# INTRODUCTION TO AFTERNOON SESSION ON THEORETICAL MODELS, FUTURE & COSMOLOGY

S.F. King (Southampton)
Les Houches Euroconference
on Neutrino Masses and Mixings
22 June, 2001

Two parts to this session

(i) models

(ii) cosmology (Leptogenesis ... )


The SNO Bombshell : $\frac{CC(^8B)_{SNO}}{BP2001} = 0.347 \pm 0.029$
(K. Heeger)

A VERY SIMPLE PICTURE IS EMERGING ...

3 Active Neutrino Flavours $\nu_e, \nu_\mu, \nu_\tau$

2 Large mixing angles

$\theta_{12} \approx \pi/4$ (LMA or LOW)

$\theta_{23} \approx \pi/4$ (Atm $\nu$'s)

1 Small mixing angle

$\theta_{13} \lesssim 0.2$ (CHOOZ, Palo Verde)

"Approximate Bi-maximal Mixing"

$$U_{MNS} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \theta_{13} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \theta_{13} \to \times e^{i\delta}$$

mass eigenstates

$$\nu_e \approx \frac{1}{\sqrt{2}}\nu_1 + \frac{1}{\sqrt{2}}\nu_2 + \theta_{13}\nu_3$$

$$\nu_\mu \approx -\frac{1}{2}\nu_1 + \frac{1}{2}\nu_2 + \frac{1}{\sqrt{2}}\nu_3$$

$$\nu_\tau \approx \frac{1}{2}\nu_1 - \frac{1}{2}\nu_2 + \frac{1}{\sqrt{2}}\nu_3$$

## UNANSWERED QUESTIONS → FUNDAMENTAL ISSUES

Why are $\nu$ masses so small?

What is pattern of $\nu$ masses? (hierarchy, inverted, degenerate)

Why are (some) $\nu$ angles large?

Is there $\not{C}\not{P}$ in lepton sector?

RH$\nu$'s + see-saw mechanism, $\not{R}_p$ SUSY, Xtra dims?

GUTs, String Theory, Flavour Symmetry

Origin of quark and lepton masses & mixing angles

Why 3 families?

Matter-antimatter asymmetry in Universe

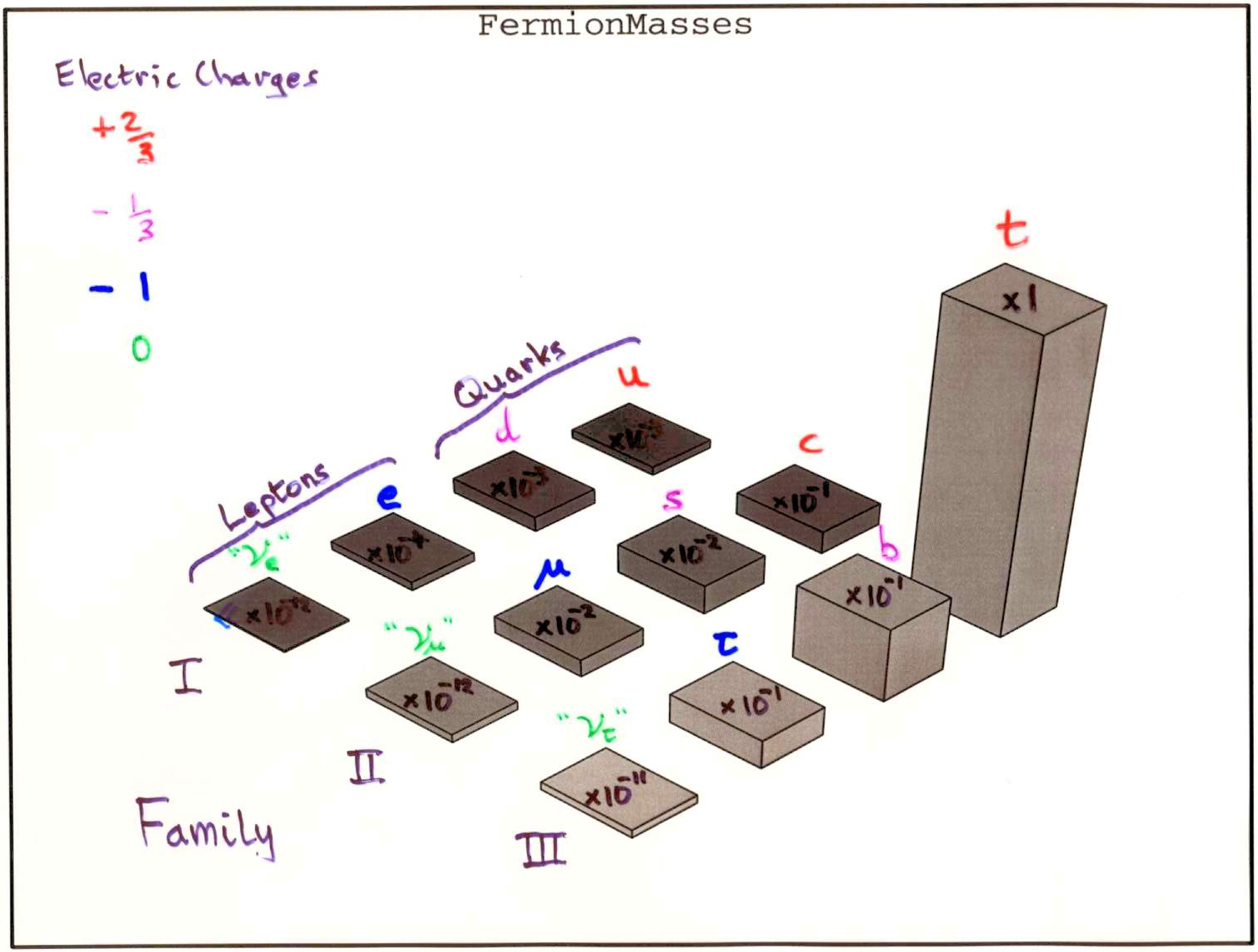
Dark matter

## UNMEASURED PARAMETERS

$\Delta m^2_{32}$ (sign)

$\Delta m^2_{21}$, $\theta_{12}$ } AMBIGUOUS SOLAR SLNS. LMA vs. LOW

$\theta_{13}$, $\delta$ } UNMEASURED


FermionMasses
Electric Charges
+2/3
-1/3
-1
0
Leptons
Quarks
"$\nu_e$"
e
d
u
"$\nu_\mu$"
$\mu$
s
c
"$\nu_\tau$"
$\tau$
b
t
×10^-4
×10^-2
×10^-1
×10^-12
×10^-2
×10^-1
×10^-11
×10^-1
×1
I
II
III
Family

# WHY ARE $\nu$ MASSES SO SMALL ?

## a) SEE-SAW MECHANISM

Gell-Mann, Ramond, Slansby
Yanagida 1979

$$\begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix}, \quad m_{LR} \ll M_{RR}$$

$$\rightarrow m_{LL} \approx m_{LR} M_{RR}^{-1} m_{LR}^T \lll m_{LR}$$

LMA friendly :) (see later)

(e.g. $M_{RR} \sim M_{GUT}$, $m_{LR} \sim M_W \rightarrow m_{LL} \sim 10^{-3}\,eV$)

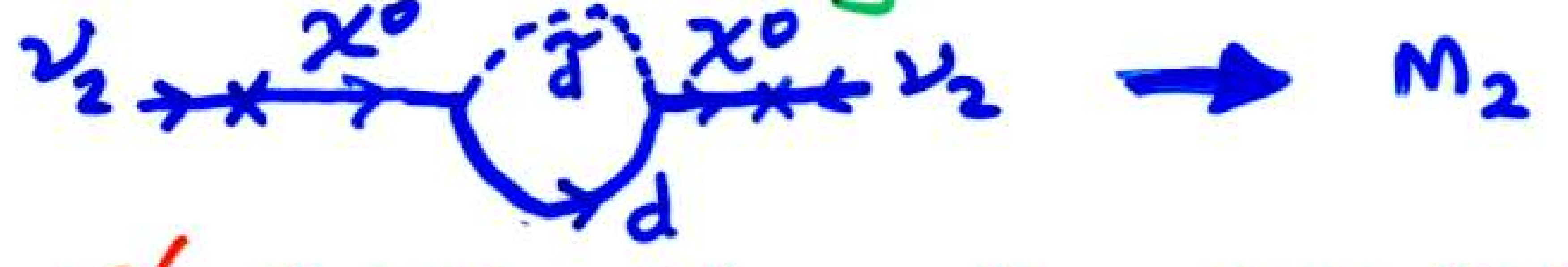
## b) $\cancel{R}_p$ SUSY models

$$W = \varepsilon_i L_i H_u + \mu H_u H_d$$

$\rightarrow$ single neutrino Majorana $m_3 \sim \varepsilon^2 / M_{SUSY}$

Then SUSY loops generate neutrino spectrum

e.g.




$\rightarrow m_2$

:( $\cancel{R}_p$ SUSY models prefer SMA MSW (Hirsch et al)

## c) Large Extra Dimensions


$$S_{bulk} = \int d^4x\, dy \left( \bar{\nu}_R \gamma^M i \partial_M \nu_R - m \bar{\nu}_R \nu_R \right)$$

$$S_{brane} = -\int d^4x \frac{1}{\sqrt{M_5}} \left( Y_\nu \bar{\nu}_R L_i H \right) + H.c.$$

Naturally small Dirac suppression

:( Large Extra Dimension models prefer SMA MSW and sterile type neutrino mixing solns (Ramond talk)

(This discussion from S.K. NPB576 (2000) 85)

# HOW TO GET LARGE MIXING ANGLES IN HIERARCHICAL SEE-SAW MODELS ?

Work in diagonal $M_{RR}$ (and charged lepton) basis

$$m_{LR} = \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix}, \quad M_{RR} = \begin{pmatrix} X' & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Y \end{pmatrix}$$

See-saw formula $m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^{T}$

$$\Rightarrow \quad m_{LL} = \begin{pmatrix} \frac{a'^2}{X'}+\frac{a^2}{X}+\frac{d^2}{Y} & \frac{a'b'}{X'}+\frac{ab}{X}+\frac{de}{Y} & \frac{a'c'}{X'}+\frac{ac}{X}+\frac{df}{Y} \\ \cdot & \frac{b'^2}{X'}+\frac{b^2}{X}+\frac{e^2}{Y} & \frac{b'c'}{X'}+\frac{bc}{X}+\frac{ef}{Y} \\ \cdot & \cdot & \frac{c'^2}{X'}+\frac{c^2}{X}+\frac{f^2}{Y} \end{pmatrix}$$

In order to obtain a natural mass hierarchy $m_2 \ll m_3$ we assume Single Right-Handed Neutrino Dominance:

$$\frac{e^2, ef, f^2}{Y} \gg \frac{b^2, bc, c^2}{X} \gg \frac{b'^2, b'c', c'^2}{X'} \rightarrow \boxed{\frac{m_1}{m_2} \ll 1}$$

$$\Rightarrow \quad \det[m_{LL}]_{23} \approx \left(\frac{e^2}{Y}\right)\left(\frac{f^2}{Y}\right) - \left(\frac{ef}{Y}\right)^2 \approx 0$$

$$\Rightarrow \quad \det[m_{LL}]_{23} = m_2 m_3 \approx 0 \Rightarrow \boxed{\frac{m_2}{m_3} \ll 1}$$

## HOW TO GET LARGE MIXING ANGLES ?

Atm. $\tan\theta_{23} \approx \frac{e}{f} \approx 1 \leftrightarrow e \approx f$

CHOOZ $\tan\theta_{13} \approx \frac{d}{\sqrt{e^2+f^2}} \ll 1 \leftrightarrow d \ll e \approx f$

LMA MSW $\tan\theta_{12} \approx \frac{\sqrt{2}\,a}{b-c} \approx 1 \leftrightarrow a \approx \frac{b-c}{\sqrt{2}}$

Note that $\theta_{23}, \theta_{13}$ are controlled by the dominant right-handed neutrino couplings $d, e, f$ whereas $\theta_{12}$ is controlled by the leading sub-dominant right-handed neutrino couplings $a, b, c$.

IT REALLY IS THAT SIMPLE ! LMA + HIERARCHY IS COMPLETELY NATURAL

HOW CAN WE ACHIEVE THE SRHND CONDITIONS AND THE YUKAWA RELATIONS

$d \ll e \approx f, \quad a \approx \frac{b-c}{\sqrt{2}}$ ?

One way is to use a U(1) Family Symmetry

Talks:
Ramond
Altarelli
Lola
Hirsch (later)

# U(1) FAMILY SYMMETRY

$$\mathcal{L} = Y^{\nu}_{ij} H_u L_i \nu^c_{Rj} + Y^{ij}_{RR} \nu^c_{Ri} \nu^c_{Rj} \Sigma$$

U(1) charges : $H_u$: 0, $L_i$: $(-3,1,1)$, $\nu^c_{Rj}$: $(9,1,-2)$, $\Sigma$: 2

$$Y^{\nu}_{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^2 & \lambda^4 \\ \lambda^{10} & \lambda^2 & 1 \\ \lambda^{10} & \lambda^2 & 1 \end{pmatrix} \sim \begin{pmatrix} a' & a & d \\ b' & b & e \\ c' & c & f \end{pmatrix}$$

$$Y_{RR} \sim \begin{pmatrix} \bar\lambda^{20} & \bar\lambda^{12} & \bar\lambda^{10} \\ \bar\lambda^{12} & \bar\lambda^{4} & \bar\lambda^{2} \\ \bar\lambda^{10} & \bar\lambda^{2} & 1 \end{pmatrix} \sim \begin{pmatrix} X' & & \\ & X & \\ & & Y \end{pmatrix}$$

Suppose $\bar\lambda = \lambda^{\frac{1}{2}}$ then, see-saw $m_{LL} = Y_\nu Y_{RR}^{-1} Y_\nu^T$,

$$m_{LL} \sim \underbrace{\begin{pmatrix} \lambda^8 & \lambda^4 & \lambda^4 \\ \lambda^4 & 1 & 1 \\ \lambda^4 & 1 & 1 \end{pmatrix}}_{\text{from } \frac{1}{Y}} + \underbrace{\begin{pmatrix} \lambda^2 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^2 \end{pmatrix}}_{\text{from } \frac{1}{X}} + \underbrace{\begin{pmatrix} \lambda^2 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^{10} & \lambda^{10} \\ \lambda^6 & \lambda^{10} & \lambda^{10} \end{pmatrix}}_{\text{from } \frac{1}{X'}}$$

Detailed "predictions":

- SRHND since $\frac{1}{Y}$ dominates 23 block of $m_{LL}$

$$\rightarrow \quad \frac{m_2}{m_3} \sim \lambda^2 \sim 5 \times 10^{-2} \; ; \; \frac{\Delta m^2_{\odot}}{\Delta m^2_{Atm}} \sim 2.5 \times 10^{-3}$$

- $m_3 \sim 5 \times 10^{-2}\,\text{eV} \rightarrow Y \equiv M_3 \sim 10^{15}\,\text{GeV}$

$$M_1 : M_2 : M_3 \sim \lambda^{10} : \lambda^2 : 1 \sim 3.10^{-7} : 5.10^{-2} : 1$$

$$\rightarrow \underbrace{M_1 \sim 3.10^{+8}\,\text{GeV}}_{\text{in Leptogenesis range}} \; , \; M_2 \sim 5.10^{13}\,\text{GeV} \; ,$$

(bit low but can vary $\lambda'$) $\quad M_3 \sim 10^{15}\,\text{GeV}$.

- $\tan\theta_{23} \sim \frac{e}{f} \sim 1$ (large Atm.)

$\tan\theta_{13} \sim \frac{d}{\sqrt{e^2+f^2}} \sim \lambda^4 \sim 2 \times 10^{-3}$ (small CHOOZ)

$\tan\theta_{12} \sim \frac{\sqrt{2}\,a}{b-c} \sim 1$ (LMA)

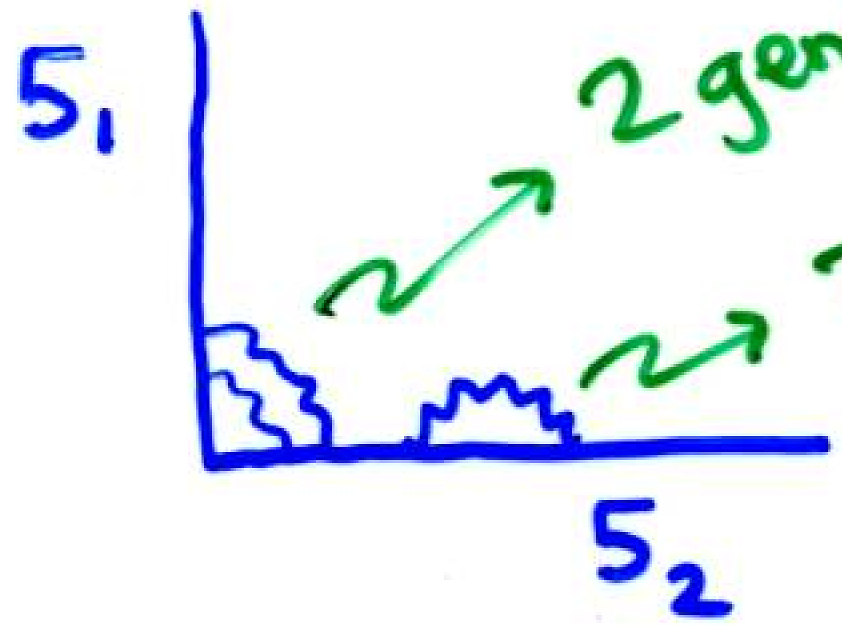
- N.B. The dominant RH$\nu$ is the heaviest in this example!

# TOWARDS A THEORY OF EVERYTHING

Every theorist has her own favourite TOE

Here's mine ... (not included in Lola talk)

Type I string theory with intersecting $D_5$ branes




2 generations from intersection

3rd (heaviest) generation from $5_2$ brane

(Shiu-Tye)

Allanach, S.K., Lola ...

→ SUSY Pati-Salam

not predicted by Shiu-Tye

$$SU(4) \times SU(2)_L \times SU(2)_R \times \overbrace{U(1)}_{\text{Family}}$$

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}^i_{L,R}$$

→ Predicts 3 Right-handed neutrinos,

Yukawa unification (large $\tan\beta \equiv \frac{\langle H_u \rangle}{\langle H_d \rangle}$)

(c.f. SO(10), but here there is

no doublet-triplet splitting problem,

no proton decay.)

SUSY Pati-Salam can describe all quark and lepton spectrum (S.K., Oliveira PRD63(2001) 095004)

Leads to neutrino sector similar to previous example :

$$Y_\nu \sim \begin{pmatrix} \lambda^{\frac{15}{2}} & \lambda^{\frac{7}{2}} & \lambda^{\frac{3}{2}} \\ \lambda^{\frac{13}{2}} & \lambda^{7/2} & 1 \\ \lambda^{\frac{13}{2}} & \lambda^{7/2} & 1 \end{pmatrix}, \quad Y_{RR} \sim \begin{pmatrix} \lambda^9 & & \\ & \lambda^5 & \\ & & 1 \end{pmatrix}$$

→ LMA MSW using SRHND,

$M_1 \sim 10^9$ GeV in leptogenesis range, etc..

The model also has some other nice predictions:

- $\tau \to \mu \gamma$ close to current limit due to $(Y_\nu)_{23} \sim 1$
- $m_h \approx 115$ GeV due to large $\tan\beta$
- $g-2$ of muon in observed range

(Blazek, S.K.)

# CONCLUSIONS

- SNO + SK consistent with LMA / LOW active $\nu$ oscillations is a (welcome) bombshell
- See-Saw can easily give LMA (SRHND) and looks very good
  - hints at SUSY + Leptogenesis
- Neutrino parameters should be measured experimentally to the best precision possible since these numbers will ultimately distinguish different TOEs (and can be known better than quark parameters which have hadronic uncertainties)