

INSIGHTS ON ν MASSES FROM LFV

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hep-ph/0106245

PLAN

- Upper limits on $BR(\tau \rightarrow \mu \gamma)$, $BR(\mu \rightarrow e \gamma)$
can be translated into
upper limits on a certain combination of
 y_D, M_R
- possibility of discriminating between
seesaw models
by improving the exp. sensitivity on those BR's?

STARTING POINT

$$m_\nu \ll m_{q,e}$$
$$\theta_{atm} \approx \pi/4$$

→ mech. responsible for m_ν different from the one at work for $m_{q,e}$

MAYBE $\nu = \text{Majorana}$, ↗

SEE-SAW ($+\nu^c$): elegant explanation

$$m_{\text{eff}}^\nu = y_D^T \frac{1}{M_R} y_D v^2$$

if $y_D \sim O(1) \rightarrow M_R \approx M_{\nu} \lesssim M_{g.c.v.}$
MSSM

CONVENTIONS:

- $\bar{R} u_D L$
- basis where $M_R, M_e = \text{diag}$

BUT even if we precisely knew $m_{\text{eff}}^\nu = U_{MNS}^* \text{diag}(m_1, m_2, m_3) U_{MNS}^+$ it wouldn't be possible to DISENTANGLE y_D and M_R

E.G. (unlike quarks) **NO ACCESS TO Left-mixing in y_D from m_{eff}^ν .**

$$m_{\text{eff}}^\nu = V_L^* d_D U_R^T \frac{1}{M_R} \underbrace{U_R d_D V_L^+}_{y_D}$$

$V_L = U_{MNS}$ ONLY IF
 $U_R = \mathbb{1}$ and/or $M_R \propto \mathbb{1}$

NEVERTHELESS from present knowledge on m_{eff}^ν

- identify the phen. acceptable PATTERNS of y_D, M_R
- useful criterion to classify them.

IN ORDER TO PROCEED?

⇒ ADDITIONAL INFORMATIONS NEEDED

CAN LFV PROVIDE ADDITIONAL INFORMATIONS?

$$BR(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11} \dots? \rightarrow 10^{-14}$$

$$BR(\tau \rightarrow \mu\gamma) < 1.1 \cdot 10^{-6} \dots? \rightarrow 10^{-9}$$

3 o.o.f.m. improvement
UNDER STUDY
→ direct searches
→ slepton decays

→ IT HAS BEEN SHOWN (also in connection with susy p.s. for a_μ)
some simple realization of the see-saw predict
 $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ at HAND

[King et al, Buchmuller et al, Ellis et al, Casas et al, Carvalho et al,
Sato et al, Hisano et al, Blazek et al, ...]

* IS IT A GENERAL FEATURE OF THE SEE-SAW?

→ I WILL ILLUSTRATE

- expected improvements ARE ABLE to provide the desired additional informations?
- susy < TeV, $\tau \rightarrow \mu\gamma$ a PROMISING TOOL to (hopefully) learn whether

y_D possesses
small ← quarks
large ← left mixings
light ν

... NOT an ACADEMIC distinction!

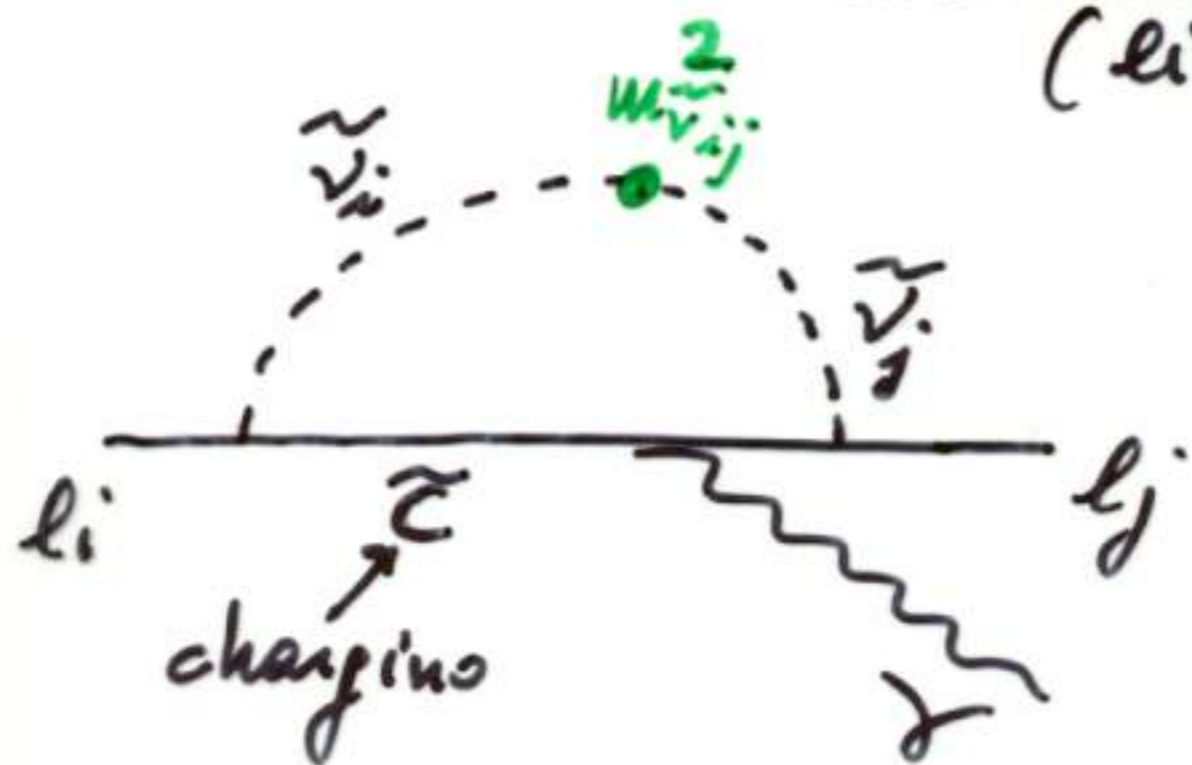
BR ($l_i \rightarrow l_j \gamma$)

- **SM** BRs \ll present bounds and **NEGLECTIBLE** due to $M_V \ll M_G$
- **MSSM** MAJOR PROBLEM: justify that ν has not already been found at a rate substantially larger than in SM

flavour dependence in soft terms \rightarrow loops with $\tilde{\nu}$ give rise to LFV because of mixing in $M_{\tilde{\nu}}^2$

E.g.: $M_{\tilde{\nu}}^2_{ij} \neq 0$

dominant contribution (like a_μ but with $m_{\tilde{\nu}}^2$)



$M_{\tilde{e}}^2_{ij} \neq 0$ **STRONGLY BOUNDED** by exp.

$A_{ij} = f(m_{\tilde{\nu}}, M_2)$
 mean $m_{\tilde{\nu}}$ gaugino $\sim t_\beta (laye t_\beta)$
 $\frac{M_{\tilde{\nu}}^2_{ij}}{M_{\tilde{\nu}}^2} \equiv \delta_{ij}$

WAY OUT: flavour indep. soft terms at $M_U \sim M_{PE}$
 $M_0^2 = M_{\tilde{L}}^2 = \dots$ universality

EVEN in this case, if ν^c are present, [Borzumati-Morino] running of $M_{\tilde{\nu}}^2$ from M_U to M_R induces $M_{\tilde{\nu}}^2_{ij} \neq 0$ due to y_D :

$$M_{\tilde{\nu}}^2_{ij} \approx -\frac{3}{8\pi^2} M_0^2 \left(y_D^+ \ln \frac{M_U}{M_R} y_D \right)_{ij}$$

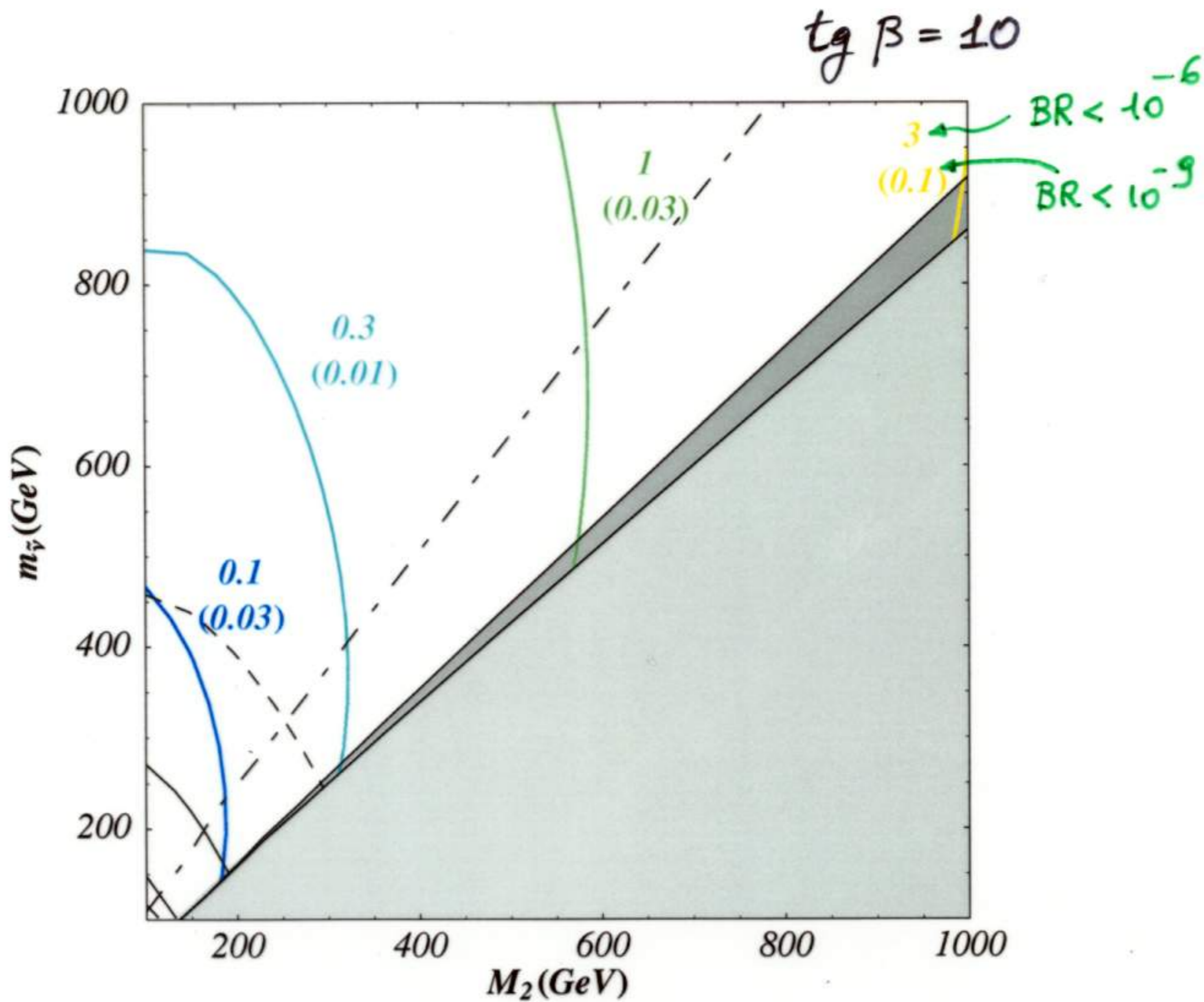
SO $BR_{ij} \leq L_{ij}^{exp}$ SUPPLIES $\left(y_D^+ \ln \frac{M_U}{M_R} y_D \right)_{ij} \leq L_{ij} (m_{\tilde{\nu}}, M_2) \propto \frac{\sqrt{L_{ij}^{exp}}}{t_\beta}$

N.B. these are upper bounds: as usual assume NO CONSPIRACY with other (eventual) LFV sources (fl. dep. soft terms, rad con in GV, unknown) (3)

$\tau \rightarrow \mu \gamma$

UPPER LIMIT ON

$$\delta_{\tau\mu} \equiv \frac{M_{\tilde{V}_{32}}^2}{M_{\tilde{V}}^2}$$

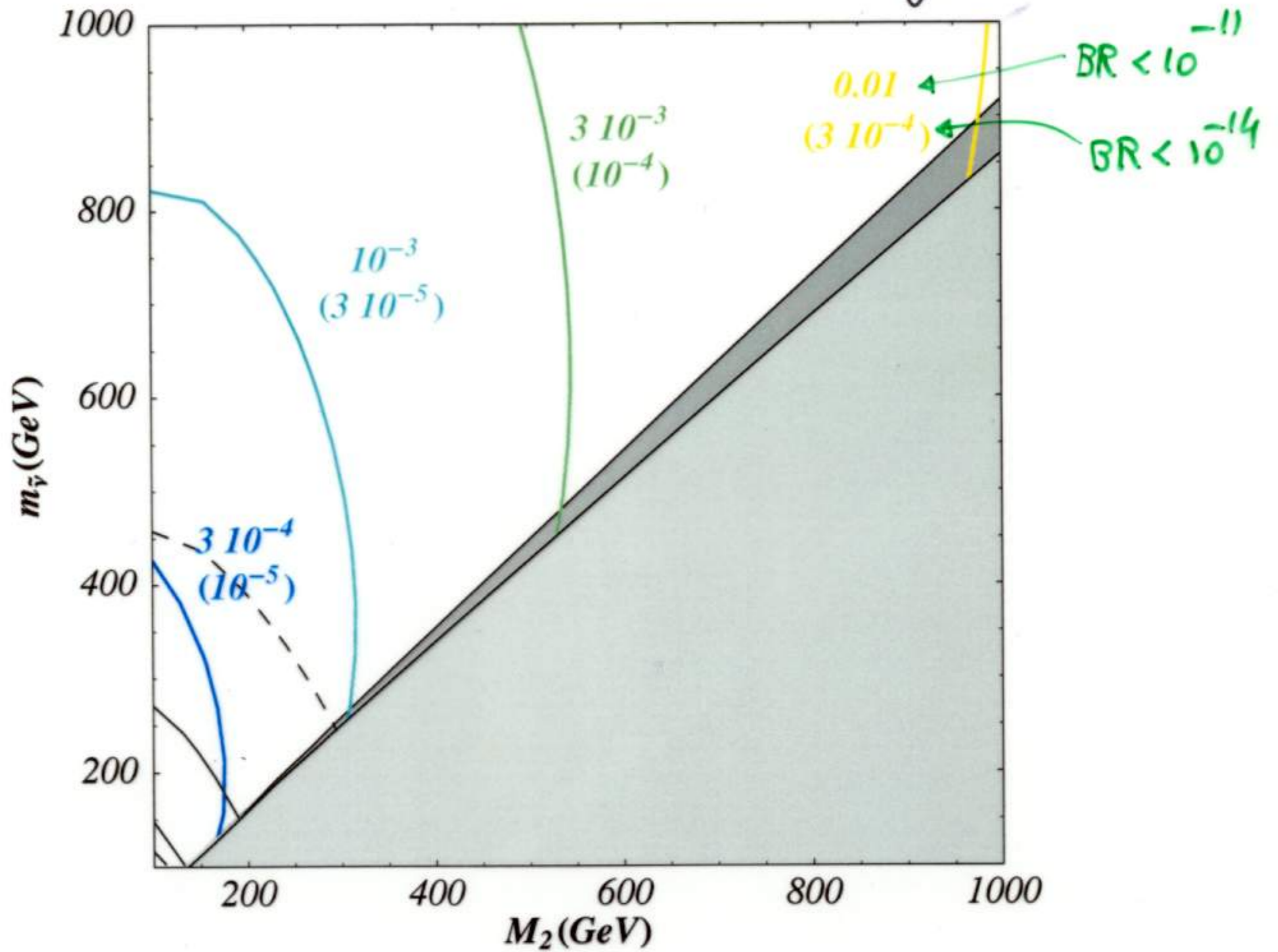


$\mu \rightarrow e\gamma$

UPPER LIMIT ON

$$\sigma_{\mu e} \equiv \frac{M_{\tilde{\nu}21}^2}{M_{\tilde{\nu}}^2}$$

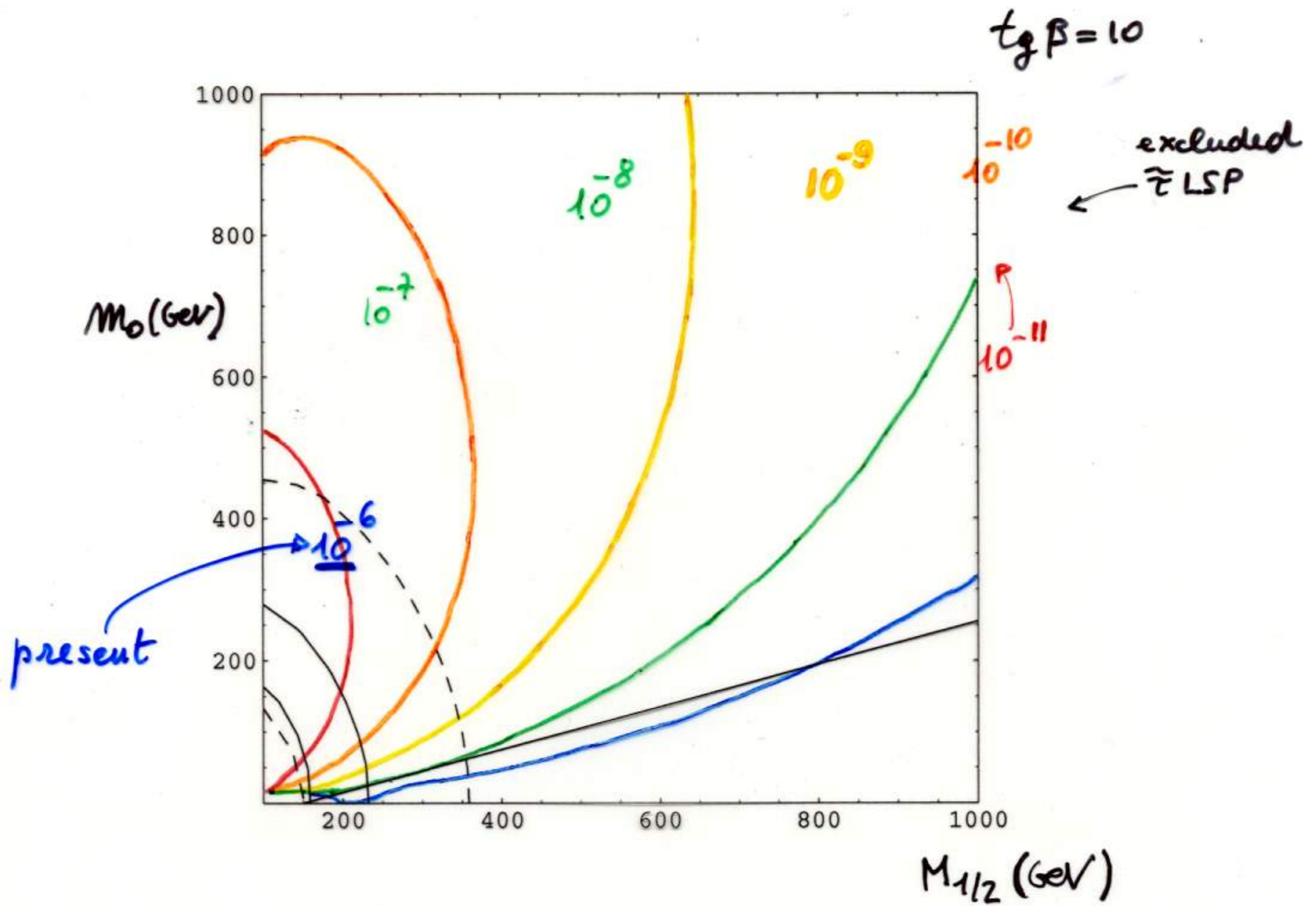
$\tan\beta = 10$



$\tau \rightarrow \mu \gamma$

fix $\left(y_0^+ \ln \frac{M_U}{M_R} y_0 \right)_{32} \approx \ln(2000) \sim 7$

and plot $BR(\tau \rightarrow \mu \gamma)$



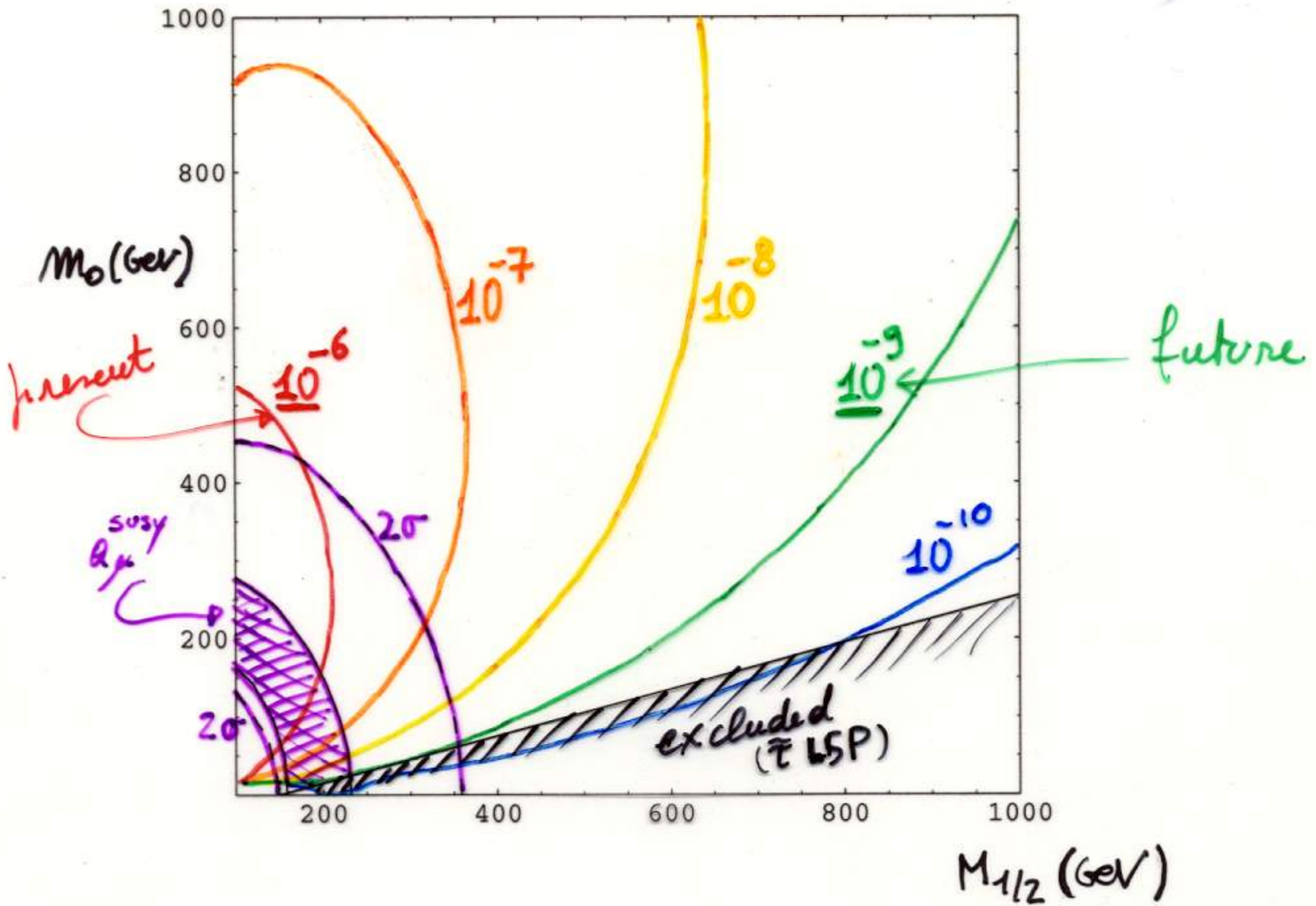
$\tau \rightarrow \mu \gamma$

fix $\left(y_D^+ \ln \frac{M_U}{M_R} y_D \right)_{32} \approx \ln(2 \cdot 10^3)$

$t_{\beta} = 10$

and plot

$BR(\tau \rightarrow \mu \gamma)$

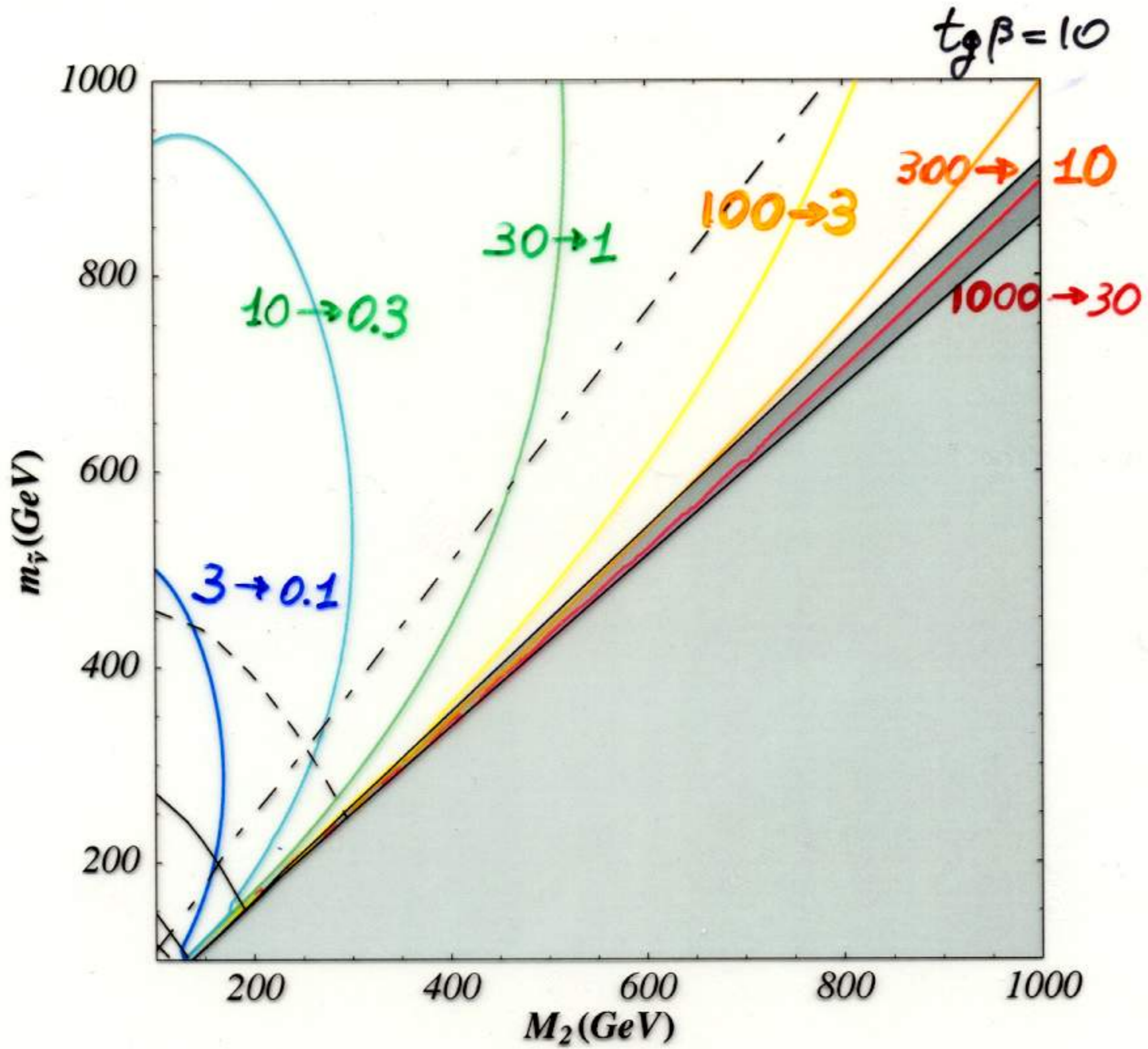


(see Ellis et al.)

$$\tau \rightarrow \mu \gamma$$

fix $BR(\tau \rightarrow \mu \gamma) \leq 10^{-6}$

and plot $L_{\tau\mu} \geq \left(y_D^+ \ln \frac{M_U y_D}{M_R y_D} \right)_{32}$



WHAT'S $(y_D^T \ln \frac{M_U}{M_R} y_D)_{ij}$?

$$y_D = \begin{pmatrix} y_{D11} & y_{D12} \times y_{D13} & y_{D13} \\ y_{D12} + y_{D22} \times y_{D23} & y_{D23} & y_{D23} \\ y_{D3} + y_{D32} \times y_{D33} & y_{D33} & y_{D33} \end{pmatrix}$$

$\ln M_U/M_1$
 $\ln M_U/M_2$
 $\ln M_U/M_3$

IT CAN HAPPEN (e.g. y_D hierarchical)
 that III gen. gives the most sizeable contr.

$$(y_D^T \ln \frac{M_U}{M_R} y_D)_{ij} \approx y_{D3i}^* y_{D3j} \ln \frac{M_U}{M_3}$$

→ some access to V_L

HOWEVER it doesn't hold in general ...

CLASSES OF SEE-SAW MODELS

Consider $3\nu_L$ with hierarchical spectrum $m_1 < m_2 < m_3$

$$m_3^2 \approx \Delta m_{atm}^2 \gg \Delta m_{sol}^2 \approx m_2^2$$

$$\theta_{atm} \sim \pi/4$$

all possible solutions

Useful CRITERION to classify the phen. allowed patterns for y_D and M_R :

which ν^c DOMINATES the 23 block of M_{eff}^{ν} ?

$$M_{atm} \approx y_{D3i}^T \frac{1}{M_i} y_{i3} \nu^2 \quad i=?$$

\equiv the contr. of the other ν^c is smaller by at least

$$\epsilon \equiv \frac{m_2}{m_3} \begin{cases} 0.1 & \text{LMA} \\ 0.04 & \text{SMA} \\ 0.006 & \text{LOW} \end{cases}$$

1) NONE

2) THE HEAVIEST, ν_3^c

3) ONE OF THE LIGHTEST, ν_1^c OR ν_2^c

CLASS 1 : NONE

E.g. 2x2

$$y_D = \begin{pmatrix} a & b \\ x & y \end{pmatrix}$$

$$M_R = \begin{pmatrix} M_2 & 0 \\ 0 & M_3 \end{pmatrix}$$

$$M_{\text{eff}}^{\nu} = \begin{pmatrix} \frac{x^2}{M_3} + \frac{a^2}{M_2} & \frac{xy}{M_3} + \frac{ab}{M_2} \\ \frac{xy}{M_3} + \frac{ab}{M_2} & \frac{y^2}{M_3} + \frac{b^2}{M_2} \end{pmatrix} \nu_u^2$$

" $y_D^2 \sim M_R$ "

→ $m_2 \sim m_3$ invoke a tuning $t \sim \epsilon$
 $t \approx \{ 10\%, 4\%, 0.6\% \}$ to have $\Delta m_{32}^2 \ll \Delta m_{21}^2$
LMA SMA LOW

→ $x \approx y$, $a \approx b$ for $\theta_{\text{atm}} \sim \pi/4$ ← **LARGE LEFT mixing in y_D**

→ $\gamma \sim O(1) \Rightarrow M_3 \sim 5 \cdot 10^{14} \text{ GeV}$

→ **LFV**: $\gamma \sim O(1) \Rightarrow \left(y_D^\dagger \ln \frac{M_u}{M_R} y_D \right)_{32} \sim 7$ **QUITE SHARP PREDICTION FOR $\tau \rightarrow \mu \gamma$**

3x3

Despite t , this is an interesting class: always arises with Abelian $U(1)$ with " $q > 0$ " (no sumy zeros,...), i.e. **simplest fl. symm.**
 [Froggatt Nielsen, Irges Leipzig Remond,]

Rcharges (ν^c charges) disappear in M_{eff}^{ν} : all ν^c contribute at the same level to m_{eff}^{ν} ;
 But, due to \ln , ν_3^c dominates ($\gamma \ln \gamma$)

LFV: $\tau \rightarrow \mu \gamma$ **SHARP**
 $\mu \rightarrow e \gamma$ **DEFINITE**: having allowed for t , then:

(i) **SMA** $y_{31} \sim 10^{-3}$ (problems with m_e)

(ii) **LMA** $y_{31} \sim 0.1$

(iii) **LOW** $y_{31} \sim 0.005$

$\left(y_D^\dagger \ln \frac{M_u}{M_R} y_D \right)_{21} \begin{matrix} \swarrow 7 \cdot 10^{-3} \text{ SMA} \\ \rightarrow 0.7 \text{ LMA} \\ \searrow 0.04 \text{ LOW} \end{matrix}$

CLASS 2: THE HEAVIEST, ν_3^c

$$y_D = \begin{pmatrix} a & b \\ x & y \end{pmatrix} \quad M_R = \begin{pmatrix} M_2 & 0 \\ 0 & M_3 \end{pmatrix} \quad m_{\text{eff}}^\nu = \begin{pmatrix} \frac{x^2}{M_3} + \frac{a^2}{M_2} & \frac{xy}{M_3} + \frac{ab}{M_2} \\ \frac{xy}{M_3} + \frac{ab}{M_2} & \frac{y^2}{M_3} + \frac{b^2}{M_2} \end{pmatrix} \nu_e^2$$

" $y_D^2 > M_R$ "

→ $M_2 \ll M_3 \rightarrow \Delta m_{\text{sol}}^2 \ll \Delta m_{\text{atm}}^2$ naturally obtained

→ $x \approx y$ for $\text{Det} m \sim \frac{1}{4} \leftarrow$ LARGE LEFT mixing in y_D

→ $y \approx 0(1) \rightarrow M_3 \sim 5 \cdot 10^{14} \text{ GeV}$

→ LFV: $y \approx 0(1) \rightarrow (y_D^{\dagger} \frac{M_R}{M_R} y_D)_{32} \sim 7$ QUITE SHARP PREDICTION FOR $\tau \rightarrow \mu \gamma$

3x3

Need for a fl. symm. more flexible than " $U(1) q \geq 0$ "

E.g.: $U(1) q \geq 0$ (susy zeros or more flavours)
[King (SRHND), Altarelli Feruglio,]

→ achieve SMA, LMA, LOW

but no sharp prediction for y_{31}
(→ look at single models)

LFV: $\tau \rightarrow \mu \gamma$ SHARP

$\mu \rightarrow e \gamma$ NOT DEFINITE PREDICTION

CLASS 3: ONE OF THE LIGHTEST, ν_1^c OR ν_2^c

$$y_D = \begin{pmatrix} a & b \\ x & y \end{pmatrix} \quad M_R = \begin{pmatrix} M_2 & 0 \\ 0 & M_3 \end{pmatrix} \quad m_{\text{eff}}^{\nu} = \begin{pmatrix} \cancel{\frac{x^2}{M_3}} + \frac{a^2}{M_2} & \cancel{\frac{xy}{M_3}} + \frac{ab}{M_2} \\ \cancel{\frac{xy}{M_3}} + \frac{ab}{M_2} & \cancel{\frac{y^2}{M_3}} + \frac{b^2}{M_2} \end{pmatrix} \nu_u^2$$

" $y_D^2 < M_R$ "

→ $m_2 \ll m_3 \rightarrow \Delta m_{\text{sol}}^2 \ll \Delta m_{\text{atm}}^2$ naturally obtained

→ $a \approx b$ for $\text{Det} m \sim \pi/4$

$x \leq y$

← NO NEED FOR LARGE LEFT mixing in y_D

→ $\sqrt{\Delta m_{\text{atm}}^2} \approx \frac{b^2}{M_2} \nu_u^2 \Rightarrow y \sim 0(1) \quad M_3 \geq M_{\text{GUT}} \sim 10^{16} \text{ GeV}$

→ LFV: NO SHARP PREDICTION FOR $\tau \rightarrow \mu \gamma, \mu \rightarrow e \gamma$ BUT CAN BE SUPPRESSED

3x3

Need for a fl. symm. which ensures " $y^2 < M$ "

E.g.: $U(1) \times U(1), U(2)$

[Altarelli-Feruglio I.M., Barbieri et al, ...]

→ can achieve SMA, LMA, LOW with small y_{31}, y_{32}

EXAMPLES OF 3

U(1)
920

$\lambda \equiv \lambda_c$ →

$$g_D = \begin{pmatrix} 3 & 2 & 0 & 0 \\ \lambda^3 & \lambda & \lambda & \\ \lambda & 0 & 0 & \\ 1 & 0 & 0 & \end{pmatrix}$$

-2
from H_U

$$M_R = \begin{pmatrix} 3 & 1 & 0 \\ \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda \\ \lambda^3 & \lambda & 1 \end{pmatrix} M_3$$

suitable for realistic SU(5)
[Altarelli, Feriolo I.M.]

canonical form + basis where $M_R, M_c = \text{diag}$

$$g_D = \begin{pmatrix} \lambda^3 & \lambda & \lambda \\ \lambda & \lambda^3 & \lambda^3 \\ 1 & \lambda^2 & \lambda^2 \end{pmatrix}$$

dominant contribution
to $\tau \rightarrow \mu\gamma$
 $(y^+_{\tau\mu\gamma})_{32} \approx \lambda^2 \ln \frac{M_U}{M_1}$

- dominance of M_1
- $\delta_{\text{atm}} \sim \pi/4$
- LOW for solar
- $\begin{cases} M_1 \sim 10^{13} \text{ GeV} \\ M_2 \sim 10^{16} \text{ GeV} \\ M_3 \sim 2 \cdot 10^{17} \text{ GeV} \end{cases}$

U(2)

$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} \text{unity}$

$$g_D = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \epsilon' & \epsilon & 1 \end{pmatrix}$$

- 1, 2 row not greater than 3
- Barbieri et al: dominance of M_1 SMA

for $\tau \rightarrow \mu\gamma$
 $(y^+_{\tau\mu\gamma})_{32} \sim \epsilon \ln \frac{M_U}{M_{3(2)}}$

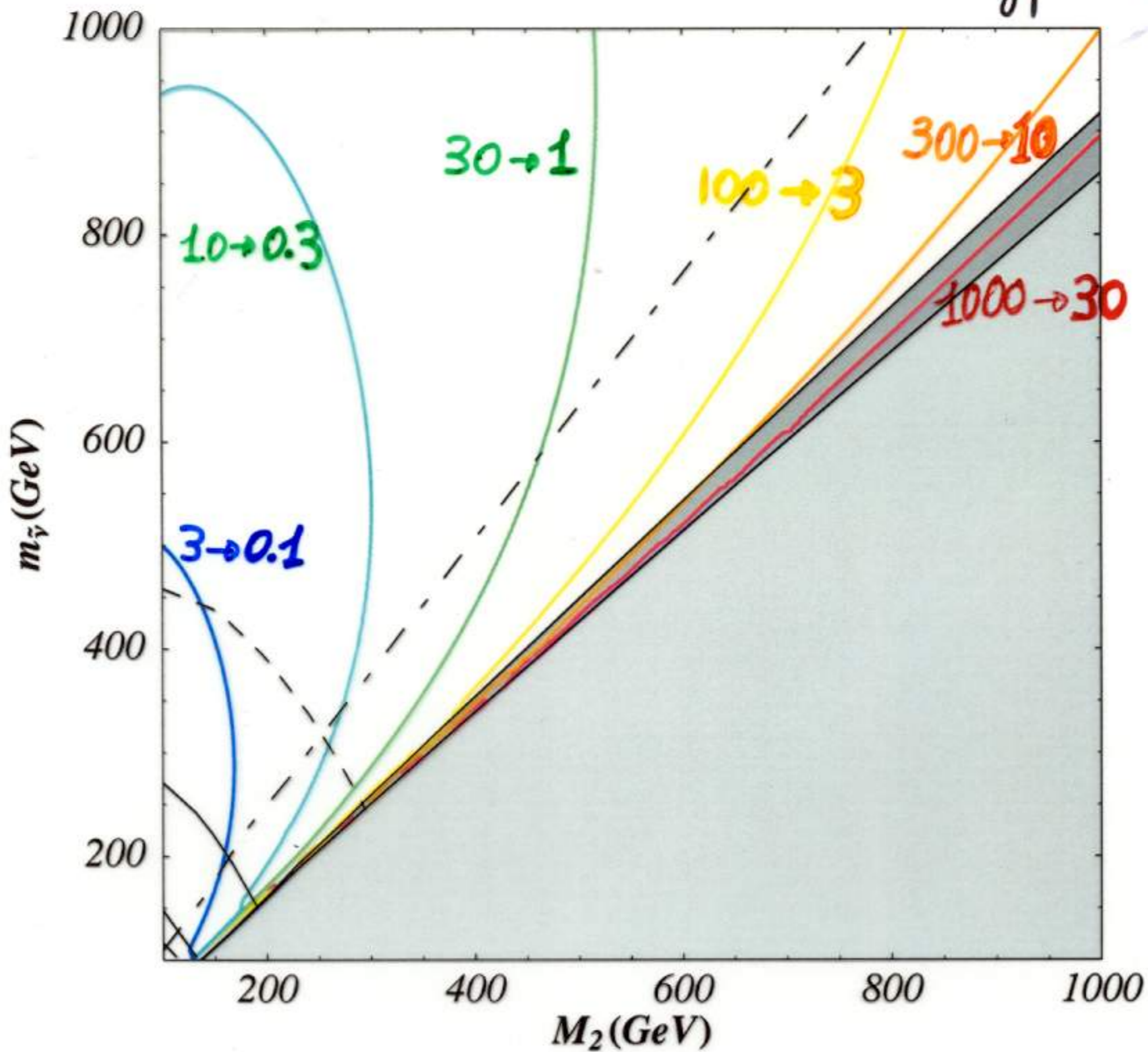
linked to $U(2) \rightarrow U(1)$

$$\tau \rightarrow \mu \gamma$$

fix $BR(\tau \rightarrow \mu \gamma) \leq 10^{-6}$
(10^{-9})

and plot $L_{\tau\mu} \geq \left(y_0^+ \ln \frac{M_U y_0}{M_R} y_0 \right)_{32}$

$$g\beta = 10$$



$$\left(y_0^+ \ln \frac{M_U y_0}{M_R} y_0 \right)_{32} \approx 0(17)$$

for class 1, 2
 \uparrow None \uparrow M_3

$\approx 0(0.4)$ in the ex. of. class 3
 \uparrow M_1, M_2

(If $\text{susy} < \text{TeV end}$)

$$\text{BR} > 10^{-9}$$

Find (M_1, M_2) .

if $\tau \rightarrow \mu \gamma$ is FOUND \Rightarrow ?

If $y_{32} y_{33} \ll 1$
go back, otherwise

$y_{32} \sim y_{33} \sim 0(1)$ $\Rightarrow V_L$ has LARGE 23 LEFT-MIXINGS

\Downarrow UNLIKE quarks! Maybe SU(5) and large R mixings in y_d

\Rightarrow DOMINANCE of M_3 or NONE in m_{eff}^2

\Downarrow ν^c are NOT NECESSARILY hierarchical

\Downarrow
 $M_{\nu^c} \div M_3 \leq M_{\text{GUT}} \text{ (g.c.u. in MSM)}$ $M_3 \sim 5 \cdot 10^{14} \text{ GeV}$

\Rightarrow FLAVOUR SYMMETRY which gives y_D^2 MORE AS hierarchical than M_R as

\Downarrow It can be

NONE \rightarrow ~~\square~~ trivial : $U(1)_{q \geq 0}$

$M_3 \rightarrow$ ~~\square~~ non trivial : $U(1)_{q \geq 0}$,
 $U(1) \times U(1)$,
 $U(2), \dots$

TO PROCEED : $\mu \rightarrow e \gamma$

Also because
(III now done. in y_{leq})

$$\frac{\text{BR}(\mu \rightarrow e \gamma)}{\text{BR}(\tau \rightarrow \mu \gamma)} = \frac{100}{17} \frac{y_{31}}{y_{33}}$$

(If $\text{susy} < \text{TeV}$ and) $\text{BR} < 10^{-9}$

if $\tau \rightarrow \mu \gamma$ is NOT FOUND

$\Rightarrow V_L$ has SMALL 23 LEFT-MIXINGS

\Downarrow LIKE quarks! Maybe LR, SO(10), ...

\Rightarrow DOMINANCE of M_1 or M_2 in m_{eff}^2

\Downarrow ν^c are STRONGLY hierarchical

\Downarrow $M_{1/2} \div M_3 \gtrsim M_{\text{GUT}}$ well for SO(10)
(g.c.u. in MSSM)

\Rightarrow FLAVOUR SYMMETRY which gives y_D^2 LESS hierarchical than M_R

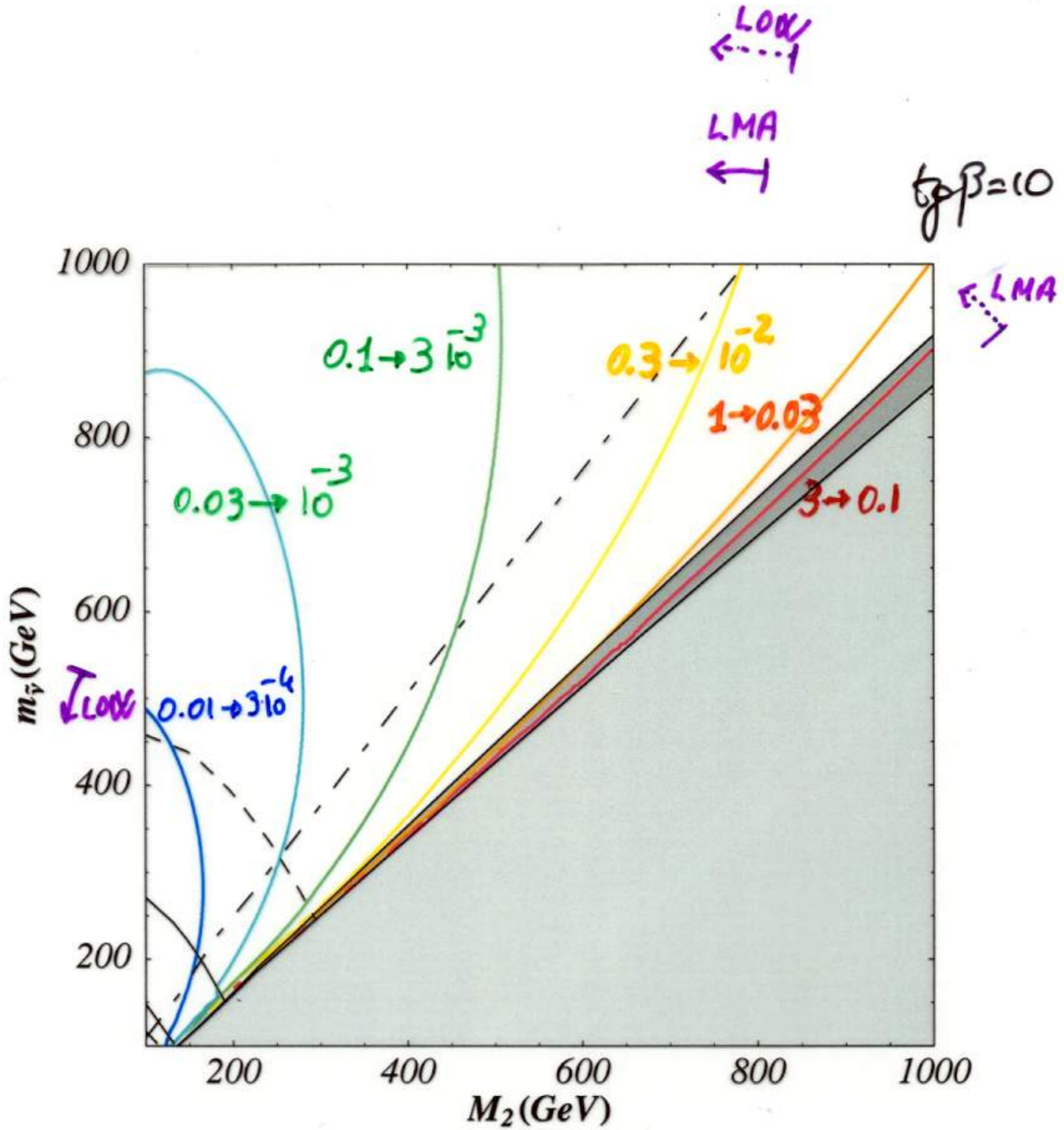
\Downarrow It can be

trivial : $U(1)_{q \geq 0}$

non trivial : $U(1)_{q \geq 0}$,
 $U(1) \times U(1)$,
 $U(2), \dots$

$\mu \rightarrow e \gamma$

fix $BR(\mu \rightarrow e \gamma) \leq 10^{-11}$ (10^{-14}) and plot $L \geq (y_D^\dagger \frac{e u M_U}{M_R} y_D)_{21}$



simplest $U(1)$ (NONE) : $(y_D^\dagger \frac{e u M_U}{M_R} y_D)_{21} \begin{cases} \nearrow 0.7 \text{ LMA} \\ \searrow 0.04 \text{ LOW} \end{cases}$

CONCLUSIONS

- Is it possible to DISCRIMINATE SEE-SAW MODELS by improving the exp. sensitivity to ch. LFV processes?

- (hierarchical spectrum)

CLASSIFYING the see-saw models according to THE MECHANISM WHICH GENERATES

$$\epsilon = \frac{m_2}{m_3} = \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}}$$

ALLOWS TO

- characterize each class through phys. properties: flavour mixing in y_D scale for ν^c masses

- study the predictions for $\tau \rightarrow \mu \gamma$, $\mu \rightarrow e \gamma$

$\tau \rightarrow \mu \gamma$ PROMISING TOOL to identify the origin of $\theta_{atm} \sim \pi/4$ (y_D or see-saw?)

$\mu \rightarrow e \gamma$ more MODEL DEPENDENT

EVEN with 10^{-3} improvement,

$\tau \rightarrow \mu \gamma$, $\mu \rightarrow e \gamma$ COULD ESCAPE DETECTION

in models where Δm_{atm}^2 is determined by one of the lightest ν^c