NEUTRINO MASS PATTERNS: Top Down Approach

Neutrino data: solar neutrinos, atmospheric, (LSND?):
strong evidence for $m_\nu \neq 0$, $\Delta L \neq 0$

- ♦ Origin of neutrino mass in SM extensions
  Which symmetries associated with a given type of texture? (ie GUTs / family symmetries,...)

  different predictions for:
  SAMSW ? LAMSW ? VO ?
  degenerate or hierarchical neutrinos?

- ♦ Renormalisation of neutrino masses and mixings
  Stability of solutions under quantum corrections
  Very large effects possible

  Massive neutrinos ⇒ lepton-flavour-violation
  Additional constraints *(talk by I.Masina)*

First indication for beyond the SM Physics!

To accommodate massive neutrinos, need to extend lepton/Higgs/... sector

A huge number of proposals in the literature

Nielsen, Froggatt, Fritzsch, Harvey, Ramond, Reiss, Dimopoulos, Hall, Binetruy, Raby, Ibanez, Ross, SL, Grossman, Nir, Shadmi, Pokorski, Savoy, Dudas, Lavignac, Petcov, Irges, Bijnens, Wetterich, Stech, Barbieri, Hall, Babu, Pati, Wilczek, Ma, Gibson, Georgi, Glashow, Albright, Anderson, Barr, Achiman, Greiner, Altarelli, Feruglio, Ellis, Shafi, King, Allanach, Kane, Strumia, Mirayama, Joshipura, Smirnov, Leontaris, Vergados, Barger, Pakvasa, Weiler, Whisnant, Smith, Weiner, Tanimoto, Jezabek, Sumino, Berezhiani, Rossi, Romanino, Kaus, Meshkov, Baltz, Mohapatra, Nussinov, Matsuda, Skadhauge, Starkman, Nomura, Yanagida, Kang, Kim, Wu, ...

To be completed!

Framework

i) How many neutrinos? depends on:

- ν **as HDM?** $\sum_i m_{\nu_i} \geq 3$ eV
- LSND?

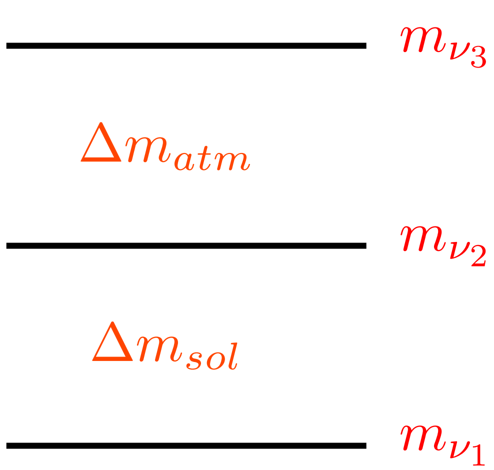
  With only 3 neutrinos, two independent Δm_{ij}

  Cannot explain all deficits simultaneously

in absence of light ν_s:

Sol. $+$ Atmo $+$ HDM $\Rightarrow m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau}$ $\mathcal{O}$(eV)

Only Sol. $+$ Atmo: can have large mass hierarchies , ie:



$m_{\nu_3} \approx \Delta m_{atmo}$

$m_{\nu_2} \approx \Delta m_{sol}$

ii) Solar (SAMWS, LAMSW, VO?)

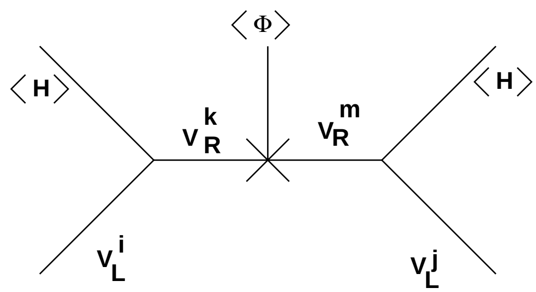
iii) Degererate or hierarchical neutrino masses?

Naturally light neutrinos by see-saw mechanism

Combine m_ν^D and M_{ν_R} to write a mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & M_{\nu_R} \end{pmatrix}$$

If $M_{\nu_R} \gg m_\nu^D$, a very light eigenvalue $m_{eff}^\nu \approx \left| \frac{(m_\nu^D)^2}{M_{\nu_R}} \right|$



v) In a given basis, lepton mixing from V_ν or $V_{\ell L}$?
(analogous to V_{CKM} for quarks: $V_{MNS} = V_\nu^\dagger V_{\ell_L}$)

How large can $V_{\nu R}$ be?

⇒ In (i) absence of cancellations and
(ii) for large neutrino hierarchies
the RH-sector does not affect V_{MNS}

Mass hierarchies and flavour symmetries

♦ Assume flavour symmetry under which different generations of fermions have different charges. Invariance under this symmetry will determine the magnitude of masses

Start discussion with a L-R symmetric model:

	Q_i	$\bar{U}_i$	$\bar{D}_i$	L_i	$\bar{E}_i$	H_2	H_1
$U(1)$	a_i	a_i	a_i	b_i	b_i	$-2a_3$	wa_3

♦ Symmetric mass matrices $+\ SU(2) \Rightarrow$
$Q_i, \bar{U}_i, \bar{D}_i$ have the same charge

♦ Up-mass matrix:
Top coupling $Q_3\bar{U}_3H_2$ 0 charge $\Rightarrow$ allowed
All other couplings forbidden

$$M^{up} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

♦ Suppose singlets θ with non-0 flavor-charges
(singlets expected in realistic models)
Then: invariant terms $Q_i\bar{U}_jH_2(<\theta>/M)^n$
n depending on flavour charges

♦ Hierarchical mass structure generated
♦ Similar for down-quark/lepton matrices

Example consistent with fermion hierarchies

$$M^{up} \propto \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix}, M^{down} \propto \begin{pmatrix} \bar\epsilon^8 & \bar\epsilon^3 & \bar\epsilon^4 \\ \bar\epsilon^3 & \bar\epsilon^2 & \bar\epsilon \\ \bar\epsilon^4 & \bar\epsilon & 1 \end{pmatrix}$$

Lepton textures with large 2-3 mixing

$$M_\ell \propto \begin{pmatrix} \bar\epsilon^5 & \bar\epsilon^3 & \bar\epsilon^{5/2} \\ \bar\epsilon^3 & \bar\epsilon & \bar\epsilon^{1/2} \\ \bar\epsilon^{5/2} & \bar\epsilon^{1/2} & 1 \end{pmatrix}, m_{eff} \propto \begin{pmatrix} \bar\epsilon^{10} & \bar\epsilon^6 & \bar\epsilon^5 \\ \bar\epsilon^6 & \bar\epsilon^2 & \bar\epsilon \\ \bar\epsilon^5 & \bar\epsilon & 1 \end{pmatrix}$$

For $Sol. + Atmo$ we need m_{ν_μ}/m_{ν_τ} $\mathcal{O}(0.01 - 0.1)$ $\surd$

$$V_{tot} = V_\nu^\dagger V_\ell = \begin{pmatrix} 1 - ... & \bar\epsilon^2 & \bar\epsilon^{5/2} \\ -\bar\epsilon^2 & 1 - ... & \sqrt{\bar\epsilon} + \bar\epsilon \\ -\bar\epsilon^{5/2} & -\sqrt{\bar\epsilon} - \bar\epsilon & 1 - ... \end{pmatrix}$$

$\mu - \tau$ mixing: $sin^2 2\theta$ up to ≈ 1 $\surd$

$(e - \mu)$ mixing: Specified by charged lepton masses!

$$V_\ell^{12} \approx \frac{M_\ell^{12}}{m_\mu} \approx \bar\epsilon^2 \approx 0.05$$

$\sin^2 2\theta$ in the small angle MSW solution $\times(?)$

★ *In models with U(1)s may not specify phases thus may not require accurate relations among masses*
$U(1)$ models naturally give large 3-neutrino hierarchies and mixing dominated by V_ℓ (M_R irrelevant for mixing)
unless precise 0-det condit. by see-saw (M_R relevant for mixing)

$SO(10)$

(i) Assume the family symmetry is combined with $SO(10)$
(ii) Use the GUT structure ONLY to constrain $U(1)$ charges
(iii) Assume no large coefficients that can regulate certain entries

• All L- and R-handed fermions in the 16 of $SO(10)$
$\Rightarrow$ all quark/lepton charges in a given generation identical

• Both MSSM Higgs fields fit in a single 10 of $SO(10)$ $\Downarrow$
For all fermions, L-R symmetric textures with similar structure
(at most different expansion parameters due to Higgs mixing)

However: $V_{\mu\tau} \approx V_{cb}$ ✗

AVOID: ie, consider effects of additional Higgs multiplets required for $SO(10) \to SU(3) \times SU(2) \times U(1)$

Generate operators with rank ≥ 4 in the mass matrices.

NOTE: Many possible operators
Choice of operator at a given entry phenomenological

$SU(5)$

Under this group we have the following relations:

$$
\begin{aligned}
Q_{(q,u^c,e^c)_i} &= Q_i^{10} \\
Q_{(l,d^c)_i} &= Q_i^{\overline{5}} \\
Q_{(\nu_R)_i} &= Q_i^{\nu_R}
\end{aligned}
$$

- M_{up} symmetric
- $M_{\ell^\pm} = M_{down}^T$
- L lepton mixing $\approx$ R down-quark one
- $M_{\ell^\pm}$,M_{down}: similar eigenvalue hiearchies

WAY OUT: Georgi-Jarlskog relations
If masses from coupling to a 45 of Higgs $m_{d_i} = 3m_{l_i}$

Can we obtain acceptable patterns of masses/mixings?

For instance,

$$
\frac{M_u}{m_t} = \begin{pmatrix} \bar{\epsilon}^6 & \bar{\epsilon}^5 & \bar{\epsilon}^3 \\ \bar{\epsilon}^5 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}, \frac{M_{down}}{m_b} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}
$$

$$
\frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon} \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}
$$

(Altarelli, Elwood, Feruglio, Irges, Ramond, ...)

Flipped $SU(5)$

$Q_{(q,d^c,\nu^c)_i} = Q_i^{10},\ \ Q_{(l,u^c)_i} = Q_i^{\bar{5}},\ \ e^c$ singlet of $SU(5)$

- Symmetric M_{down}
- $m_\nu^D = M_{up}^T$
- R-H up-quarks connected to L-H charged-leptons $\Downarrow$
  constrained if we require large lepton mixing

$SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$

Particles placed in $(3,3,1)$, $(\bar{3},1,\bar{3})$ and $(1,3,\bar{3})$ as:

$$\begin{pmatrix} u \\ d \\ D \end{pmatrix}_L \quad \begin{pmatrix} \bar{u} & \bar{d} & \bar{D} \end{pmatrix}_L \quad \begin{pmatrix} \ell^c & L & e^- \\ L^c & \ell & \nu \\ e^+ & \nu^c & N \end{pmatrix}_L$$

- Symmetric lepton mass matrices

(as in L-R symm. models)

- Asymmetric up and down, with similar structure

but can have different expansion parameters

Remember: in L-R sym. need cancellations for correct V_{cb}

$$\frac{M_u}{m_t} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \frac{M_{down}}{m_b} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}$$

For $\epsilon = \bar{\epsilon}^2$ viable hierarchies, $V_{cb} \simeq m_s/m_b$

GUTS and U(1) symmetries symmary

- Assume (close-to-) maximal lepton mixing
- textures determined by U(1) and GUT-fermion structure

without additional help from Higgs or heavy GUT fields

	SU(5)	SO(10)	flip.SU(5)	$SU(3)^3$	L-R-sym.
m_{up}	Sym.	Sym. (≈ all fer)	Asym.	Asym.	Sym.
m_d	Asym.	Sym.	Sym.	Asym.	Sym.
$m_{\ell^\pm}$	$m_{\ell^\pm} = m_d^T$	Sym.	Asym. corel. to up	Sym.	Sym.
m_ν	uncorrel. (ν_R singl.)	Sym.	$m_\nu^D = m_{up}^T$	Sym.	Sym.
V_{cb}	$V_{\mu\tau} \gg V_{cb}$ $\surd$	$V_{\mu\tau} \approx V_{cb}$ $\times$	VERY large $V_{\mu\tau} \gg V_{cb}$ $\times$	$V_{\mu\tau} \gg V_{cb}$ $\surd$	large $V_{\mu\tau} \gg V_{cb}$?
m_{up}	a bit high ?			$\surd$	$\surd$

A "$\times$" implies that this simple framework has to be extended (and additional model dependance introduced) in order to obtain acceptable fermion mass patterns.

Models with Abelian Flavour Symmetries

- Large splitting between fermion masses

  Naturally leads to large neutrino hierarchies

- Unknown phases/order unity coefficients $\Downarrow$

  Difficult to obtain naturally degenerate neutrinos

- In many models lepton hierarchies consistent with mostly SAMSW but LAMSW possible, ie by see-saw conditions

Models with non-Abelian flavour symmetries

- Degenerate ν and $\ell^{\pm}$ textures assuming

  ie that the lepton fields are SO(3) triplets

- Subsequently break SO(3) so as:

  large charged lepton splitting/ small neutrino splitting

- Favour almost-degenerate neutrino textures

- Textures with (almost)-bimaximal mixing predicted

  LAMSW / VO oscillations for solar neutrinos

Effects of radiative corrections on neutrino masses and mixing

Babu, Leung, Pantaleone, Chankowski, Plucienník Tanimoto, Ellis, SL, Pokorski, Haba, Okamura, Siguira, Casas, Espinosa, Ibarra, Navarro...

For i, j, generation indices

$$\frac{1}{m_{eff}^{ij}}\frac{d}{dt}m_{eff}^{ij} = \frac{1}{8\pi^2}\left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2 + \lambda_j^2)\right)$$

$$16\pi^2\frac{d}{dt}\sin^2 2\theta_{23} = 2\sin^2 2\theta_{23}(1 - 2\sin^2\theta_{23})\lambda_\tau^2\frac{m_{eff}^{33} + m_{eff}^{22}}{m_{eff}^{33} - m_{eff}^{22}}$$

$\sin^2 2\theta_{23}$ affected by quantum corrections if:

(i) λ_τ large (large $\tan\beta$) (ii) $m_{eff}^{33} - m_{eff}^{22}$ small

Semi-analytic and numerical studies $\Rightarrow$

The mixing can even be amplified/destroyed!

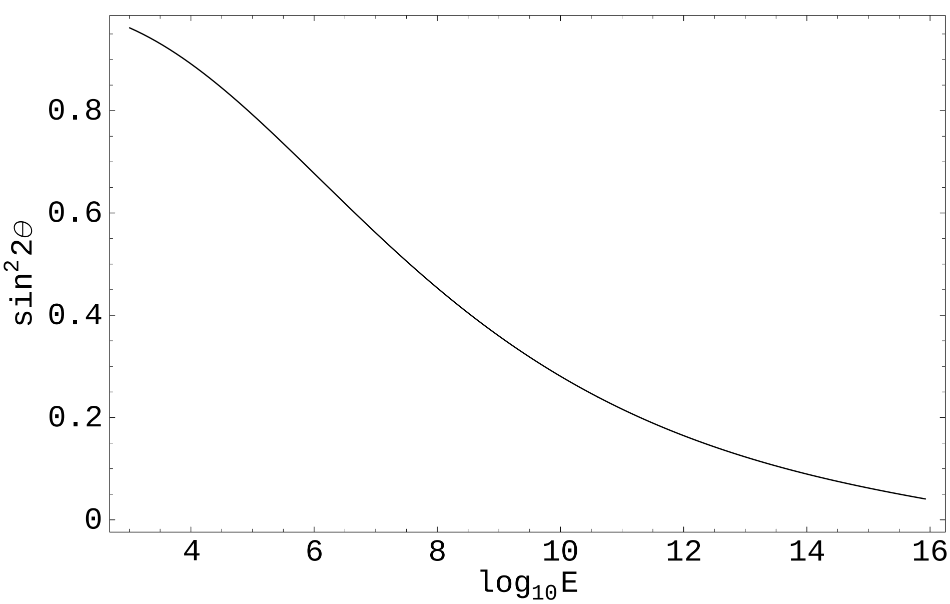
Existence of fixed-point solutions for mixing
(Chankowski, Krolikowski, Pokorski)

i.e: Un extreme case!

$$m_{eff} = \begin{pmatrix} 1-x & x^2 \\ x^2 & 1+x \end{pmatrix}$$

$x \approx 0.2$, $\approx M_{GUT} = 10^{16}$ GeV

$\lambda_t = \lambda_b = \lambda_\tau = 2.0$



$$\frac{1}{m^{ij}_{eff}}\frac{d}{dt}m^{ij}_{eff} = \frac{1}{8\pi^2}\left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2 + \lambda_j^2)\right)$$

$$(i, j = e, \mu, \tau)$$

$$\frac{m^{ij}_{eff}}{m^{ij}_{eff,0}} = exp\left\{\frac{1}{8\pi^2}\int_{t_0}^{t}\left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2 + \lambda_j^2)\right)\right\}$$

$$\equiv I_g \cdot I_t \cdot \sqrt{I_i} \cdot \sqrt{I_j}$$

1. The relative structure in m_{eff} is only modified by the leptonic Yukawa couplings

2. On the contrary, the gauge and top couplings give only an overall scaling factor

$$m_{eff} \propto \begin{pmatrix} m_0^{11}\ I_e & m_0^{12}\ \sqrt{I_\mu}\ \sqrt{I_e} & m_0^{13}\ \sqrt{I_e}\ \sqrt{I_\tau} \\ m_0^{21}\ \sqrt{I_\mu}\ \sqrt{I_e} & m_0^{22}\ I_\mu & m_0^{23}\ \sqrt{I_\mu}\ \sqrt{I_\tau} \\ m_0^{31}\ \sqrt{I_e}\ \sqrt{I_\tau} & m_0^{32}\ \sqrt{I_\mu}\ \sqrt{I_\tau} & m_0^{33}\ I_\tau \end{pmatrix}$$

Effects on mass einenvalues (masses of physical neutrinos) *[Ellis, SL]*

ex: Start from the solution with 3 exactly degenerate neutrinos (eigenvalues: 1,-1,1) at $m_{\rm GUT}$ et calculate the radiative corrections on eigenvalues:

$\lambda_\tau^{\rm GUT}$	m_3	m_2	m_1
3.0	0.866	-0.952	0.997
1.2	0.903	-0.966	0.998
0.48	0.962	-0.987	0.9996
0.10	0.9478	-0.9993	0.99998
0.013	0.99998	-0.99999	1.00000

$m_{eff}\ \mathcal{O}(1\ {\rm eV}),\ M_N = 10^{13}$ GeV

Need to worry about stability of neutrino textures, especially for degenerate neutrinos

the latter may require that:

(a) we start with *slightly* non-degenerate neutrinos at high scales (need to motivate by symmetries)

(b) or the structure of m_ν^D is such that it stabilises texture

For approaches with stable degenerate neutrino solutions see:

Barbieri, Ross, Strumia

Casas, Espinosa, Ibarra, Navaro

Neutrino masses and Yukawa unification

Effect of running of λ_N on RGEs:

Vissani, Smirnov
Brignole, Murayama, Rattazzi

$$\begin{aligned}
16\pi^2 \frac{d}{dt}\lambda_t &= \left(6\lambda_t^2 + \lambda_N^2 - G_U\right)\lambda_t \\
16\pi^2 \frac{d}{dt}\lambda_N &= \left(4\lambda_N^2 + 3\lambda_t^2 - G_N\right)\lambda_N \\
16\pi^2 \frac{d}{dt}\lambda_b &= \left(\lambda_t^2 - G_D\right)\lambda_b \\
16\pi^2 \frac{d}{dt}\lambda_\tau &= \left(\lambda_N^2 - G_E\right)\lambda_\tau
\end{aligned}$$

Run affects (decreases) λ_τ but not λ_b

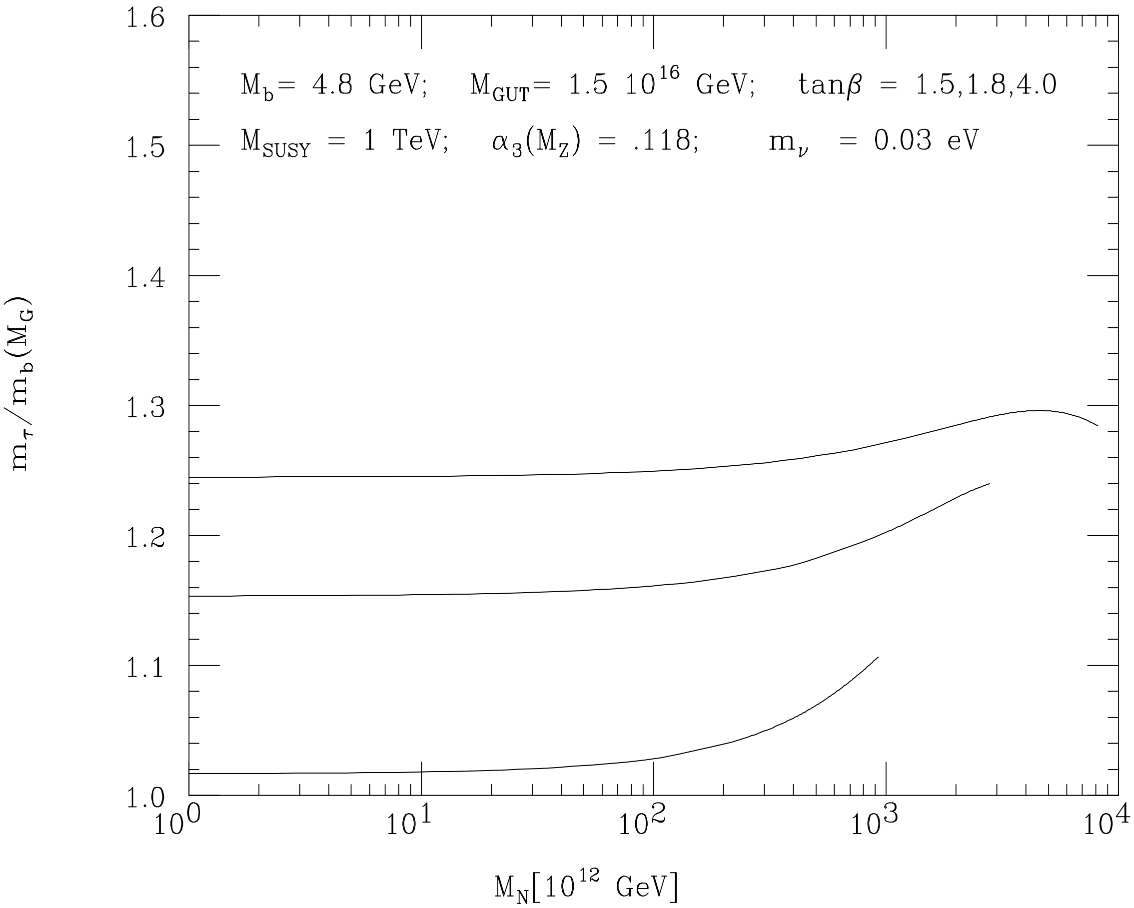
small $tan\beta$: $\approx 10\%$ effect for $M_N = 10^{13-14}$ GeV
large $tan\beta$: λ_b fp + large m_b corrections $\Rightarrow$ no effect

$b - \tau$ unif. + small $tan\beta$ $\Rightarrow$ large mixing
(Leontaris, SL, Ross, Carena, Ellis, Wagner)

- $b - \tau$ equality at m_{GUT} refers to $(m_\ell^0)_{33} = (m_D^0)_{33}$
- No prediction from GUT for complete matrices

- Assume textures where

before diagonalisation: $(m_\ell^0)_{33} = (m_D^0)_{33}$
after diagonalisation: $(m_\ell^0)_{33} \neq (m_D^0)_{33}$



- small λ_N ($\Rightarrow$ small M_N), $b-\tau$ unif. at λ_t fixed point

- As λ_N increases (M_N increases from see-saw)
  λ_N lowers λ_τ with respect to λ_b
  need to start with higher $\lambda_\tau/\lambda_b(M_{GUT})$

Suppose that in basis where m_{down} is diagonal:

$$m_\ell^0 = \begin{pmatrix} x^2 & x \\ x & 1 \end{pmatrix} m_0 \qquad \text{Then } \frac{m_\tau^0}{m_b^0} = 1 + x^2$$

large mixing in charged-lepton sector

The additional mixing to match SK, provided by νs

- As $\tan\beta$ increases, away from λ_t f.p. and $b-\tau$ unif.
- For M_N close to m_{GUT} effects decrease due to
  $\ln(M_N/m_{GUT})$ dependence
  Visible unless λ_N non-perturb. before peak is reached

LFV IN RARE DECAYS AND CONVERSIONS

In SM extensions with $\Delta L_i \neq 0$, non-zero rates for processes such as:

$$\mu \to e\gamma$$
$$\tau \to \mu\gamma$$
$\mu - e$ conversion on nuclei

In SUSY, large rates mediated by gaugino-slepton loops. Even if:

$$M_{\rm GUT} : m_{\tilde{\ell},\tilde{\nu}} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad {\rm REGs} \longrightarrow \begin{pmatrix} 1 & \star & \star \\ \star & 1 & \star \\ \star & \star & 1 \end{pmatrix}$$

corrections in the basis where $(\lambda_\ell^\dagger \lambda_\ell)_i^j$ is diagonal, ie:

$$\delta m_{\tilde{\ell}} \propto \frac{1}{16\pi} \ln \frac{M_{\rm GUT}}{M_N} \lambda_\nu^\dagger \lambda_\nu m_{\rm SUSY}^2$$

INFO: The larger the μ-e lepton mixing and the neutrino masses, the larger the rates for LFV

SUMMARY – CONCLUSIONS

- **Super-Kamiokande data**
  also finds: ν_μ/ν_e in atmosph. $<$ expected

- **Evidence for Neutrino Oscillations**
  Neutrino masses / $\Delta L \neq 0$ $\Downarrow$
  Extensions of SM (L-R, GUTS, $R\!\!\!/_p$, ...)

- **Implications for underlying theory?**
  Predictions for neutrino textures from flavour and GUT symmetries

  with different answers to

  which solution for solar neutrinos?
  hierarchical neutrino masses/degenerate neutrinos?

- **Quantum corrections on neutrino masses/mixing**
  Change of GUT mixing and δm^2

Different models "prefer" different solutions of the solar and atmospheric neutrino deficits.
A very large number of proposals in literature.
Which is true?
The new data can now help us to exclude/constrain many of the existing models