input



$\Delta m^2_{21} = 10^{-4}\,\mathrm{eV}^2$ (left plot), $\Delta m^2_{21} = 10^{-5}\,\mathrm{eV}^2$ (right plot)

Sub-Leading Parameters – $\sin^2 2\theta_{13}$

L-E optimization



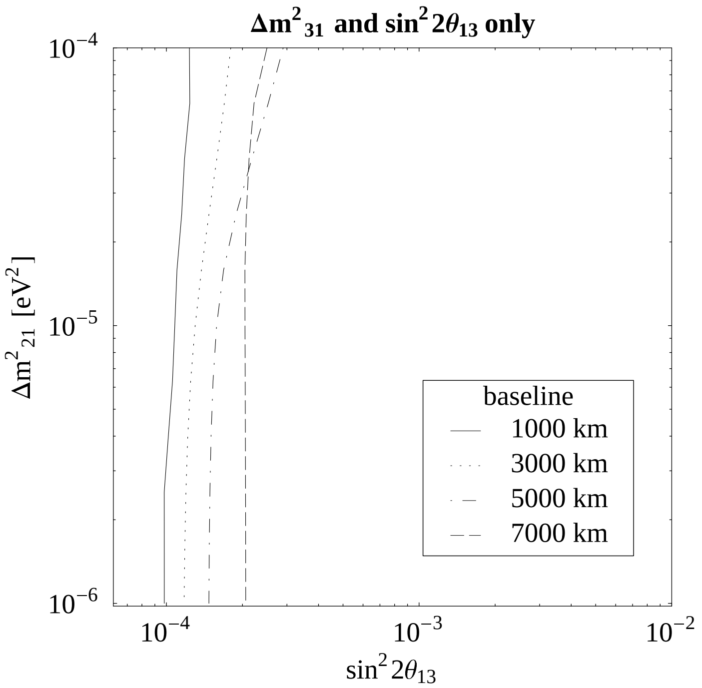
$\delta_{\rm CP}$ unknown

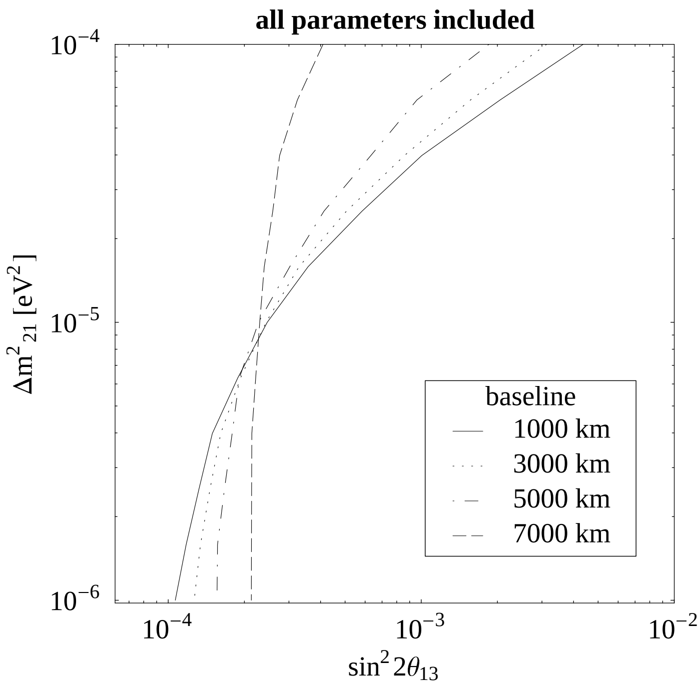
- large θ_{13}: $3000\,\mathrm{km} < L < 8000\,\mathrm{km}$ similar
- $\sin^2 2\theta_{13} \lesssim 10^{-3}$: $7000\,\mathrm{km} < L < 8000\,\mathrm{km}$ best

small $\theta_{13} \Rightarrow$ large baseline

Sub-Leading Parameters – $\sin^2 2\theta_{13}$

$\sin^2 2\theta_{13}$ sensitivity limit





$$E_\mu = 50\,\text{GeV}$$

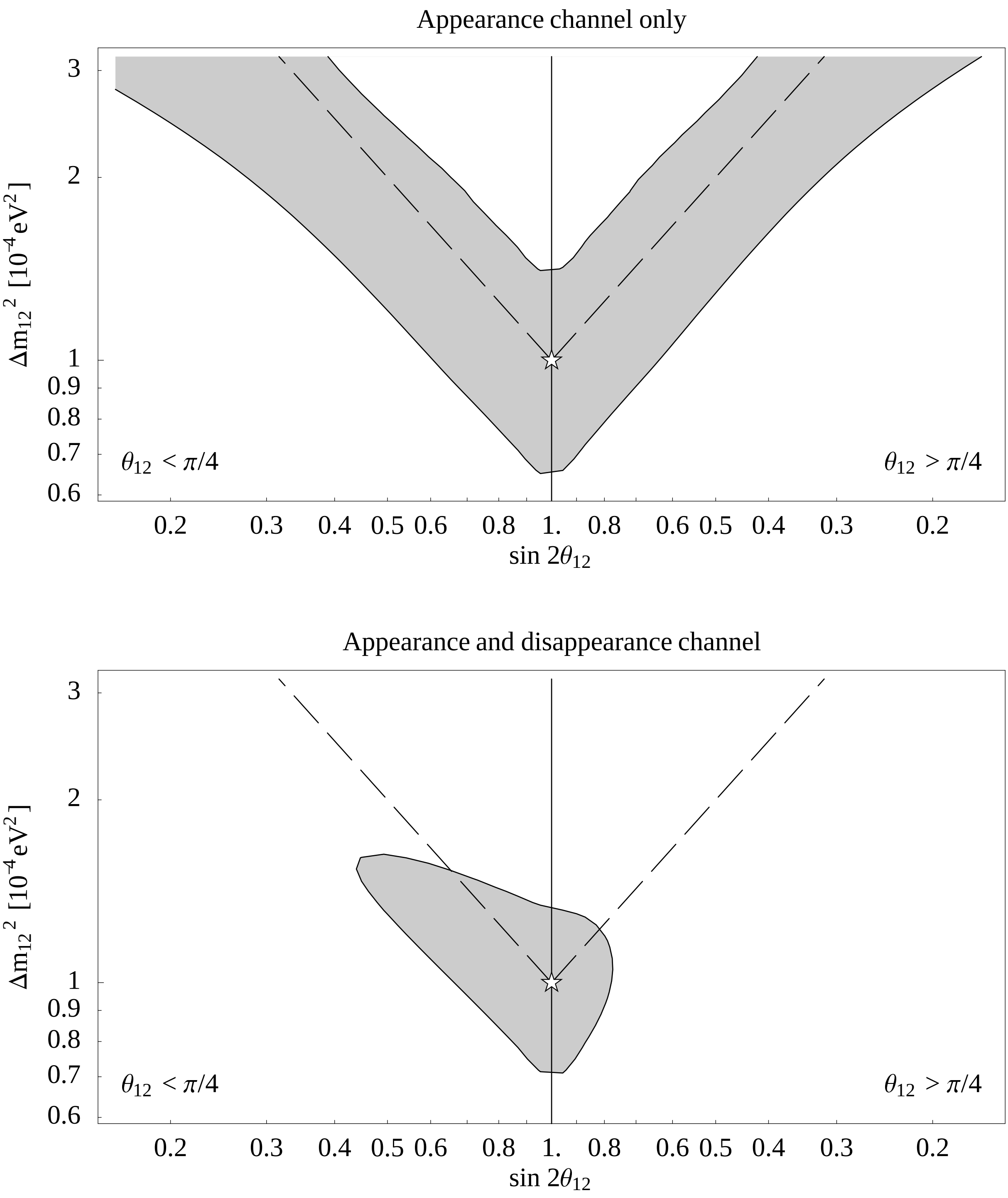
- left plot: only two parameter fit of $\sin^2 2\theta_{13}$ and Δm^2_{21}
- right plot: all correlations included

In the LMA-case, correlations with
θ_{12} and $\delta_{\rm CP}$
deteriorate sensitivity at high values of Δm^2_{21}.

(This effect is weaker at longer baselines.)

Small-Δm^2 effects – Δm^2_{21}, θ_{12}

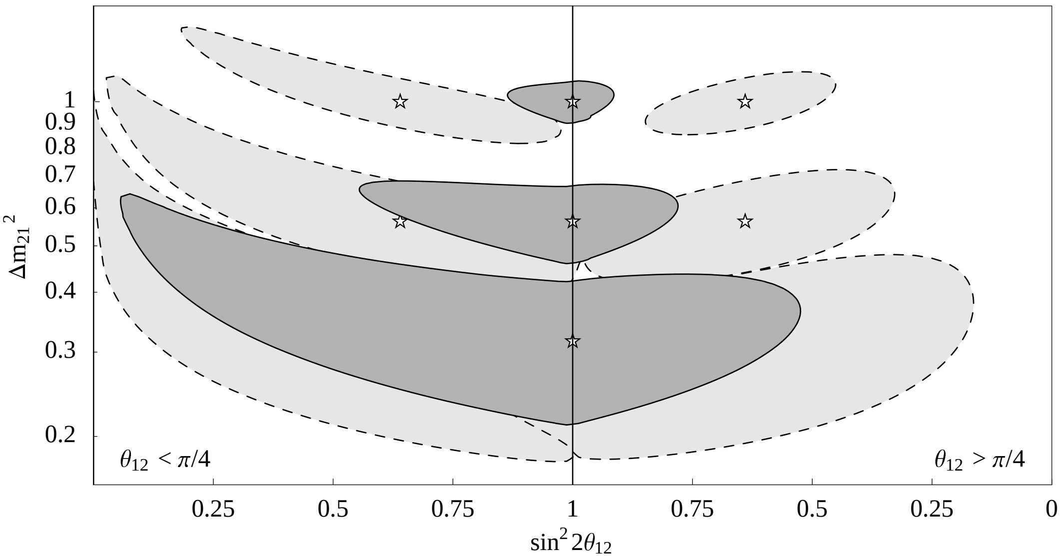
correlation of Δm^2_{21} and θ_{12}



$E = 50\,\mathrm{GeV},\ L = 3\,000\,\mathrm{km}$

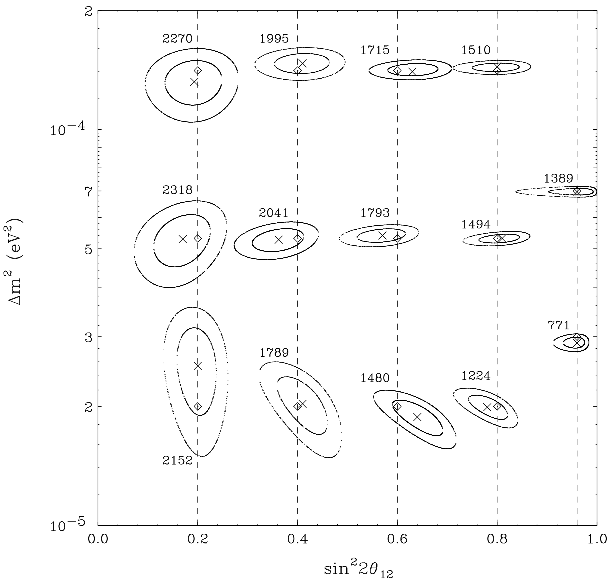
Small-Δm^2 effects – Δm^2_{21}, θ_{12}

determination of Δm^2_{21} and θ_{12}



$$E = 50\,\mathrm{GeV},\ L = 3\,000\,\mathrm{km},\ N_\mu m_{\mathrm{kt}} = 2 \cdot 10^{22}(!)$$

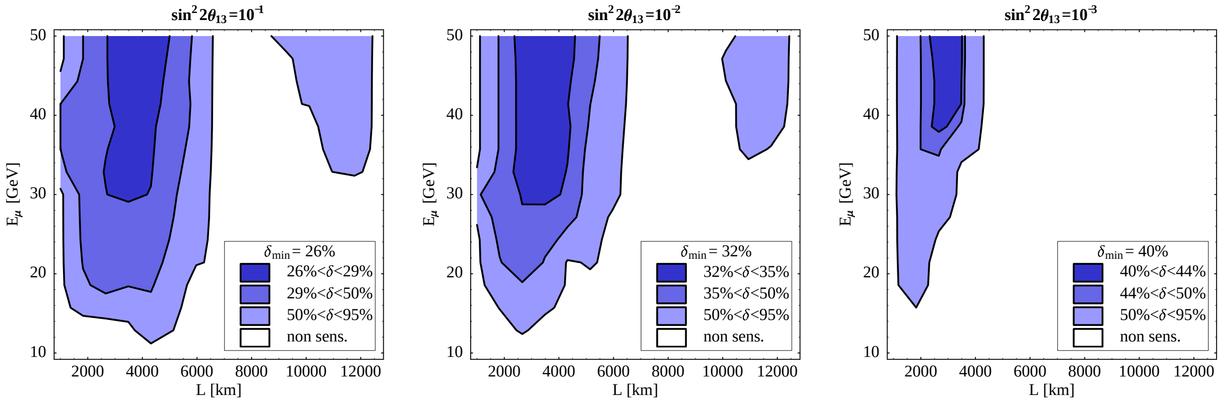
accuracy of KamLand:



V. Barger, D. Marfatia and B.P. Wood, Phys. Lett. **B498** (2001) 53.

Small-Δm^2 effects – $\delta_{\rm CP}$

L-E optimization for $\delta_{\rm CP}$

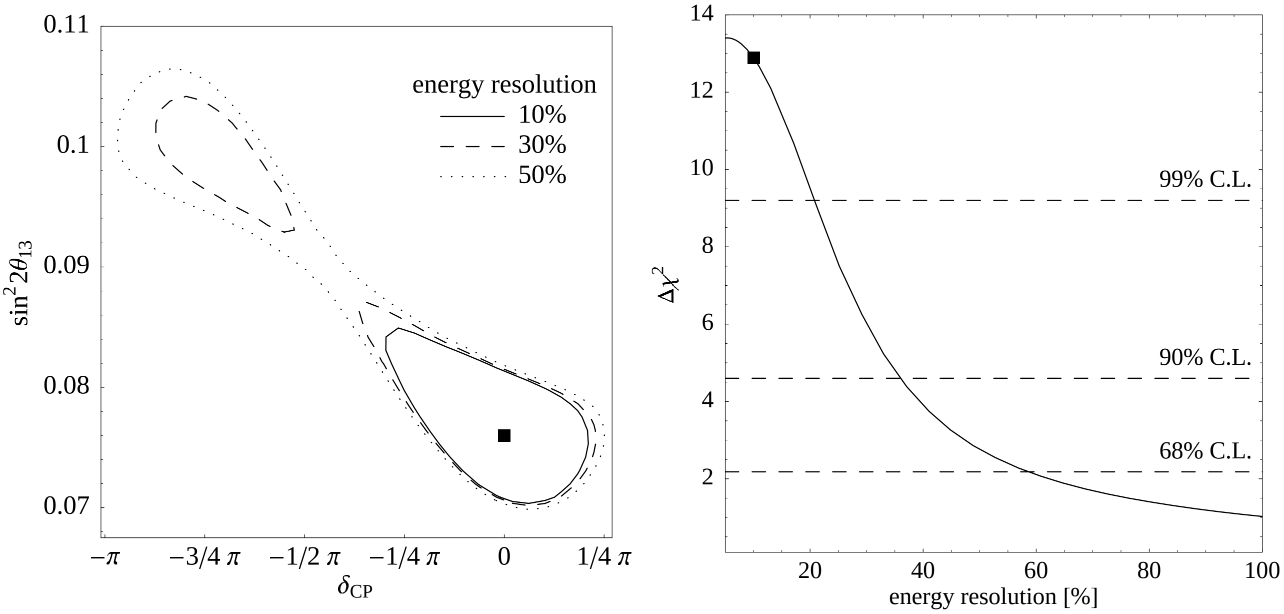


- baselines around $3000\,\mathrm{km}$ best
- higher energies prefered
- optimal baseline depends on the value of Δm^2_{31}

Δm^2_{31}	$\sin^2 2\theta_{13}$	baseline	beam energy
$6.0 \cdot 10^{-3}\,\mathrm{eV}^2$	10^{-1}	$2400\,\mathrm{km} \lesssim L \lesssim 4200\,\mathrm{km}$	$E_\mu \gtrsim 40\,\mathrm{GeV}$
	10^{-2}	$2400\,\mathrm{km} \lesssim L \lesssim 4000\,\mathrm{km}$	$E_\mu \gtrsim 40\,\mathrm{GeV}$
	10^{-3}	$1800\,\mathrm{km} \lesssim L \lesssim 2700\,\mathrm{km}$	$E_\mu \gtrsim 45\,\mathrm{GeV}$
$3.5 \cdot 10^{-3}\,\mathrm{eV}^2$	10^{-1}	$2800\,\mathrm{km} \lesssim L \lesssim 4500\,\mathrm{km}$	$E_\mu \gtrsim 30\,\mathrm{GeV}$
	10^{-2}	$2500\,\mathrm{km} \lesssim L \lesssim 4500\,\mathrm{km}$	$E_\mu \gtrsim 30\,\mathrm{GeV}$
	10^{-3}	$2500\,\mathrm{km} \lesssim L \lesssim 3500\,\mathrm{km}$	$E_\mu \gtrsim 40\,\mathrm{GeV}$
$1.0 \cdot 10^{-3}\,\mathrm{eV}^2$	10^{-1}	no sens.	no sens.
	10^{-2}	no sens.	no sens.
	10^{-3}	no sens.	no sens.

Small-Δm^2 effects – $\delta_{\rm CP}$

correlation of $\delta_{\rm CP}$ with θ_{13}



$$E = 50\,\mathrm{GeV},\ L = 3\,000\,\mathrm{km},\ \Delta m^2_{21} = 10^{-4}\,\mathrm{eV}^2$$

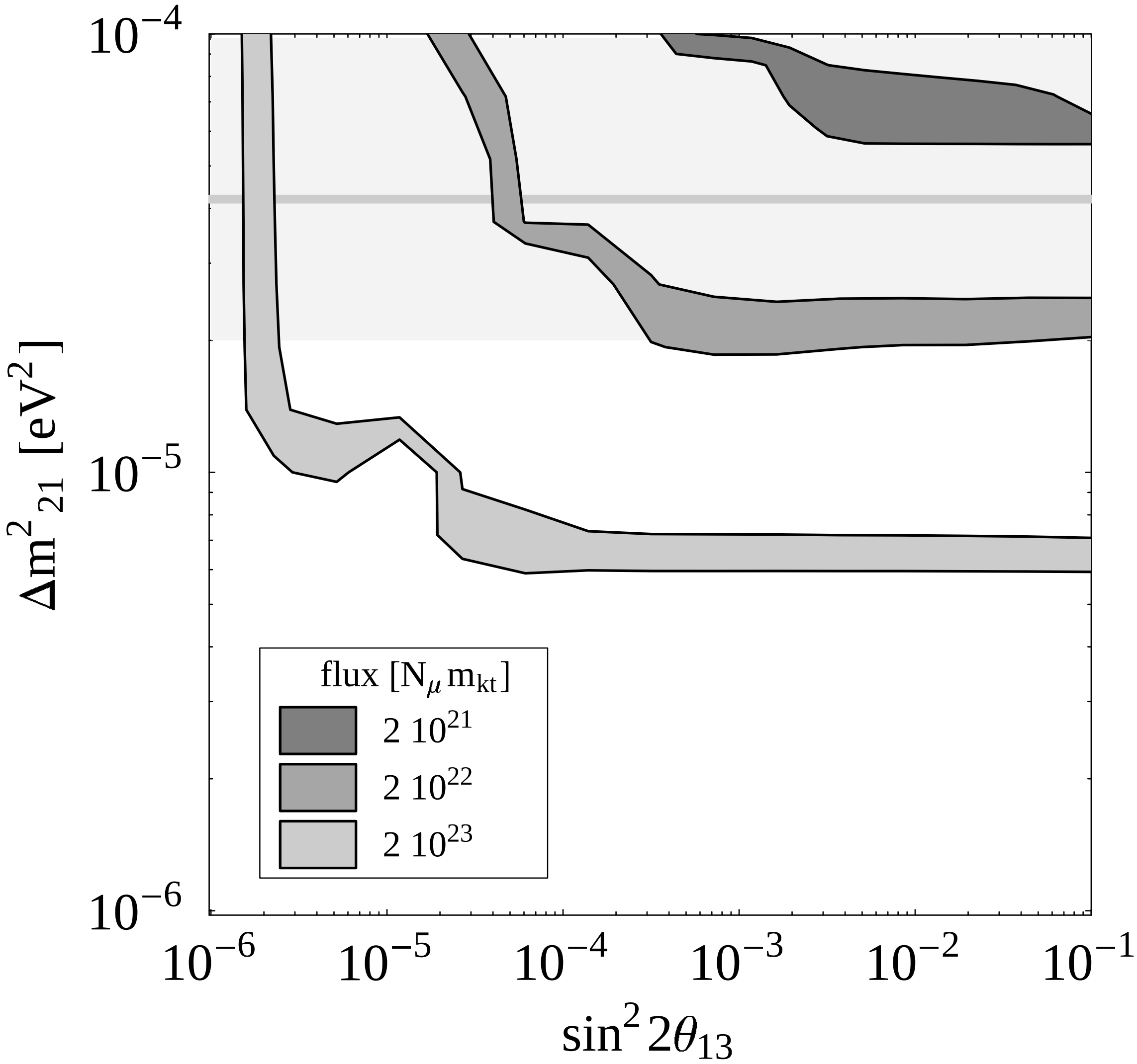
- two degenerate solutions[1]
- depends strongly on energy resolution
- degeneracy lifted for energy resolution better than 25%

degeneracy $\longleftrightarrow$ spectral information

[1] J. Burguet-Castell *et al.*, hep-ph/0103258.

Small-Δm^2 effects – $\delta_{\rm CP}$

sensitivity limit for $\delta_{\rm CP}$



$E = 50\,\mathrm{GeV}$, $L = 3\,000\,\mathrm{km}$

$2 \cdot 10^{20}$ muons $\otimes$ $10\,\mathrm{kt}$ is the minimum

Conclusions

Statistics

- the information in the event rate spectra should be optimally exploited
- parameter correlations are important

Leading Parameters

- errors on Δm^2_{31} and θ_{23} about 8%
- $L_{min} \simeq 3\,000\,\mathrm{km}$
- $E > 30\,\mathrm{GeV}$

Sub-Leading Parameters

- strong correlation with Δm^2_{21} or δ_{CP} depending on δ_{CP}
- higher baselines around $7\,000\,\mathrm{km}$ are most sensitive
- KamLand input only for $\cos\delta_{\mathrm{CP}} \geq 0$ helpful

Small-Δm^2 effects

- sensitivity to Δm^2_{21} and θ_{12} is much worse than at e.g. KamLand
- sensitivity to δ_{CP} best at $50\,\mathrm{GeV}$ and $\simeq 3\,000\,\mathrm{km}$
- sufficient energy resolution lifts degeneracy between δ_{CP} and θ_{13}
- KamLand input does not improve the results on δ_{CP}
- at least $N_\mu m_{\mathrm{kt}} = 2 \cdot 10^{21}$ required