Baryogenesis via lepton number violation in Anti-GUT model ‡

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Overview:

- *What is Anti-GUT Model?*
- *Froggatt-Nielsen Mechanism and Mass matrices*
- *Baryogenesis and Sakharov conditions*
- *Baryogenesis via Lepton Number violation*
- *Results and "efficiency" of this model*
- *Conclusion and Future Looking*

‡ Holger Bech Nielsen and YT, Nucl. Phys. B **588** (2000) 281; Nucl. Phys. B **604** (2001) 405; Phys. Lett. B **507** (2001) 241; in preparation

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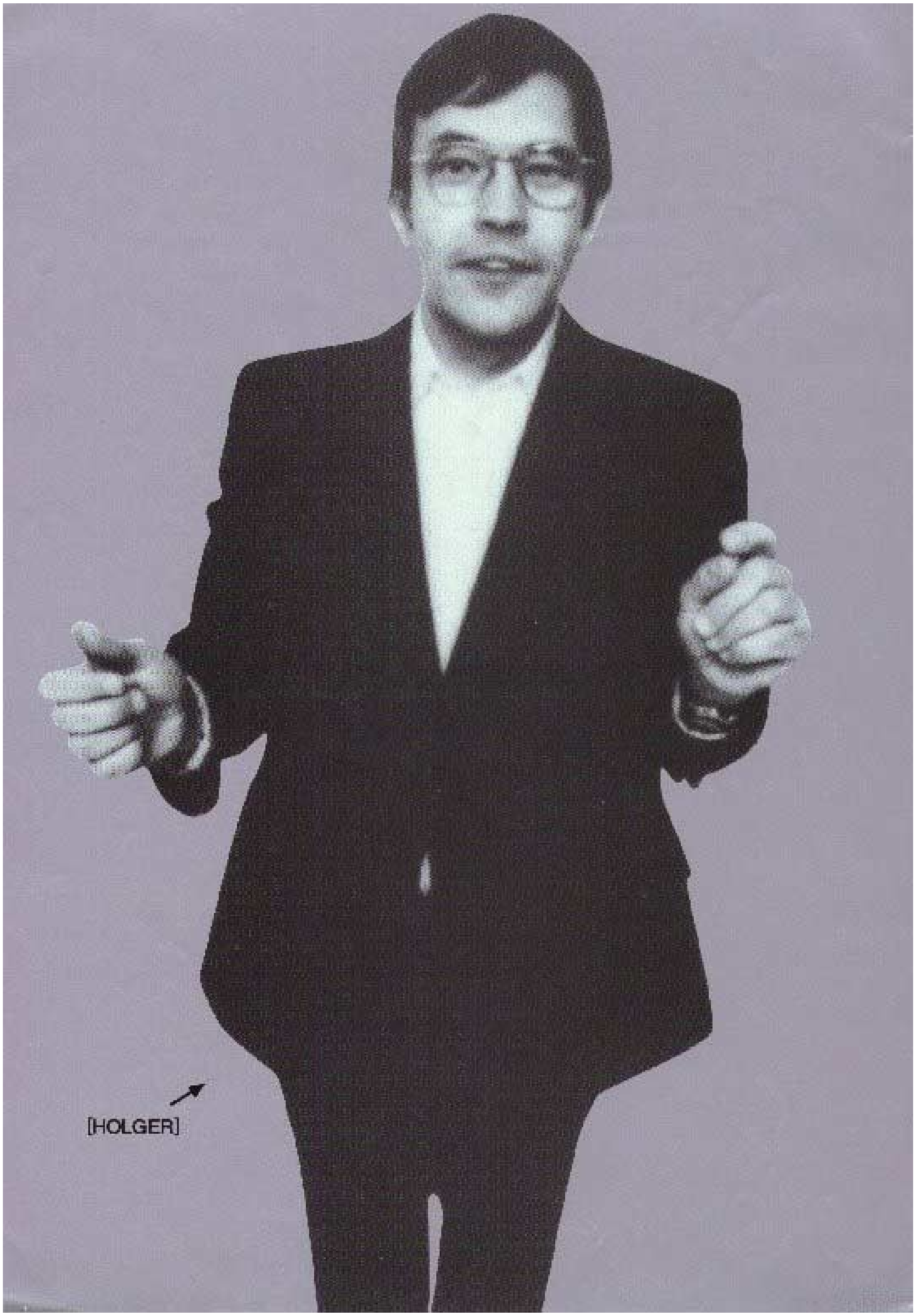

[HOLGER]

Anti-Grand Unification Model

Anti-GUT assumptions:

1. It should only contain transformations transforming the known 45 ($=$ 3 generations of 15 Weyl particles each) Weyl fermions into each other unitarily, $+$ three right-handed Majorana neutrinos ($i.e.$ the gauge group must be a subgroup of $U(48)$).

2. It should be anomaly-free even without using the Green-Schwarz anomaly cancellation mechanism. [M.B. Green and J. Schwarz, Phys. Lett. **B149** (1984) 117]

3. It should NOT unify the irreducible representations under the SM gauge group, $SMG = SU(3) \otimes SU(2) \otimes U(1)$, at the weak scale.

4. It should be as big as possible under the foregoing assumptions.

Introduce of right-handed Neutrinos

See-saw Neutrinos – Majorana Neutrinos

[M. Gell-Mann, P. Ramond, R. Slansky; T. Yanagida (1979)]

- They are Not mass protected by SM

- They can not have No SM quantum numbers

They are so heavy and have to be washed out at high temperature (about 10^{12} GeV) and therefore we can not see now.

$\Rightarrow$ We can "see" them only effectively: Baryogenesis

Introduce of the new scale:

Introduce a new Higgs field ϕ_{B-L}, *i.e.* the $B-L$ quantum numbers are gauged and are broken by ϕ_{B-L}. ($\langle\phi_{B-L}\rangle \approx 10^{12}$ GeV).

The Anti-GUT gauge group it the non-simple gauge group (near Planck scale $\approx 10^{19}$ GeV):

$$\underset{i=1,2,3}{\times} (SMG_i \times U(1)_{B-L,i})$$

This large gauge group is broken down spontaneously to the SM group by 6 Higgs particles. family specific gauge group, each generation $\subset SO(10)$

Weakhyperchages from $U(1)$ quantum numbers

1. The charge quantisation rule:

$$\frac{d_i}{2} + \frac{t_i}{3} + \frac{y_i}{2} = 0 \pmod 1 ,$$

   where t_i is the triality of the $SU(3)$ representation:

$$\begin{cases} t_i = 1 \pmod 3 & \text{for quarks} \\ t_i = -1 \pmod 3 & \text{for anti} - \text{quarks} \\ t_i = 0 \pmod 3 & \text{for gluons} \end{cases}$$

   and d is the "duality" of the $SU(2)$ representation:

$$\begin{cases} d_i = 0 & \text{for integer spin } SU(2)_i \text{ repre.} \\ d_i = 1 & \text{for half} - \text{integer spin } SU(2)_i \text{ repre.} \end{cases}$$

2. always take the smallest allowed representation

We find the quantum charges from anomaly calculations:

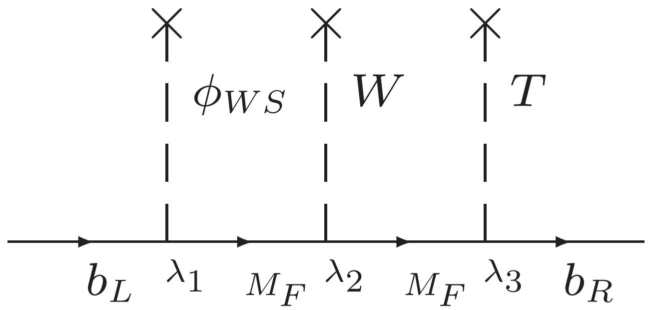
Table 1: The all $U(1)$ quantum charges in extended Anti-GUT

	SMG_1	SMG_2	SMG_3	$U_{B-L,1}$	$U_{B-L,2}$	$U_{B-L,3}$
u_L, d_L	$\frac{1}{6}$	0	0	$\frac{1}{3}$	0	0
u_R	$\frac{2}{3}$	0	0	$\frac{1}{3}$	0	0
d_R	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	0	0
e_L, ν_{e_L}	$-\frac{1}{2}$	0	0	-1	0	0
e_R	-1	0	0	-1	0	0
ν_{e_R}	0	0	0	-1	0	0
c_L, s_L	0	$\frac{1}{6}$	0	0	$\frac{1}{3}$	0
c_R	0	$\frac{2}{3}$	0	0	$\frac{1}{3}$	0
s_R	0	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	0
μ_L, ν_{μ_L}	0	$-\frac{1}{2}$	0	0	-1	0
μ_R	0	-1	0	0	-1	0
ν_{μ_R}	0	0	0	0	-1	0
t_L, b_L	0	0	$\frac{1}{6}$	0	0	$\frac{1}{3}$
t_R	0	0	$\frac{2}{3}$	0	0	$\frac{1}{3}$
b_R	0	0	$-\frac{1}{3}$	0	0	$\frac{1}{3}$
τ_L, ν_{τ_L}	0	0	$-\frac{1}{2}$	0	0	-1
τ_R	0	0	-1	0	0	-1
ν_{τ_R}	0	0	0	0	0	-1
ϕ_{WS}	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{2}{3}$	1	$-\frac{1}{3}$
S	$\frac{1}{6}$	$-\frac{1}{6}$	0	$-\frac{2}{3}$	$\frac{2}{3}$	0
W	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	-1	$\frac{1}{3}$
ξ	$\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	0
T	0	$-\frac{1}{6}$	$\frac{1}{6}$	0	0	0
χ	0	0	0	0	-1	1
ϕ_{B-L}	$-\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{2}{3}$	$-\frac{2}{3}$	2

Froggatt-Nielsen mechanism and Mass matrices

[C.D. Froggatt and H.B. Nielsen, Nucl. Phys. **B147** (1979) 277]

To get the mass matrices we use Froggatt-Nielsen mechanism:



Tree diagram for bottom quark mass in the AGUT model. The crosses indicate the couplings of the Higgs fields to the vacuum and the fundamental Yukawa couplings λ_i are of *order unity complex random numbers*.
$\longrightarrow$ this model can only predict order of magnitude.

More technical correction – "Factorial Factor Correction" (correction for Feynman diagram):

$$\phi_{WS}\, S^\alpha\, W^\beta\, T^\gamma\, \xi^\delta\, \chi^\epsilon$$
$$\to \sqrt{\frac{(\alpha+\beta+\gamma+\delta+\epsilon+1)!}{\alpha!\,\beta!\,\gamma!\,\delta!\,\epsilon!}}\,\phi_{WS}\, S^\alpha\, W^\beta\, T^\gamma\, \xi^\delta\, \chi^\epsilon$$

Random walk in complex plane

the uct-quarks:

$$M_U \simeq \frac{\langle \phi_{\mathrm{WS}} \rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}SWT^2\xi^2 & 6\sqrt{10}SWT^2\xi & 6\sqrt{35}S^2W^2T\xi \\ 12\sqrt{105}S^2WT^2\xi^3 & 2\sqrt{3}WT^2 & 2\sqrt{15}SW^2T \\ 2\sqrt{35}S^3\xi^3 & \sqrt{2}S & \sqrt{6}WT \end{pmatrix}$$

the dsb-quarks:

$$M_D \simeq \frac{\langle \phi_{\mathrm{WS}} \rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}SWT^2\xi^2 & 6\sqrt{10}SWT^2\xi & 2\sqrt{105}S^2T^3\xi \\ 2\sqrt{15}WT^2\xi & 2\sqrt{3}WT^2 & 2\sqrt{5}ST^3 \\ 6\sqrt{210}SW^2T^4\xi & 2\sqrt{210}SW^2T^4 & \sqrt{6}WT \end{pmatrix}$$

the charged leptons:

$$M_E \simeq \frac{\langle \phi_{\mathrm{WS}} \rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}SWT^2\xi^2 & 60\sqrt{14}S^3WT^2\xi^3 & 60\sqrt{2002}S^3WT^4\xi^3\chi \\ 6\sqrt{30030}S^4WT^2\xi^5 & 2\sqrt{3}WT^2 & \sqrt{210}WT^4\chi \\ 36\sqrt{3233230}S^7W^2T^4\xi^5 & 30\sqrt{14}S^3W^2T^4 & \sqrt{6}WT \end{pmatrix}$$

Table 2: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
m_u	3.1 MeV	4 MeV
m_d	6.6 MeV	9 MeV
m_e	0.76 MeV	0.5 MeV
m_c	1.29 GeV	1.4 GeV
m_s	390 MeV	200 MeV
m_μ	85 MeV	105 MeV
M_t	179 GeV	180 GeV
m_b	7.8 GeV	6.3 GeV
m_τ	1.29 GeV	1.78 GeV
V_{us}	0.21	0.22
V_{cb}	0.023	0.041
V_{ub}	0.0050	0.0035
J_{CP}	1.04×10^{-5}	$2-3.5 \times 10^{-5}$
$\tilde{\chi}^2$	1.46	—

The VEV of these Higgs fields (in Planck unit):

$$\langle S \rangle = 0.721 \quad , \quad \langle W \rangle = 0.0945$$

$$\langle T \rangle = 0.0522 \quad , \quad \langle \xi \rangle = 0.0331$$

All fantasy Higgs field have VEV $\sim 1/10$ in Planck unit.

Neutrino mass matrices

the Dirac neutrinos:

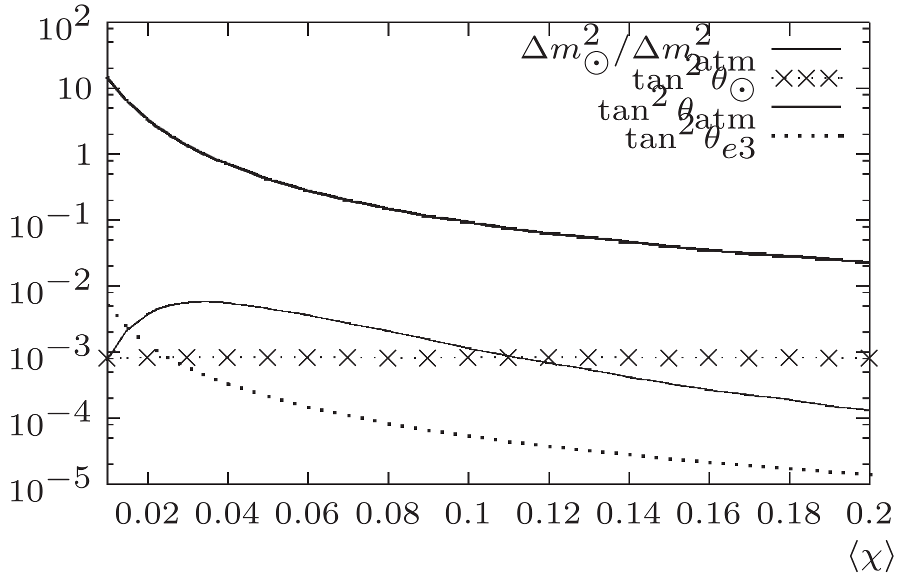
$$M_\nu^D \simeq \frac{\langle \phi_{\rm WS} \rangle}{\sqrt{2}} \begin{pmatrix} 6\sqrt{35}\, S\, W\, T^2\, \xi^2 & 60\sqrt{14}\, S^3\, W\, T^2\, \xi^3 & 60\sqrt{154}\, S^3\, W\, T^2\, \xi^3\, \chi \\ 6\sqrt{35}\, S^2\, W\, T^2\, \xi & 2\sqrt{3}\, W\, T^2 & 2\sqrt{15}\, W\, T^2\, \chi \\ 6\sqrt{70}\, S^2\, W\, T\, \xi\, \chi & 2\sqrt{6} W\, T \chi & \sqrt{6} W\, T \end{pmatrix}$$

and the Majorana neutrinos:

$$M_R \simeq \langle \phi_{\rm B-L} \rangle \begin{pmatrix} 2\sqrt{210} S^3 \chi^2 \xi^2 & \sqrt{15} S \chi^2 \xi & \sqrt{6} S \chi \xi \\ \sqrt{15} S \chi^2 \xi & \sqrt{6} S \chi^2 & \sqrt{\frac{3}{2}} S \chi \\ \sqrt{6} S \chi \xi & \sqrt{\frac{3}{2}} S \chi & S \end{pmatrix}$$

Note that the random complex order of unity which are supposed to multiply all the mass matrix elements are not here represented.

Numerically calculation of the neutrino oscillation



We find from figure:

$$\begin{aligned}
\langle\chi\rangle &= 0.0345 \\
\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} &= 5.8\,{}^{+30}_{-5} \times 10^{-3} \\
\tan^2\theta_\odot &= 8.3\,{}^{+21}_{-6} \times 10^{-4} \\
\tan^2\theta_{e3} &= 4.3\,{}^{+11}_{-3} \times 10^{-4} \\
\tan^2\theta_{\rm atm} &= 0.97\,{}^{+2.5}_{-0.7} \\
\sum_{i=1}^{3} U^2_{ei}\, m_i \ {\rm eV} &= 5.9\,{}^{+5.3}_{-2.8} \times 10^{-5}\ {\rm eV}
\end{aligned}$$

Baryogenesis and Sakharov conditions

The cosmological baryon asymmetry – the ratio of the baryon density to the entropy density of the Universe:

$$Y_B = \frac{n_B}{s} = (0.1 - 1) \times 10^{-10}$$

Sakharov conditions

[A. D. Sakharov, JETP Lett. 5 (1976) 24]

How does occur Matter-Anti-matter asymmetry in Expanding Universe ?

- *Baryon number violation*
- C *and* CP *violation*
- *out-of-equilibrium*

Fukugita-Yanagida scenario

[M. Fukugita and T. Yanagida, Phys. Lett. **B147** (1986) 45]

Right-handed Majorana neutrino decay in Lepton number violating way
$\Rightarrow$ B is violation due to Sphaleron process.

[V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **B155** (1985) 36]

Why Fukugida-Yanagida method?

- *SM Higgs mass* $\sim 115\ \mathrm{GeV}$ *(?)*

- $MSSM$ *is consistent in electroweak phase transition scenario*
  Mass of right-handed stop $\lesssim M_t$
  Mass of left-handed stop $\gtrsim 1\ \mathrm{TeV}$

[M. Laine, hep–ph/0010275; M. Carena $et\ al.$, hep–ph/0011055]

CP Violation of Majorana Decay

[M. A. Luty, Phys. Rev. **D45** (1992) 455; W. Buchmüller and M. Plümacher, Phys. Lett. **B431** (1998) 354; L. Covi, E. Roulet and F. Vissani, Phys. Lett. **B384** (1996) 169]

$$\epsilon_i \equiv \frac{\Gamma_{N_{R_i}\ell} - \Gamma_{N_{R_i}\bar{\ell}}}{\Gamma_{N_{R_i}\ell} + \Gamma_{N_{R_i}\bar{\ell}}},$$

where $\Gamma_{N_{R_i}\ell} \equiv \sum_{\alpha,\beta} \Gamma(N_{R_i} \to \ell^\alpha \phi_{WS}^\beta)$ and $\Gamma_{N_{R_i}\bar{\ell}} \equiv \sum_{\alpha,\beta} \Gamma(N_{R_i} \to \bar{\ell}^\alpha \phi_{WS}^{\beta\dagger})$ are the N_R decay rates (in the N_{R_i} rest frame).

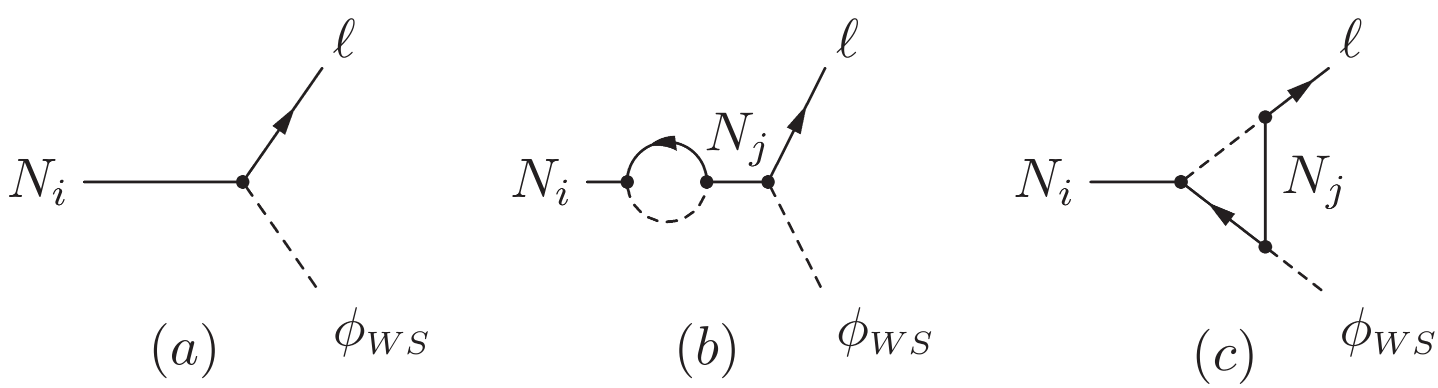


Figure 1: Tree level (a), self-energy (b) and vertex (c) diagrams contributing to heavy Majorana neutrino decays.

$$\epsilon_i = \frac{1}{4\pi \left\langle \phi_{WS} \right\rangle^2 ((M_\nu^D)^\dagger M_\nu^D)_{ii}} \sum_{j \neq i} \mathrm{Im}[((M_\nu^D)^\dagger M_\nu^D)^2_{ji}] \times \left[f\left(\frac{M_j^2}{M_i^2}\right) + g\left(\frac{M_j^2}{M_i^2}\right) \right]$$

where the function, $f(x)$, comes from the one-loop vertex contribution and the other function, $g(x)$, comes from the self-energy contribution. These functions can be calculated in perturbation theory only for differences between Majorana neutrino masses which are sufficiently large compare to its decay widths, $i.e.$ the mass splittings satisfy the condition, $|M_i - M_j| \gg |\Gamma_i - \Gamma_j|$:

$$\begin{aligned} \text{vertex :} \qquad f(x) &= \sqrt{x}\left[1 - (1+x)\ln\frac{1+x}{x}\right], \\ \text{self} - \text{energy :} \qquad g(x) &= \frac{\sqrt{x}}{1-x}. \end{aligned}$$

The crucial parameter – ratio of decay width and Hubble constant ($i = 1, 2, 3$)

$$\begin{aligned} K_i &\equiv \frac{\Gamma_i}{2H}\Big|_{T=M_i} \\ &= \frac{M_{\mathrm{Planck}}}{1.66\,\langle\phi_{WS}\rangle^2\, 8\pi g_{*\,i}^{1/2}} \frac{((M_\nu^D)^\dagger M_\nu^D)_{ii}}{M_i}\,, \end{aligned}$$

where Γ_i is width of the flavour i Majorana neutrino, M_i is its mass and $g_{*\,i}$ is degree of freedom (~ 100 in SM).

$$\begin{aligned} g_{*\,i} &= \sum_{j=\mathrm{bosons}} g_j \left(\frac{T_j}{T}\right)^4 + \frac{7}{8} \sum_{j=\mathrm{fermions}} g_j \left(\frac{T_j}{T}\right)^4 \\ &= \underbrace{28 + \frac{7}{8}\cdot 90}_{\text{Standrad Model}} + \underbrace{\frac{7}{4}\cdot i}_{\text{see-saw particles}} \\ &= \begin{cases} 108.5 & : \quad i = 1 \\ 110.25 & : \quad i = 2 \\ 112 & : \quad i = 3 \end{cases}\,, \end{aligned}$$

here T_j denotes the effective temperature of any species j.

Moreover, we should note here that due to the electroweak sphaleron effect, the baryon number asymmetry Y_B is related to the lepton number asymmetry Y_L: [J. A. Harvey and M. S. Turner, Phys. Rev. **D42** (1990) 3344.]

$$Y_B = a\, Y_{B-L} = \frac{a}{a-1} Y_L$$
$$\text{with } \ a = \frac{8N_f + 4N_H}{22N_f + 13N_H}\,,$$

where N_f is the number of generations and N_H the number of Higgs doublets, this reads in the SM $a = 28/79$.

Dilution factor – κ_i: [E. W. Kolb and M. S. Turner, *The Early Universe*; A. Pilaftsis, Int. J. Mod. Phys. **A14** (1999) 1811.]

$$\begin{aligned}
10 \lesssim K_i \lesssim 10^6 : \qquad \kappa_i &= -\frac{0.3}{K_i (\ln K_i)^{\frac{3}{5}}} \\
1 \lesssim K_i \lesssim 10 : \qquad \kappa_i &= -\frac{1}{2\,K_i}\,, \\
0 \lesssim K_i \lesssim 1 : \qquad \kappa_i &= -\frac{1}{6}\,.
\end{aligned}$$

Due to the random number couplings we should use more smooth dilution factor in the range $0 \lesssim K_i \lesssim 10$:

$$0 \lesssim K_i \lesssim 10 : \qquad \kappa_i = -\frac{1}{2\sqrt{{K_i}^2 + 9}}$$

Using Dirac- and Majorana- mass matrices:

$$\begin{aligned} \epsilon_3 &= 6.8 \times 10^{-9} \qquad K_3 = 1.06 \\ \epsilon_2 &= 6.0 \times 10^{-9} \qquad K_2 = 4.29 \\ \epsilon_1 &= 4.8 \times 10^{-10} \quad K_1 = 19.8 \end{aligned}$$

We have now all informations which we need to calculate Bayogenesis!

$$\begin{aligned} Y_B &= \sum_{i=1}^{3} Y_{B,\,i} \\ &= \sum_{i=1}^{3} \kappa_i \frac{\epsilon_i}{g_{*\,i}} = 1.46^{+5.87}_{-1.17} \times 10^{-11}\,. \end{aligned}$$

Conservation of the $B - L$-quantum charge

A priori the excess of $(B - L)$ quantum number risk to be diluted or washed out before the "accidental" $(B - L)$ conservation of the SM sets in.

we define a new "new charge" $(\widehat{B - L})_3$ *(proto* $(B - L)_3$*) which is washed away much more slowly.*

In the era until the lightest right-handed neutrino has become so hard to produce:

- *no more inverse decay processes producing* M_1
- 2*-by-*2 *scatterings are supposed negligible*

Then, we have effective conservation of the $(\widehat{B - L})_3$.

Correction of the dulition factor: $\sim exp(-K_3 \sin^2\theta_R)$, and $K_2 \sim 4$, therefore our calculation is o.k.

What do we learn?

All mass matrix elements are suppressed due to the top quark mass.
This means

$$\epsilon_{3\,\mathrm{No-SUSY}} \approx \epsilon_{1\,\mathrm{SUSY}}$$

Baryogenesis comes Heaviest Majorana Neutrino decay!!.

In the SUSY case the Baryogenesis comes from the lightest Majorana neutrino decay.
But No Gravitino problem – not so strong wash-out for heaviest Majorana neutrino. [for example: T. Asaka *et al.*, Phys. Rev. **D61** (2000) 083512]

Mass matrix of Buchmüller, Plümacher and Yanagida type – $SU(5) \times U(1)_{FN}$

The case with a nonparallel family structure where the chiral $U(1)_{FN}$ charges are different for the $\mathbf{5}^*$-plets and the $\mathbf{10}$-plets of the same family: ($g_* \sim 200$ and $\epsilon^2 \sim 1/300$)

$$
\begin{aligned}
h_\nu &\sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \end{pmatrix} \\
h_r &\sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \\
m_{\nu_{ij}} &\sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \frac{v_2^2}{\langle R \rangle} .
\end{aligned}
$$

Then

$$\epsilon_1 \sim \frac{3}{16\,\pi}\,\epsilon^4$$

$$\Rightarrow Y_B \sim \kappa \times 10^{-8}$$

"Efficiency" of this model

- quark and charged lepton masses: 9
- quark and charged mixing angles (+ Jarlskog triangle): 4
- mass square differences for neutrino: 2
- mixing angles for neutrino: 3
- Baryogenesis and neutrinoless double beta decay: 2

Total predictions are 20

However we have taken into the predictions two quantities namely, $\tan^2\theta_{e3}$ and "effective" Majorana neutrino mass for which only experimental upper bounds exist.

Number of parameters

	"Yukawa"	"Neutrino"	# of parameters	# of predictions
Standard Model	14	7	20	—
"Old" Anti-GUT	4	—*	4†	9
Anti-GUT	6		6	14

* The "old" Anti-GUT can not predict the neutrino oscillation ($0\nu2\beta$) either Baryogenesis (without Majorana neutrinos).

† here we have not counted the neutrino oscillation parameters.

Conclusion

We have presented a model in which it is assumed that:

- An under some condition maximal gauge group $\underset{i=1,2,3}{\times}(SMG_i \times U(1)_{B-L,i})$.
- the set of fermion and scalar fields given by the table.
- All couplings and mass parameters are in Planck scale of order unity, so that only VEVs are of different order of magnitudes (in the string scale). The order of one couplings can be treated statistically ($i.e.$ as random numbers).

The predictions of Anti-GUT (*only order of magnitude*):

$$
\begin{aligned}
\frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} &= 5.8{}^{+30}_{-5} \times 10^{-3} \\
\tan^2\theta_\odot &= 8.3{}^{+21}_{-6} \times 10^{-4} \\
\tan^2\theta_{e3} &= 4.3{}^{+11}_{-3} \times 10^{-4} \\
\tan^2\theta_{\rm atm} &= 0.97{}^{+2.5}_{-0.7} \\
Y_B &= 1.46{}^{+5.87}_{-1.17} \times 10^{-11} \\
|\langle m_{ee}\rangle| &= 5.9{}^{+5.3}_{-2.8} \times 10^{-5}\ {\rm eV}\,.
\end{aligned}
$$

Note that the Anti-GUT model is very successful to describe neutrino oscillations, their mixing angles $(0\nu 2\beta)$ and also Baryogenesis.

Future looking

- Baryogenesis using Boltzmann equation
- Proton decay – too stable!
- LMA-MSW ?
- Dark matter study (Monopole??)
- Anti-GUT $\Rightarrow$ SUSY-GUT?

  If SUSY particles exist ...