Non-adiabatic oscillations of (supernova) neutrinos

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based on collaboration with *R. Tomàs*, *J.W.F. Valle*,

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- hep-ph/0106???

1. Oscillation of supernova neutrinos
2. Resonance versus adiabaticity conditions
3. Crossing probability in the WKB approximation
4. Likelihood analysis of SN 1987A neutrino signal
5. Conclusions

1. Some basics about supernova neutrinos

• interaction strength of neutrinos with neutrons varies like

$$\kappa(\nu_e) > \kappa(\bar{\nu}_e) > \kappa(\nu_{\mu,\tau}, \bar{\nu}_{\mu,\tau})$$

$\Rightarrow$ $\bar{\nu}_{\mu,\tau}$ decouple deeper in the core
$\Rightarrow$ $\bar{\nu}_{\mu,\tau}$ are more energetic than $\bar{\nu}_e$ neutrinos

Simulations give

$$\left.\begin{array}{l} 14 \le \langle E_{\bar{\nu}_e} \rangle \le 17 \text{ MeV} \\ 24 \le \langle E_{\bar{\nu}_{\mu,\tau}} \rangle \le 27 \text{ MeV} \end{array}\right\} \Rightarrow \tau = \frac{\langle E_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle E_{\bar{\nu}_e} \rangle} = 1.4 - 2$$

$$1.5 \times 10^{53} \text{ erg} \le E_b \le 4.5 \times 10^{53} \text{ erg}$$

• equipartition: same luminosity for all flavours (spectral temperatures do not fix flux, $\mathcal{L}_{\nu_i} \not\propto R^2 T_i^4$)

• power law potential profile $A(r) = 2EV(r) \propto r^{-3}$.

• 19 $\bar{\nu}_e$ neutrino from SN 1987A detected; also from future galactic SN mainly $\bar{\nu}_e$ neutrinos

Survival probability of $\bar{\nu}_e$ neutrinos

- probability of $\bar{\nu}_e$ to arrive at surface of the Earth

$$P_{\bar{e}\bar{e}} = P^S_{\bar{e}1} P^E_{1\bar{e}} + P^S_{\bar{e}2} P^E_{2\bar{e}} = (1 - P_c)\cos^2\theta + P_c \sin^2\theta$$

- P_c is level-crossing probability; with

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta |\mathrm{d}\ln A/\mathrm{d}x|_{\mathrm{res}}}$$

$\gamma \gg 1$: adiabatic evolution $\Leftrightarrow P_c = 0$
$\gamma \ll 1$: non-adiabatic evolution $\Leftrightarrow P_c = \sin^2\vartheta$

How to include non-adiabatic effects for $\gamma \sim 1$?

WKB approximation for P_c

Semi-classical crossing probability

$$\ln P_c = -\frac{1}{E}\,\Im\int_{x_1(A_1)}^{x_2(A_2)} \mathrm{d}x\,\Delta_m$$

$$\Delta_m = \left[(A-\Delta m^2\cos 2\vartheta)^2 + (\Delta m^2\sin 2\vartheta)^2\right]^{1/2},$$

where $A_2 = \Delta m^2(\cos 2\vartheta + \mathrm{i}\sin 2\vartheta)$ is branch point of Δ_m in the upper complex x plane.

For arbitrary power-law profiles:

$$P_c = \exp\left(-\frac{\gamma\mathcal{F}_n\pi}{2}\right)$$

[*Kuo, Pantaleone '89*]

Problems and limitations:

1. γ evaluated at $\vartheta_\mathrm{m} = \pi/4$ makes sense only in resonant region

2. $\mathcal{F}_n$ given by KP converges only for $\vartheta < \pi/8$

what about dark side or anti-neutrinos?

2.a Where is adiabaticity maximal violated?

define pmva as the point where

$$\frac{d|\tilde{\psi}_i|^2}{dr}(\vartheta_\mathrm{m})$$

has its maximum.

$$\frac{\mathrm{d}}{\mathrm{d}\vartheta_\mathrm{m}}\begin{pmatrix}\tilde{\psi}_1\\ \tilde{\psi}_2\end{pmatrix}=\begin{pmatrix}\frac{\mathrm{i}\Delta_\mathrm{m}}{4E\vartheta'_\mathrm{m}} & -1\\ 1 & -\frac{\mathrm{i}\Delta_\mathrm{m}}{4E\vartheta'_\mathrm{m}}\end{pmatrix}\begin{pmatrix}\tilde{\psi}_1\\ \tilde{\psi}_2\end{pmatrix}$$

differentiate

$$\frac{\Delta_\mathrm{m}}{\vartheta'_\mathrm{m}}=\frac{2\Delta m^2\sin^2 2\vartheta}{\sin^3 2\vartheta_\mathrm{m}}\frac{1}{\mathrm{d}A/\mathrm{d}r},$$

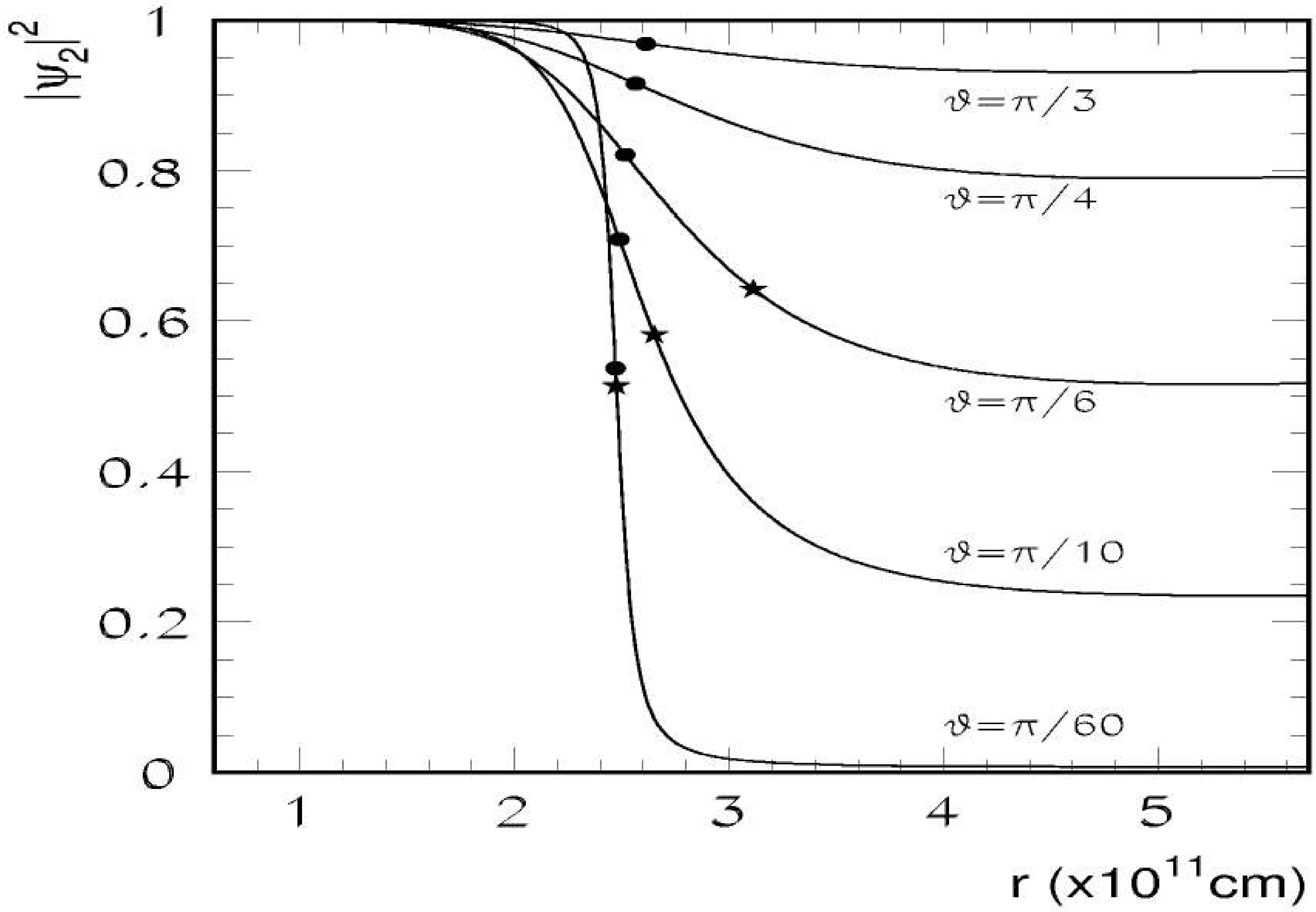
for a power law profile, $A(r)\propto r^n$, the minimum is at

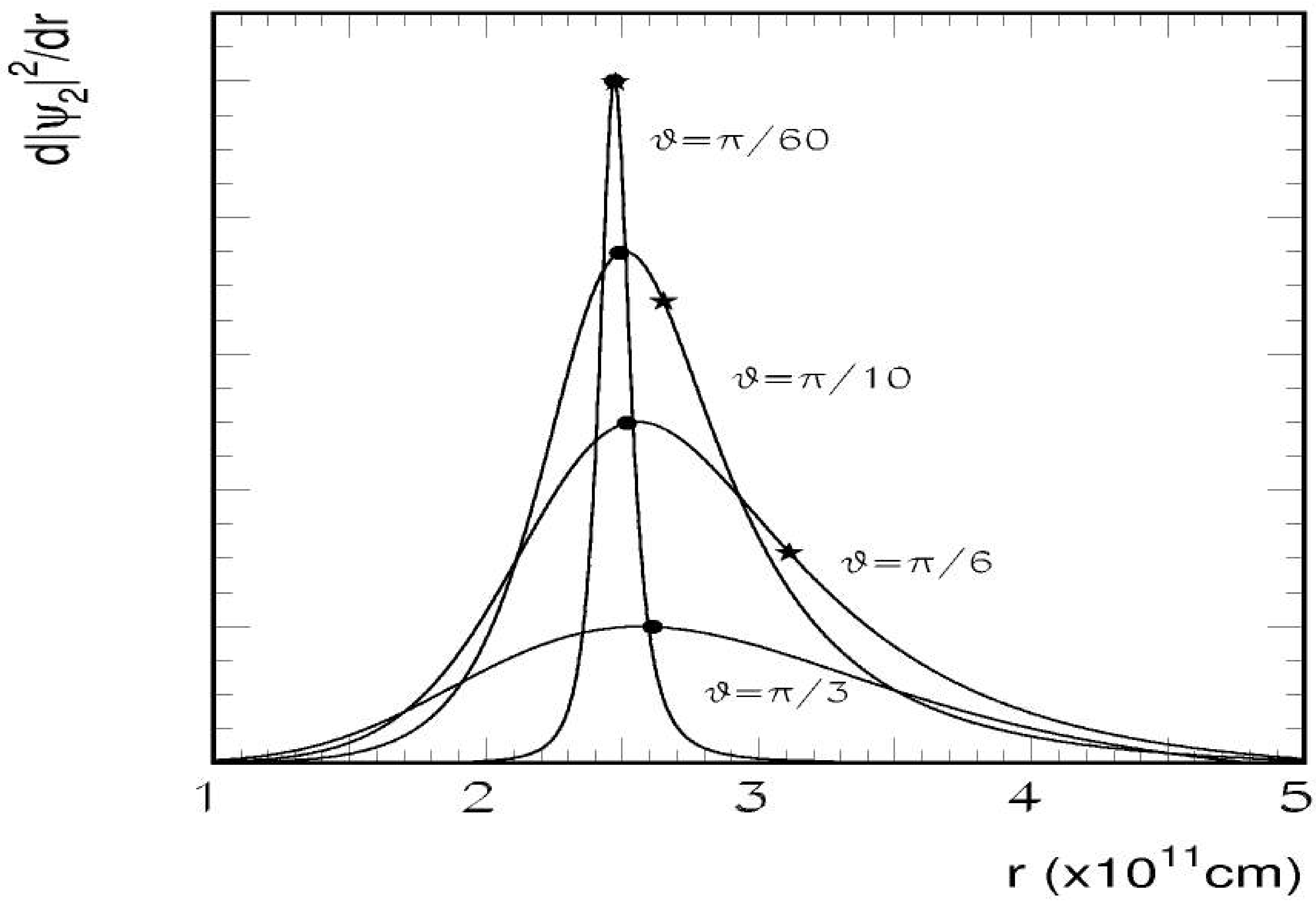
$$\cot(2\vartheta_\mathrm{m}-2\vartheta)+2\cot(2\vartheta_\mathrm{m})$$
$$-\ \frac{1}{n}\left[\cot(2\vartheta_\mathrm{m}-2\vartheta)-\cot(2\vartheta_\mathrm{m})\right]=0\,.$$

$\Rightarrow$ pmva is at $\vartheta_\mathrm{m}=\pi/4$ only for $n=1$

for $n\to\pm\infty$ we recover exponential profile
[*Friedland, hep-ph/0010231*]

Resonance point versus pmva for $n = -3$:





- $\star$ resonance point $\vartheta_m = \pi/4$
- $\bullet$ pmva

2.b When does the neutrino evolve adiabatically?

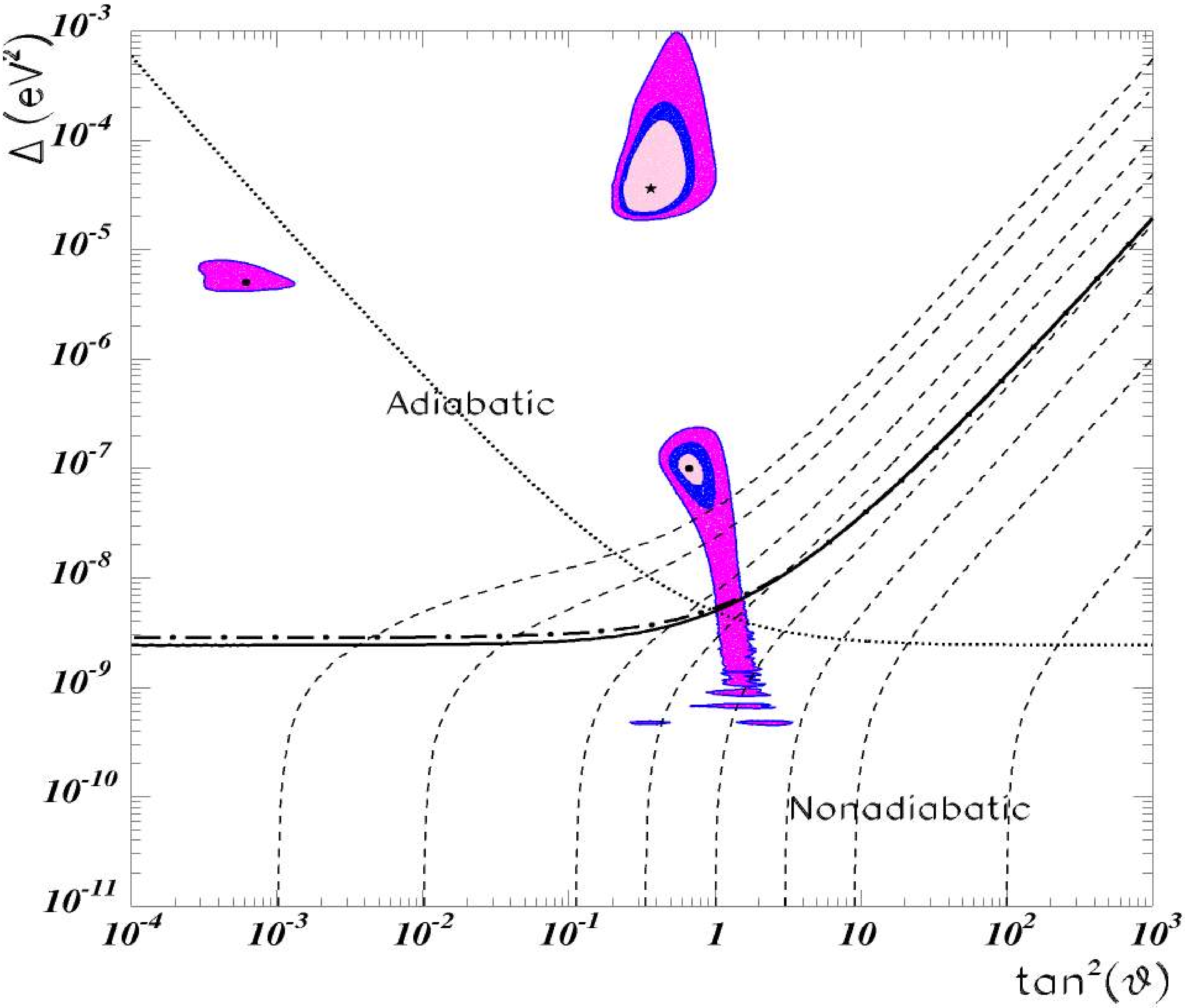
Need global measure for the accumulated non-adiabatic effects along the trajectory from $\vartheta_{\mathrm{m}} \approx \pi/2$ to ϑ. For a non-adiabatic evolution, require

$$|\int_{\pi/2}^{\vartheta} \mathrm{d}\vartheta_{\mathrm{m}}\, \tilde{\psi}_1| = \varepsilon\, |\int_{\pi/2}^{\vartheta} \mathrm{d}\vartheta_{\mathrm{m}}\, \frac{4E\vartheta'_{\mathrm{m}}}{\Delta_{\mathrm{m}}}\, \tilde{\psi}_2|$$

and $\varepsilon \ll 1$.

With $\tilde{\psi}_1 \approx \cos\vartheta_{\mathrm{m}}$, $\tilde{\psi}_2 \approx \sin\vartheta_{\mathrm{m}}$ and eq. for $\Delta_{\mathrm{m}}/\vartheta'_{\mathrm{m}}$, borderline of the non-adiabatic region follows for $A = 2EV_0(r/R_0)^n$ as

$$\frac{\Delta m^2}{E} = \left\{\varepsilon\, \frac{f(\vartheta)}{\sin^2(2\vartheta)(1-\sin\vartheta)}\, \frac{2n(2V_0)^{1/n}}{R_0}\right\}^{\frac{n}{n+1}},$$



supernova progenitor profile $n = -3$,
dashed line: constant survival probability $P_{\overline{e}\overline{e}}$

New representation for P_c:

start directly from

$$\ln P_c = -\frac{1}{E}\,\Im \int_{x_1(A_1)}^{x_2(A_2)} \mathrm{d}x\,\Delta_m\,,$$

but use as integration path circle centered at zero starting at $A_1 = \Delta$ and going to $A_2 = \Delta e^{2i\vartheta}$.

Substitute $x = R_0(\Delta/A_0)^{1/n} e^{i\phi}$
$\Rightarrow \vartheta$ dependence of P_c can be factored the into functions $\mathcal{G}_n$,

$$\ln P_c = -\kappa_n \mathcal{G}_n(\vartheta)\,,$$

with

$$\kappa_n = \left(\frac{\Delta}{E}\right)\left(\frac{\Delta}{A_0}\right)^{1/n} R_0$$

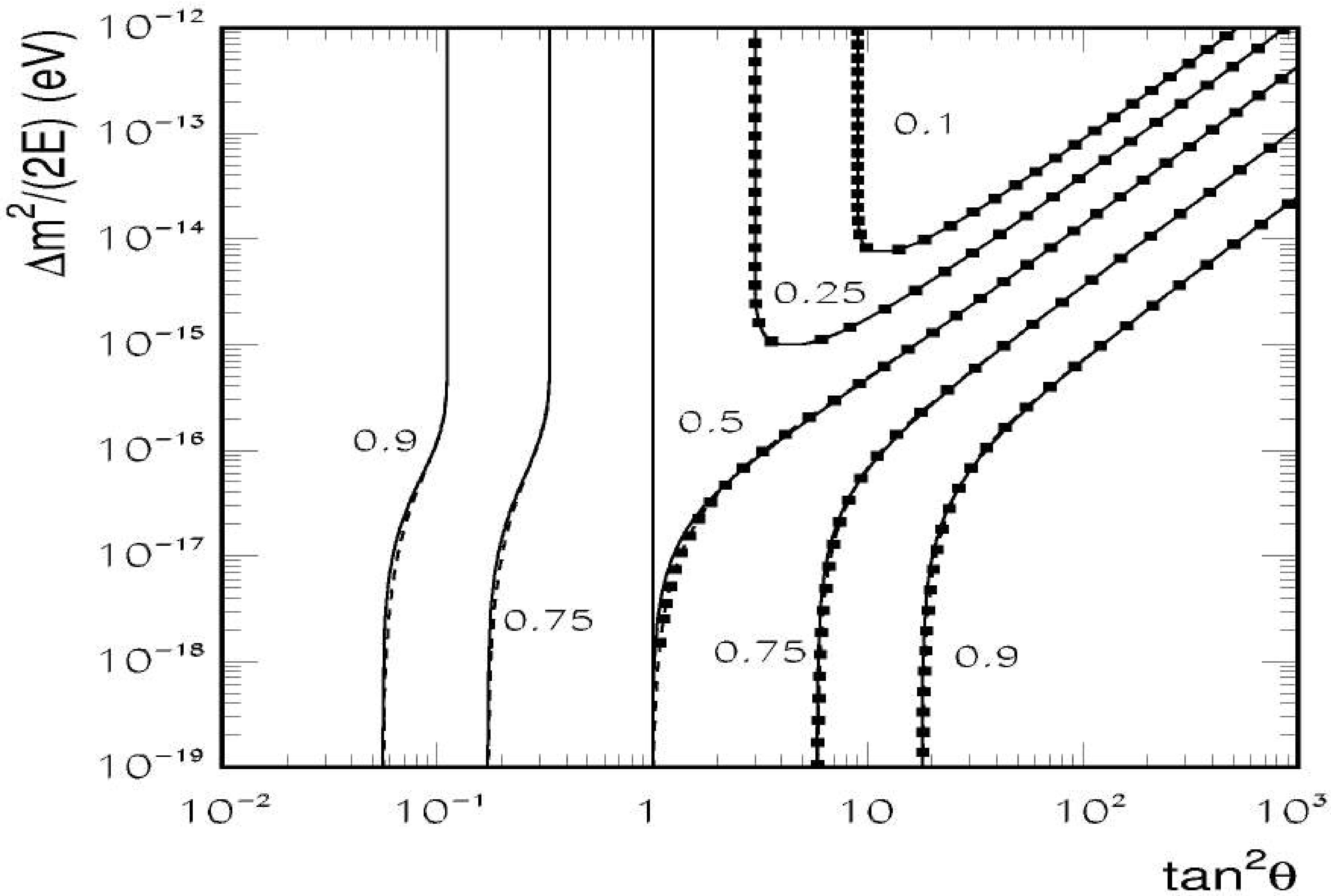
and

$$\mathcal{G}_n(\vartheta) = \left| \Re \int_0^{2\vartheta/n} \mathrm{d}\phi\, e^{i\phi} \left[\left(e^{in\phi} - C\right)^2 + S^2\right]^{1/2} \right|.$$

- $\mathcal{G}_n$ are well suited for numerical evaluation
- correspond to a neutrino state propagating in the physical part of the x plane, $x > 0$
- valid for all ϑ.

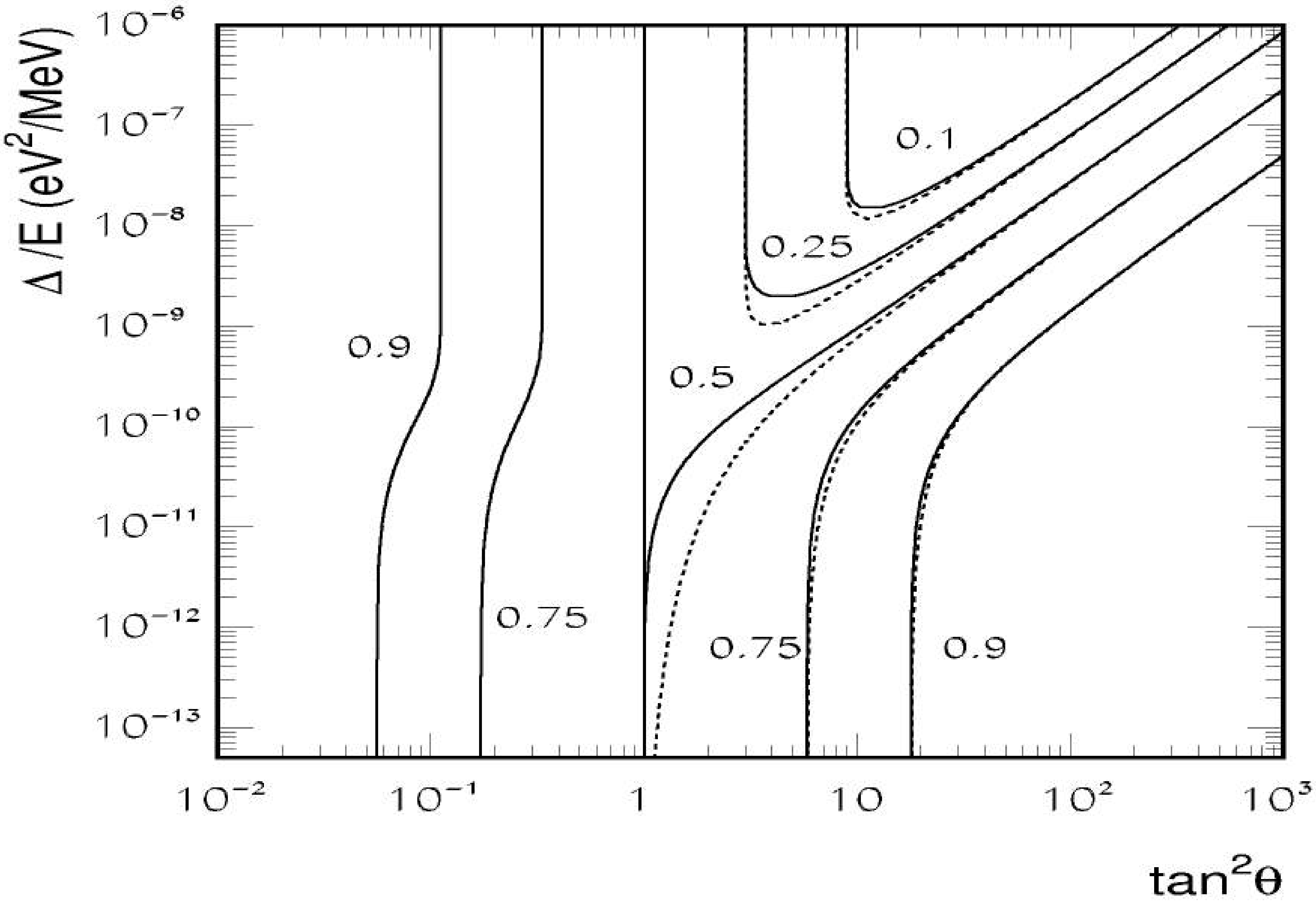
Comparison of the different methods:

Contours of constant survival probability $P_{\overline{e}\overline{e}}$ for a typical SN profile:



numerical: solid line
$\mathcal{G}_{-3}$: dashed line
$\mathcal{F}_{-3}$: squares

Comparison with the usual linear "approximation" $\mathcal{F}_{-3} = 1$ together with $A \propto r^{-3}$:



numerical: solid line

$\mathcal{F}_{-3} \sim 1$ together with $A \propto r^{-3}$: dashed line

$\Rightarrow$ good approximation for $\tan^2 \vartheta \gtrsim 5$

N.B.: WKB formalism works only for small non-adiabatic perturbation.

Generally, use instead of P_{LSZ}

$$P_c = \frac{\exp\left(-\frac{\pi\gamma_n}{2}\mathcal{F}_n\right) - \exp\left(-\frac{\pi\gamma_n}{2}\mathcal{F}'_n\right)}{1 - \exp\left(-\frac{\pi\gamma_n}{2}\mathcal{F}'_n\right)},$$

where $\mathcal{F}'_n = \mathcal{F}_n / \sin^2\vartheta$.

4. Likelihood analysis of SN 1987A signal

• probability of $\bar{\nu}_e$ to arrive at surface of the Earth

$$P_{\bar{e}\bar{e}} = P^S_{\bar{e}1} P^E_{1\bar{e}} + P^S_{\bar{e}2} P^E_{2\bar{e}} = (1 - P_c)\cos^2\theta + P_c \sin^2\theta$$

with

$$P_c = \exp(-\kappa \mathcal{G})$$

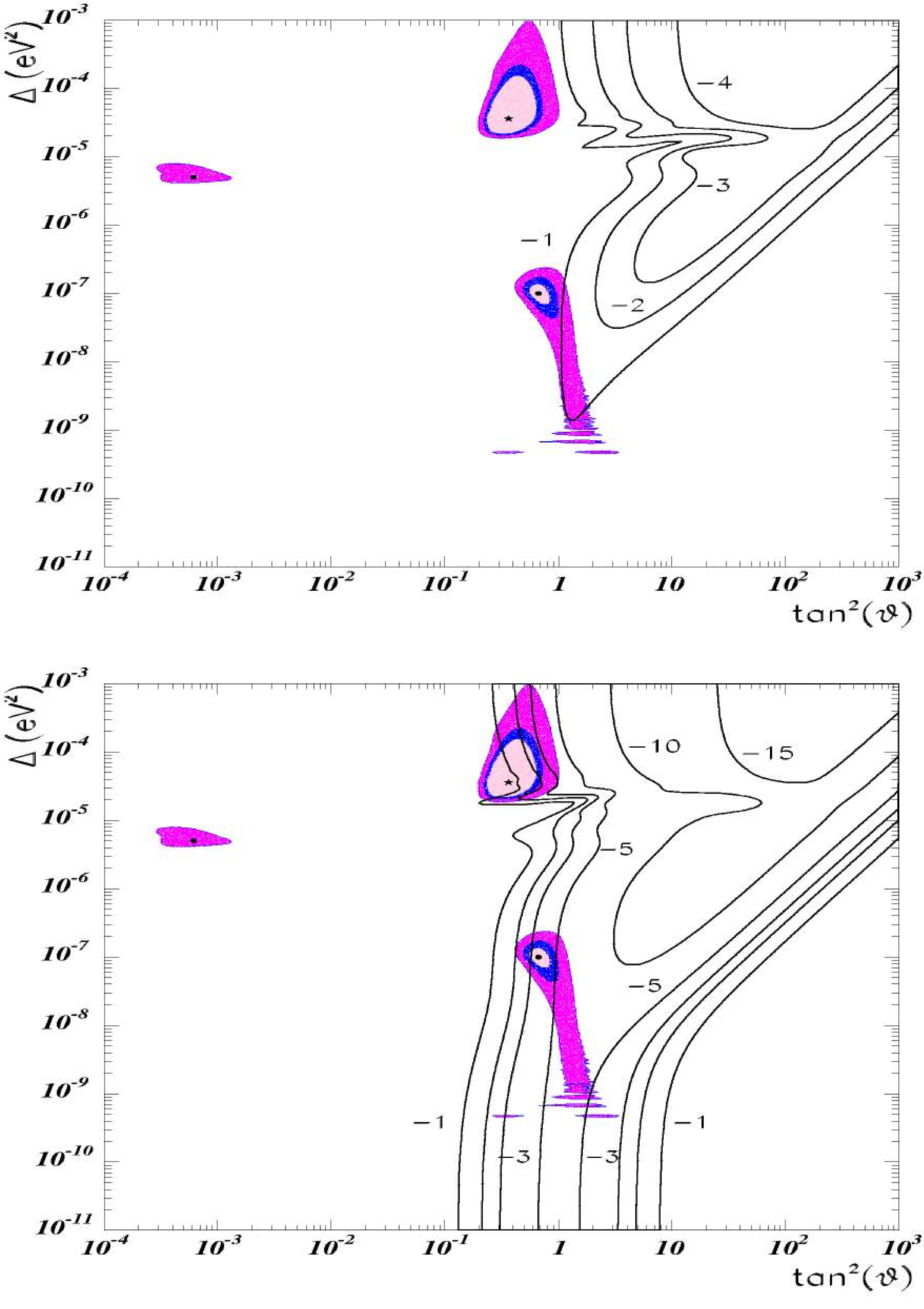
• then $\bar{\nu}$ had to cross the mantle of the Earth,
⇒ box potential, i.e. replace $P^E_{i\bar{e}} = |\langle \bar{\nu}_i | \bar{\nu}_e \rangle|^2$ with

$$\begin{aligned}
\cos^2\theta &\rightarrow \cos^2\theta + \sin 2\theta' \sin(2\theta - 2\theta') \sin^2(\pi d / l'_{\rm osc}) \\
\sin^2\theta &\rightarrow \sin^2\theta - \sin 2\theta' \sin(2\theta - 2\theta') \sin^2(\pi d / l'_{\rm osc}) \,,
\end{aligned}$$

• we derive CL for parameters α or test different hypotheses α and α' with the likelihood function

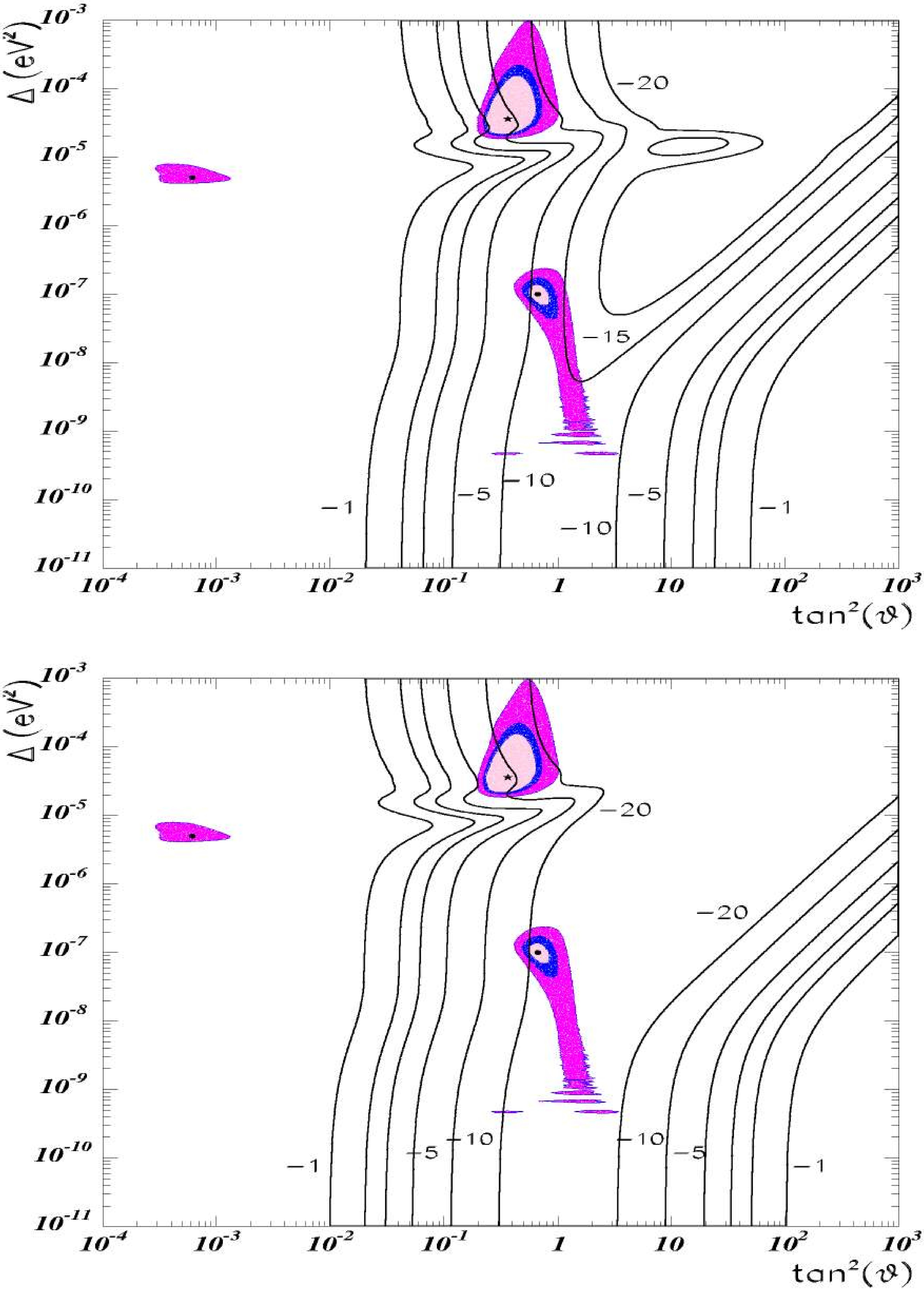
$$\mathcal{L}(\alpha) \propto \exp\left(-\int n(E,\alpha)\, dE\right) \prod_{i=1}^{N_{\rm obs}} n(E_i, \alpha)$$

Results of SN1987A fit



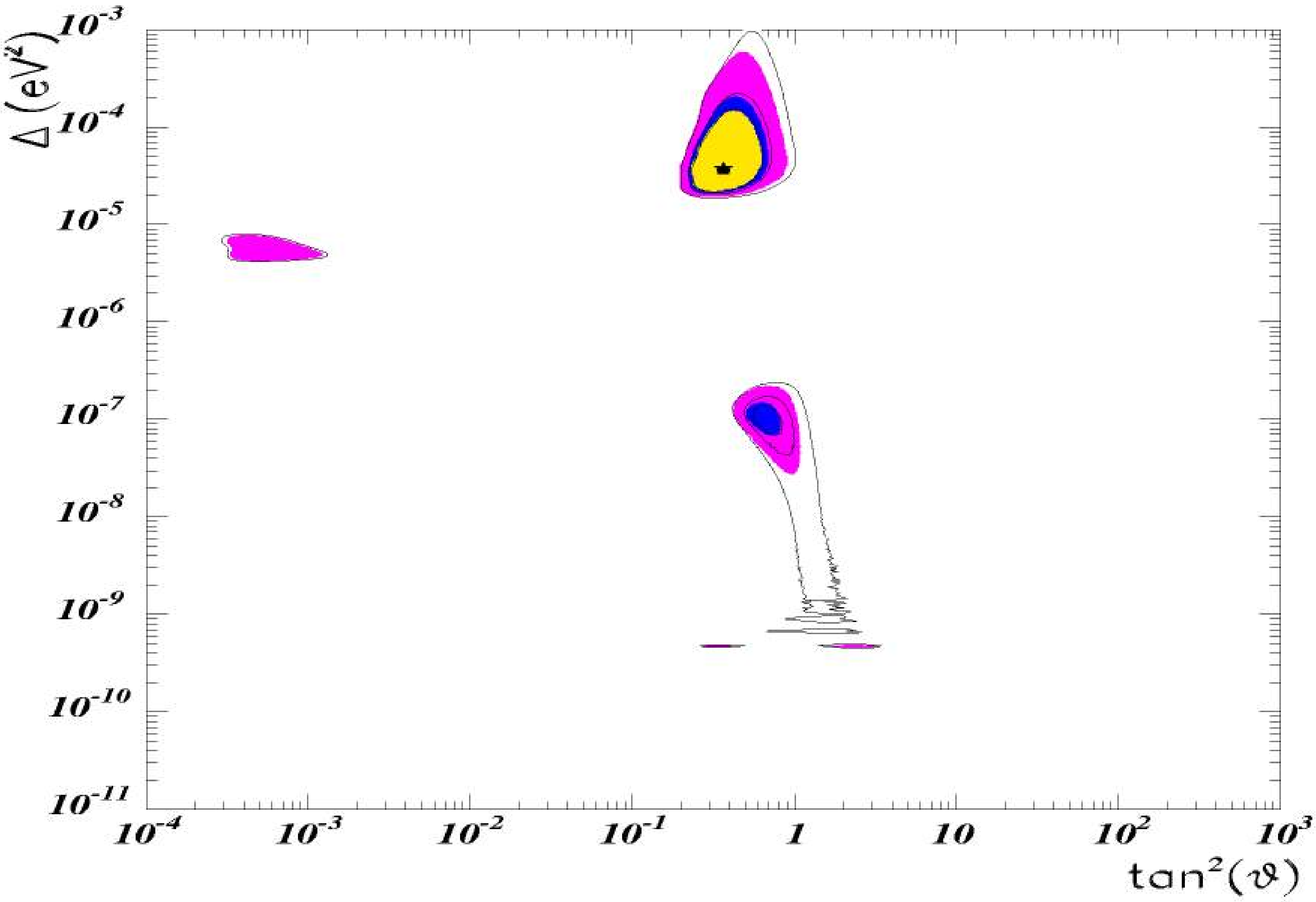
$E_{\rm b} = 1.5 \times 10^{53}$ erg, $\langle E_{\bar{\nu}_{\rm e}} \rangle = 12$ MeV
$\tau = 1.4$ (top), $\tau = 1.7$ (bottom).

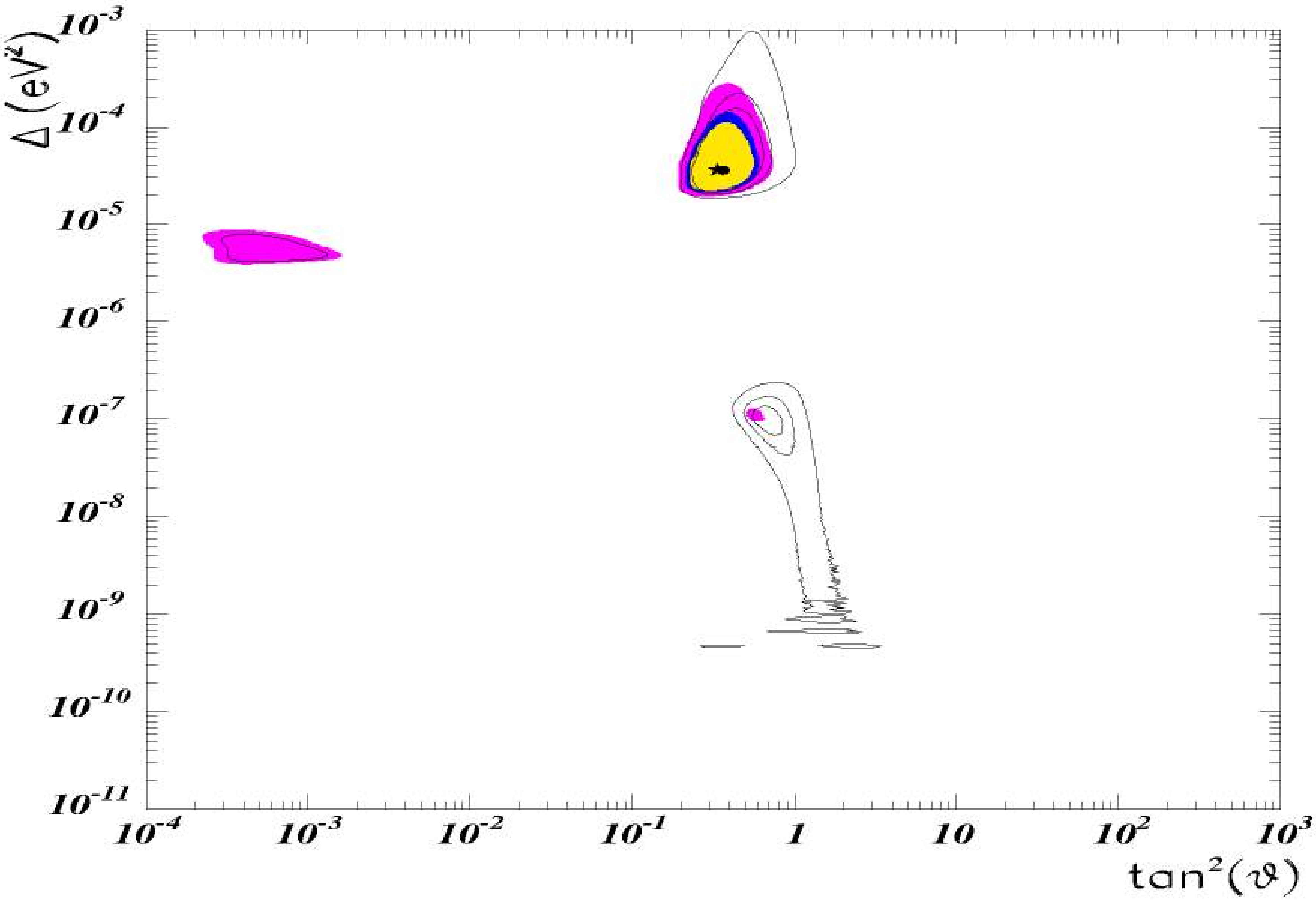
Results of SN1987A fit: $\ln(R) = \ln\left(\frac{\mathcal{L}(\vartheta,\Delta m^2)}{\mathcal{L}(SMA)}\right)$



$E_{\rm b} = 3 \times 10^{53}$ erg, $\langle E_{\bar{\nu}_{\rm e}} \rangle = 14$ MeV
$\tau = 1.4$ (top), $\tau = 1.7$ (bottom).

Results of combined solar + SN1987A fit





$E_\mathrm{b} = 1.5 \times 10^{53}$ erg, $\langle E_{\bar{\nu}_\mathrm{e}} \rangle = 12$ MeV
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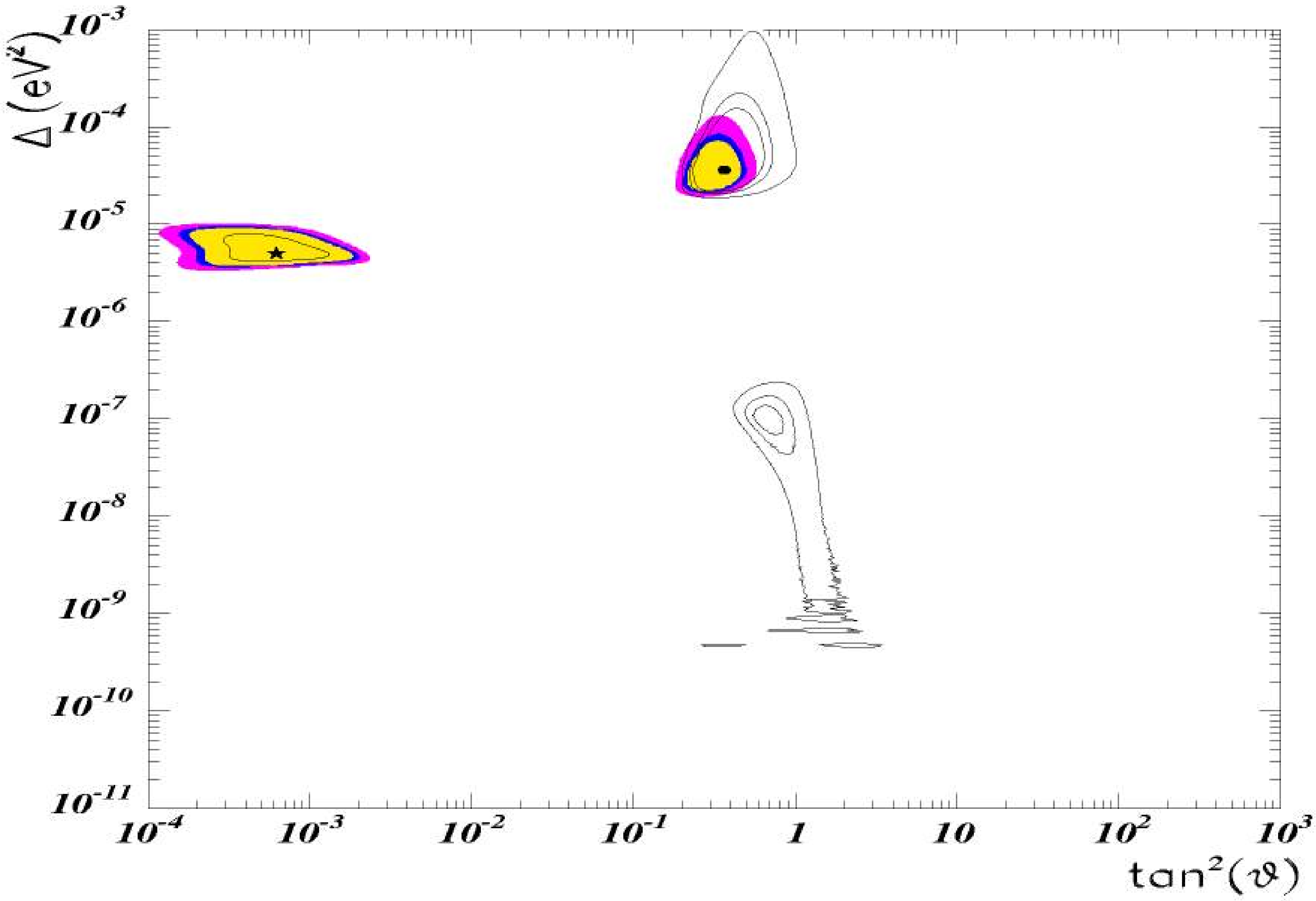
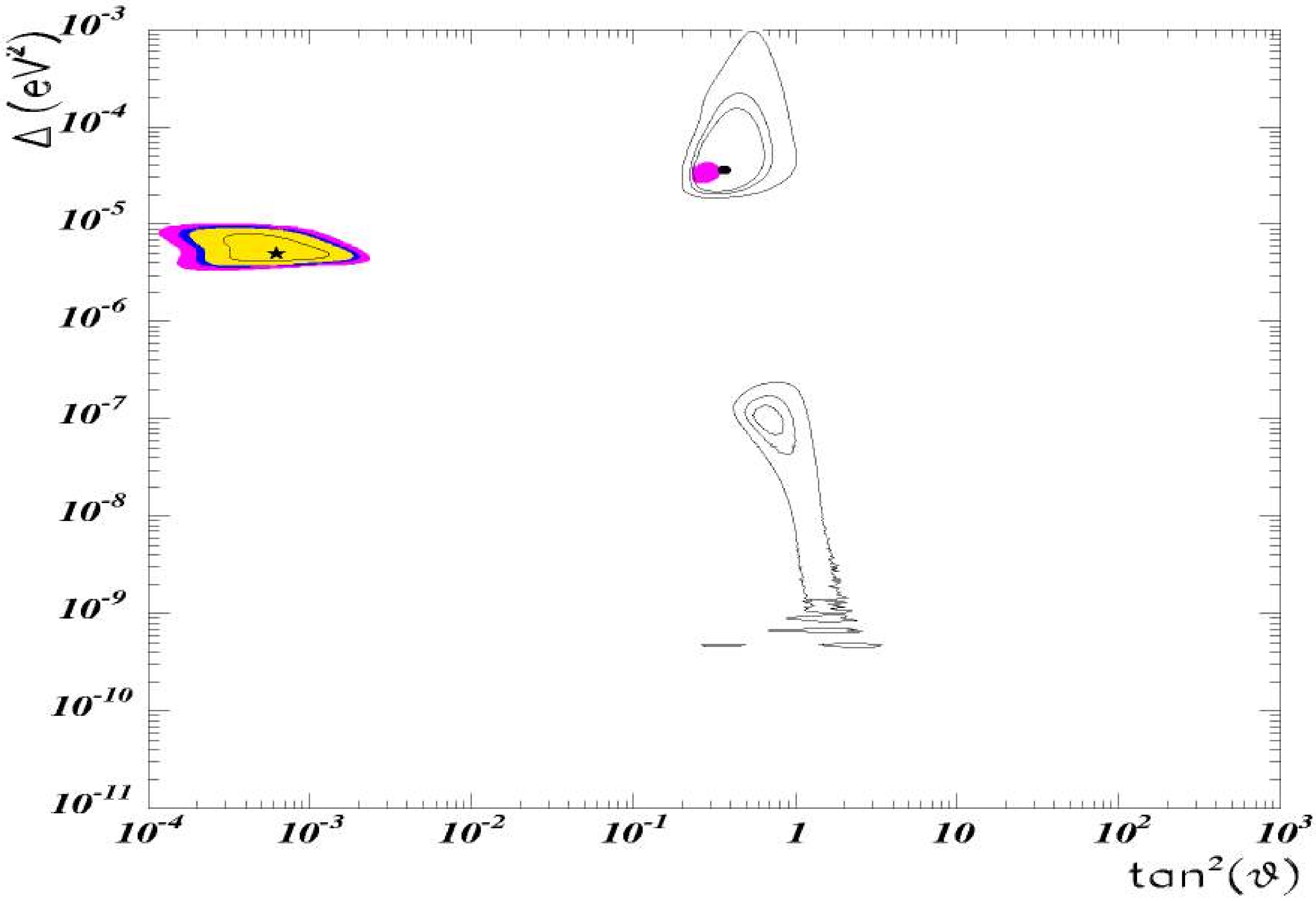
Results of combined solar + SN1987A fit





$E_b = 3 \times 10^{53}$ erg, $\langle E_{\bar{\nu}_e} \rangle = 14$ MeV
$\tau = 1.4$ (top), $\tau = 1.7$ (bottom).

5. Conclusions:

- resonance point and pmva coincide only for linear profile
- local (⇒ pmva) and global (⇒ adiabaticity condition) non-adiabaticity should be distinguished
- crossing probability can be calculated within the WKB approximation also in the non-resonant region for arbitrary power law profile
- combined fit of solar and SN 1987A data
  1. disfavours VO solution at 99% CL for all reasonable SN parameter
  2. allows LOW solution at 95% CL only for abnormal low values of E_{b}, $\langle E_{\bar{\nu}_{\mathrm{e}}} \rangle$ and τ
  3. favours low Δ values in LMA-MSW
  4. improves generally fit of SMA-MSW