Texture models, neutrino observables and Leptogenesis

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1. Neutrino oscillation data and neutrino masses

2. Texture models, $U(1)$ family symmetries and $\mathcal{O}(1)$ coefficients

3. Neutrino observables and Leptogenesis

4. Summary

* M. Hirsch and S.F. King, in preparation

Neutrino masses: Experimental facts

A) Limits (PDG2000):

$$m_{\nu_e} \le 3 \text{ eV}$$

$$m_{\nu_\mu} \le 190 \text{ keV}$$

$$m_{\nu_\tau} \le 18.2 \text{ MeV}$$

$$\langle m_\nu \rangle \le (0.2 - 0.6) \text{ eV}$$

B) "Hints" on non-zero neutrino masses:

⇒ Atmospheric neutrinos:

Ratio of ν_e/ν_μ events disagrees with expectation, SuperKamiokande confirms earlier experiments with high statistics

⇒ Solar neutrinos

5 experiments (using 3 experimental techniques) observe less neutrinos than expected

⇒ neutrinos as hot dark matter (HDM)

Small ($\mathcal{O}[eV]$) neutrino mass might help structure formation

⇒ LSND experiment

Neutrino masses and the seesaw mechanism

⇒ In the SM neutrinos are massless, because $N \equiv \nu_R$ does not exist

⇒ Suppose N exists, add the following mass terms:

$$\mathcal{L} = m_D \overline{\nu}_L N + M_M N N$$

⇒ Gives the following mass matrix (one generation notation):

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & M_M \end{pmatrix}.$$

⇒ Assuming $m_D \ll M_M$, one arrives at the famous seesaw formula:

$$m_\nu \simeq \frac{m_D^2}{M_M}$$

⇒ Smallness of observed neutrino masses explained by large mass scale M_M

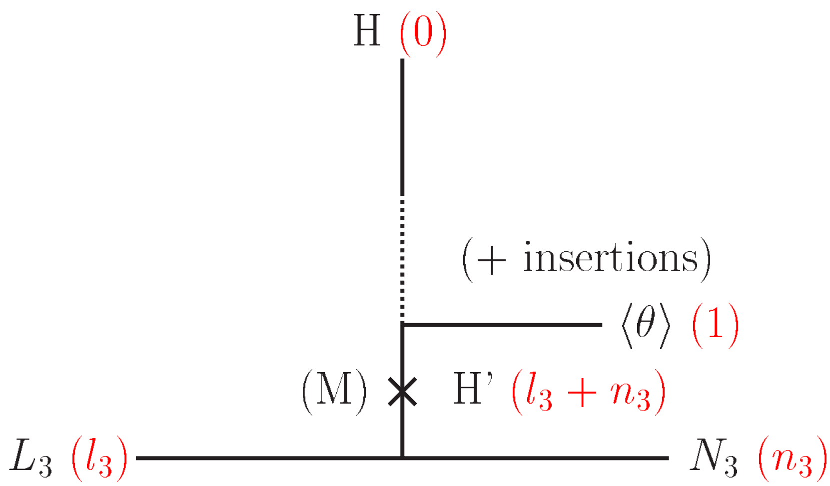
⇒ However, seesaw mechanism alone does not fix relative size of different entries in the mass matrix

Frogatt-Nielson mechanism

Assume some heavy singlet θ exists. Additional exotic vector matter with mass M_V allows an expansion parameter λ to be generated by a Froggatt-Nielsen mechanism,

$$\frac{<\theta>}{M_V} = \frac{<\bar{\theta}>}{M_V} = \lambda \approx 0.22$$

Assign $U(1)$ charges to L_i, N_{Ri}, etc:



The above diagram generates mass term:

$$m_{L_3,N_3} \sim \lambda^{|l_3+n_3|}\langle H\rangle$$

Even if coupling at vertex is of $\mathcal{O}(1)$,
strong suppression can be generated!

$U(1)$ Family Symmetry and Textures

Using basic idea of Frogatt-Nielsen mechanism, assign flavour charges (FC) to all fields, leading to mass matrices of the form:

$$Y_\nu \sim \begin{pmatrix} a_{11}\lambda^{|l_1+n_1|} & a_{12}\lambda^{|l_1+n_2|} & a_{13}\lambda^{|l_1+n_3|} \\ a_{21}\lambda^{|l_2+n_1|} & a_{22}\lambda^{|l_2+n_2|} & a_{23}\lambda^{|l_2+n_3|} \\ a_{31}\lambda^{|l_3+n_1|} & a_{32}\lambda^{|l_3+n_2|} & a_{33}\lambda^{|l_3+n_3|} \end{pmatrix}$$

$$M_{RR} \sim \begin{pmatrix} A_{11}\lambda^{|2n_1+\sigma|} & A_{12}\lambda^{|n_1+n_2+\sigma|} & A_{13}\lambda^{|n_1+n_3+\sigma|} \\ A_{12}\lambda^{|n_1+n_2+\sigma|} & A_{22}\lambda^{|2n_2+\sigma|} & A_{23}\lambda^{|n_2+n_3+\sigma|} \\ A_{13}\lambda^{|n_1+n_3\sigma|} & A_{23}\lambda^{|n_2+n_3+\sigma|} & A_{33}\lambda^{|2n_3+\sigma|} \end{pmatrix}$$

$\Rightarrow$ Since λ is $\lambda \ll 1$, a high power in the exponent leads to very small entries in the mass matrices, so-called texture zeros

Example (FC1): $l_1 = -2$, $l_2 = 0$, $l_3 = 0$, $n_1 = -2$, $n_2 = 1$, $n_3 = 0$ and $\sigma = 0$, leads to (after the seesaw):

$$m_{LL}^{FC1} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} + \mathcal{O}\begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^2 \end{pmatrix}.$$

$\Rightarrow$ Advantage: Smallness of mass and relative size of entries can be easily fixed

$\Rightarrow$ Disdvantage: Coefficients a_{ij} and A_{ij} (assumed to be $\mathcal{O}(1)$ couplings) not predicted

$\mathcal{O}(1)$ coefficients

Basic idea:

⇒ Since $\mathcal{O}(1)$ coefficients not predicted, assume them to be a random numbers

⇒ Choose interval for coefficients such that texture structure of mass matrix is not destroyed

⇒ Run a huge sample of models in a computer program

⇒ Plot the *logarithmically binned* distributions with correct relative normalisation for each model

⇒ A model is then considered to be a "good" model, if the peaks of the distributions coincide with (or are close to) the preferred experimental value

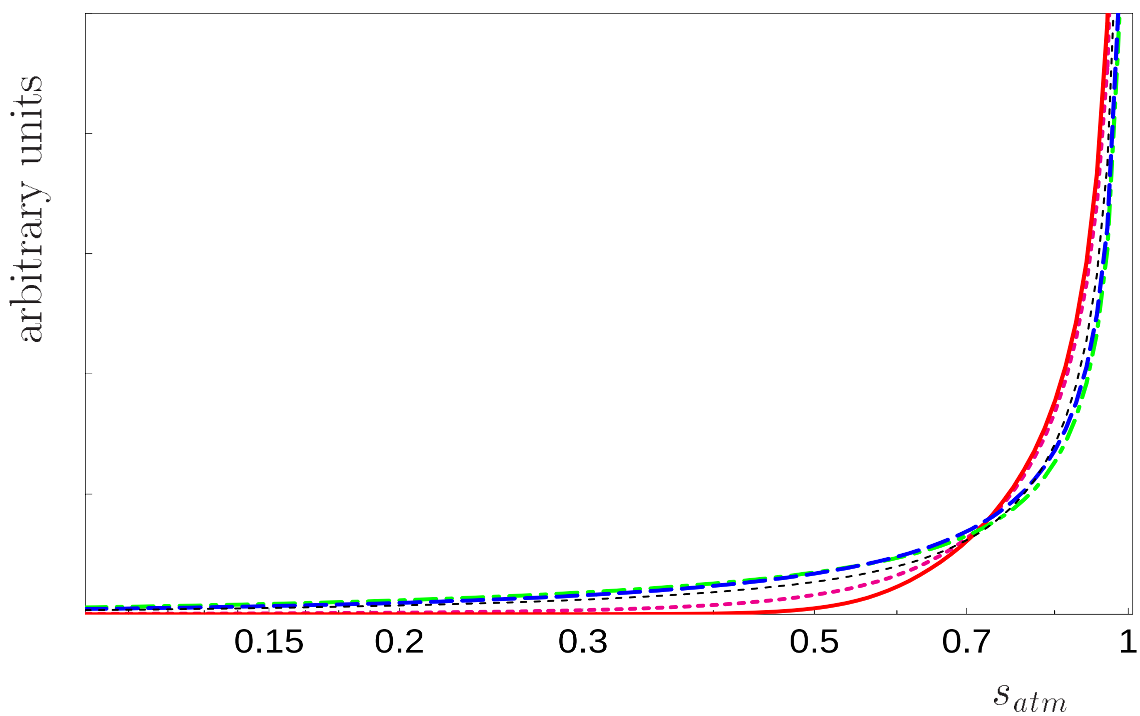
The atmospheric angle

Figure: The atmospheric angle for 5 different models (10^8 random sets per model):

a) red: FC1,

b) FC2 (dot-dashes), c) FC3 (thick dots), d) FC4 (thin dots),

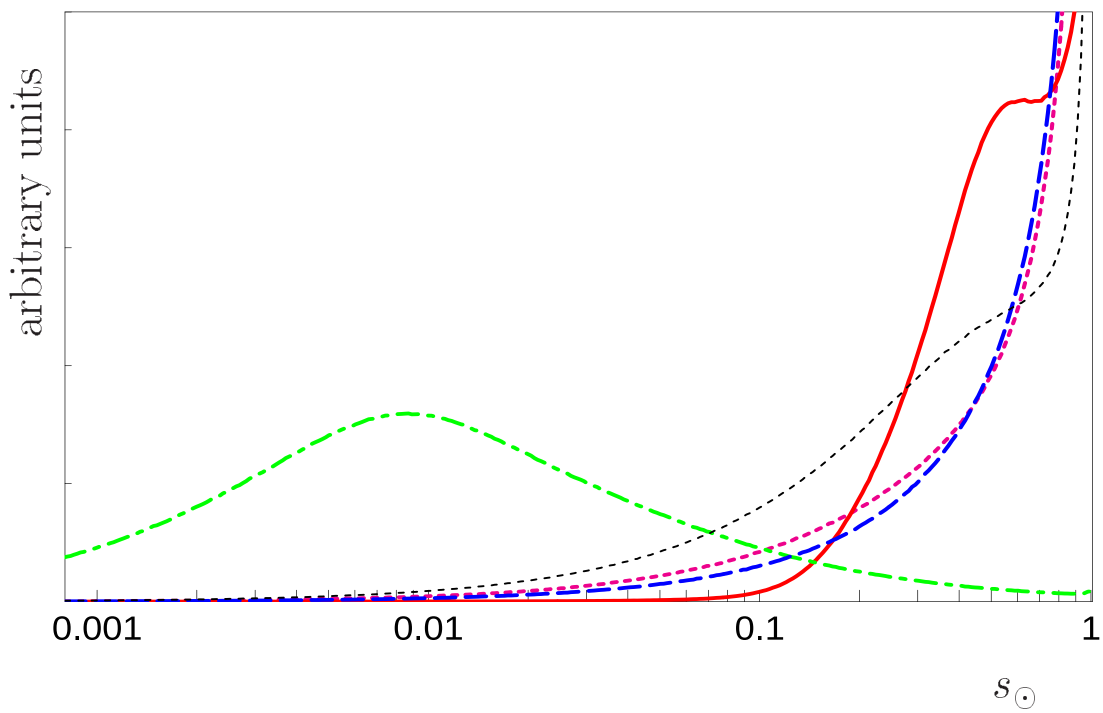
e) blue: neutrino mass anarchy (No structure in neutrino mass matrix).



It is easy to generate large atmospheric angle!

The solar angle

Figure: The solar angle for 5 different models:
a) red: FC1, b) FC2 (dot-dashes), c) FC3 (thick dots),
d) FC4 (thin dots), e) blue: neutrino mass anarchy.

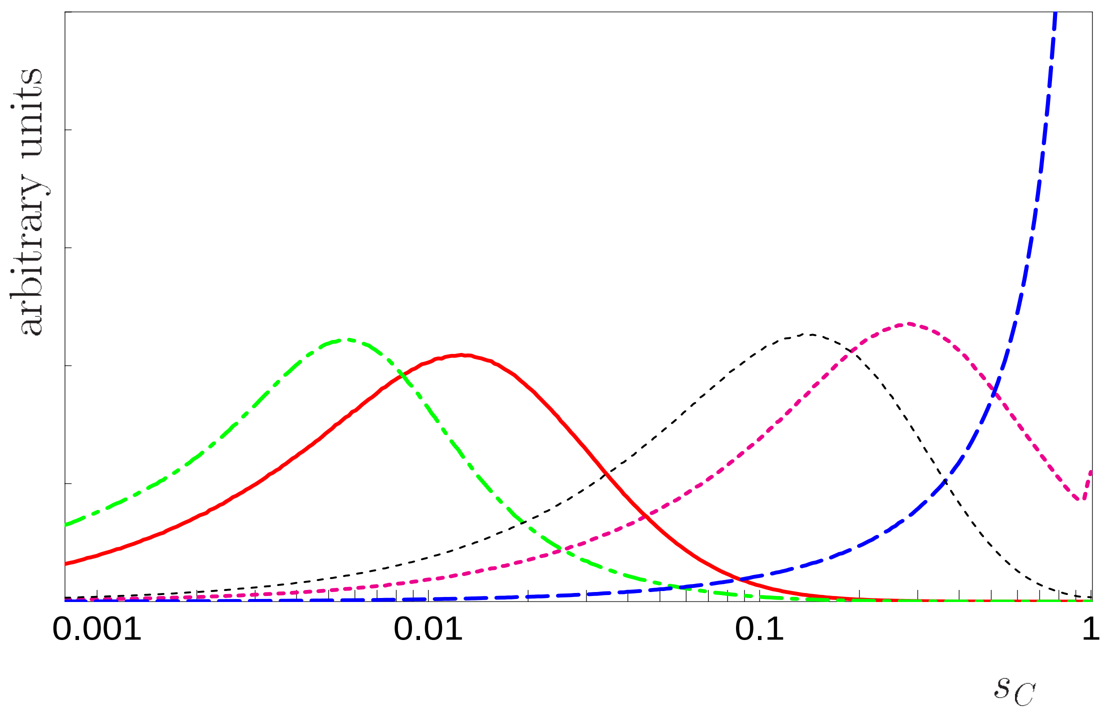


Anarchy prefers large solar angle

Flavour models can be constructed for either,
large (FC1, FC3 and FC4) or small (FC2) solar angle

The "Chooz angle"

Figure: $s_C = 4|U_{e3}|^2(1 - |U_{e3}|^2)$ for 5 different models:
a) red: FC1, b) FC2 (dot-dashes), c) FC3 (thick dots),
d) FC4 (thin dots), e) blue: neutrino mass anarchy.



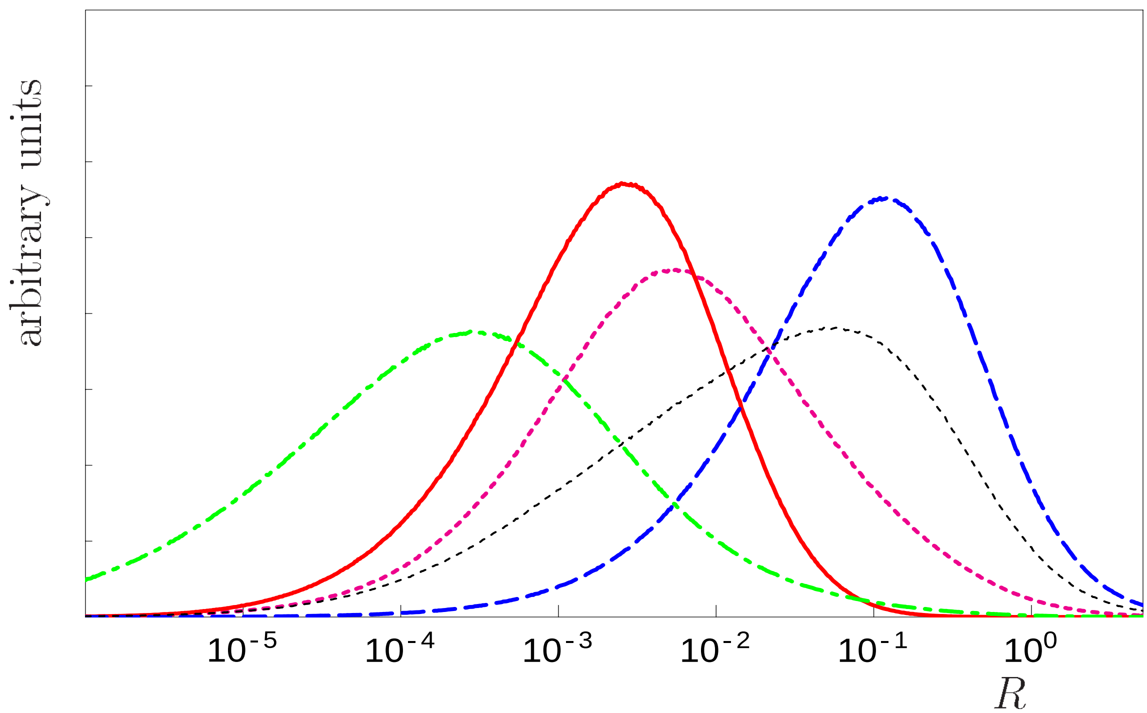
Experimentally: $s_C \leq (0.1 - 0.3)$ **in SK region**

Anarchy prefers large s_C, peaks at $s_C = 1$!

Chooz angle important discriminator!

Ratio of Δm^2's

Figure: $R \equiv |\Delta m^2_{12}|/|\Delta m^2_{23}|$ for 5 different models:
a) red: FC1, b) FC2 (dot-dashes), c) FC3 (thick dots),
d) FC4 (thin dots), e) blue: neutrino mass anarchy.

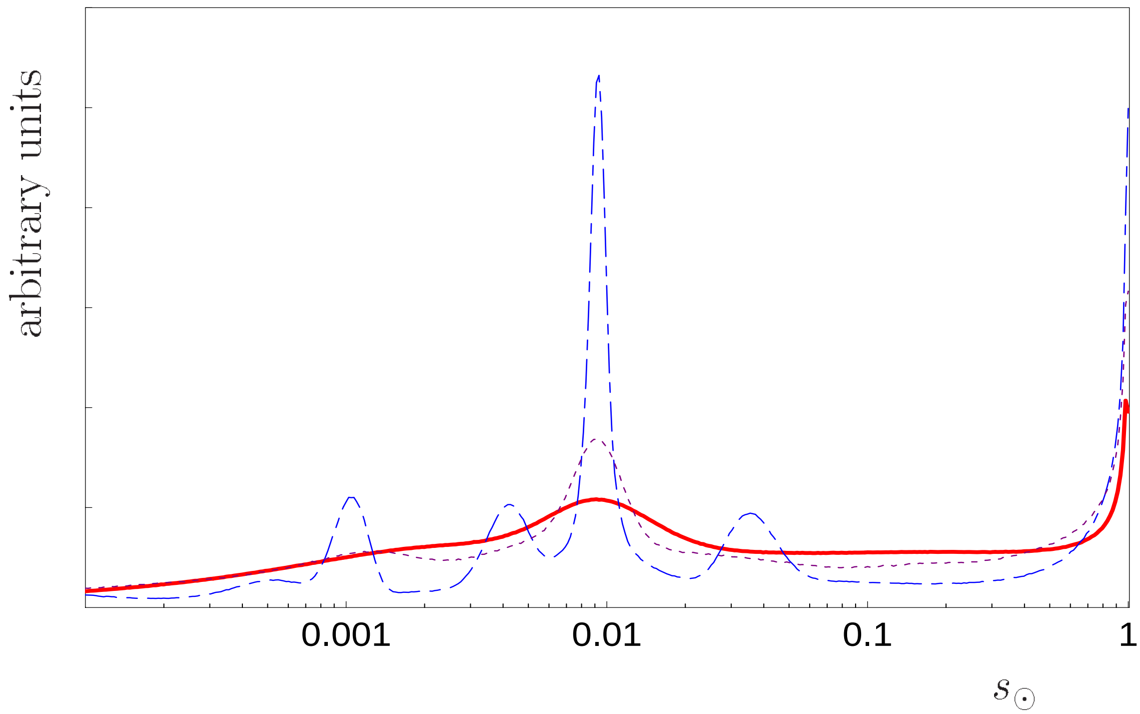


Spread in R huge!
Coefficients a_{ij} and A_{ij} can not be neglected!

Small values of R disfavour neutrino mass anarchy

Variation in range of coefficients

Figure: Solar angle for 3 different ranges of coefficients for the model FC2:
a) red: $\pm[\sqrt{2\lambda}, 1/\sqrt{2\lambda}]$, b) magenta: $\pm[0.82, 1.18]$,
c) blue $\pm[0.95, 1.05]$.



Choice of coefficients very important!

$\Rightarrow$ Theoretical work in texture models should concentrate on calculation of coefficients!

Leptogenesis

$\Rightarrow$ CP violation in decay of lightest N_R comes from interference between tree-level and one-loop amplitude:

$$\begin{aligned}\epsilon &= \frac{\Gamma(N_{R1} \to L_j + H_2) - \Gamma(N_{R1}^\dagger \to L_j{}^\dagger + H_2^\dagger)}{\Gamma(N_{R1} \to L_j + H_2) + \Gamma(N_{R1}^\dagger \to L_j{}^\dagger + H_2^\dagger)} \\ &= \frac{1}{8\pi (Y_\nu^\dagger Y_\nu)_{11}} \sum_{i\neq 1} Im\left(\left[(Y_\nu^\dagger Y_\nu)_{1i}\right]^2\right)\left(f(\frac{M_1^2}{M_i^2}) + g(\frac{M_1^2}{M_i^2})\right)\end{aligned}$$

where

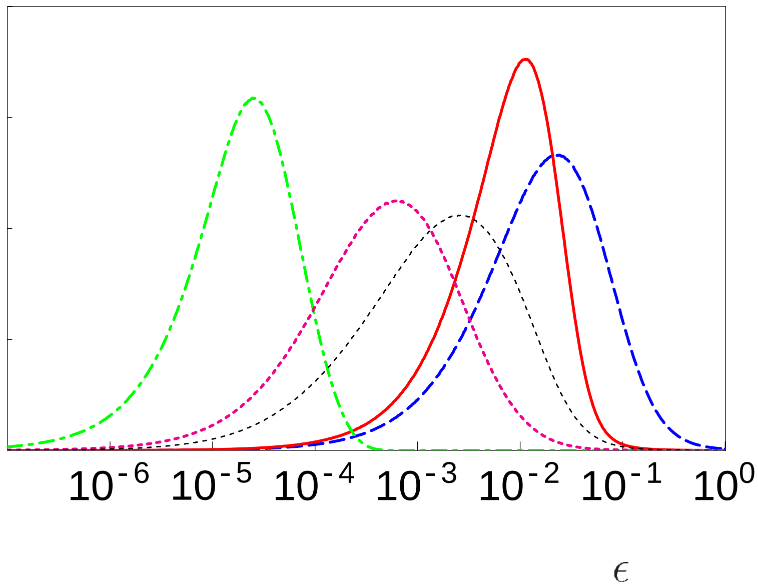
$$f(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right], \quad g(x) = \frac{\sqrt{x}}{1-x}.$$

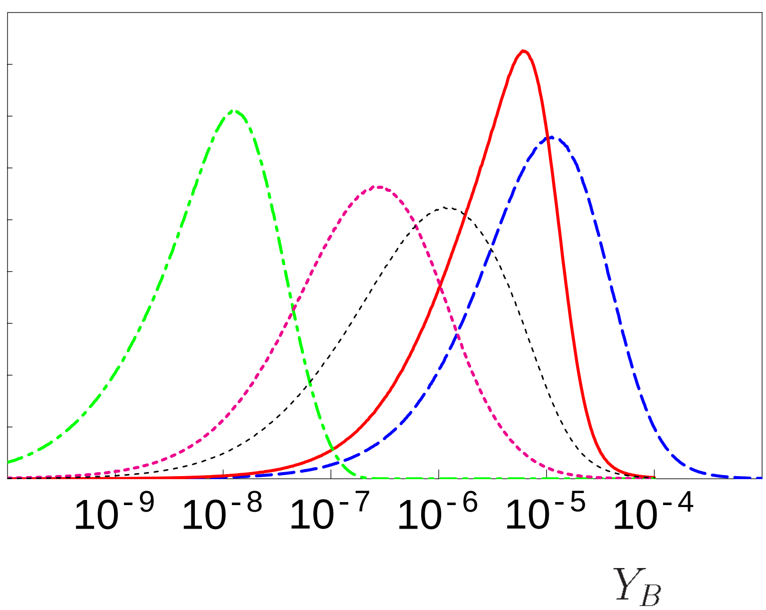
$\Rightarrow$ Texture models fix order of magnitude of Y_ν

$\Rightarrow$ Taking into account $\mathcal{O}(1)$ coefficients ϵ can be calculated like any low-energy observable

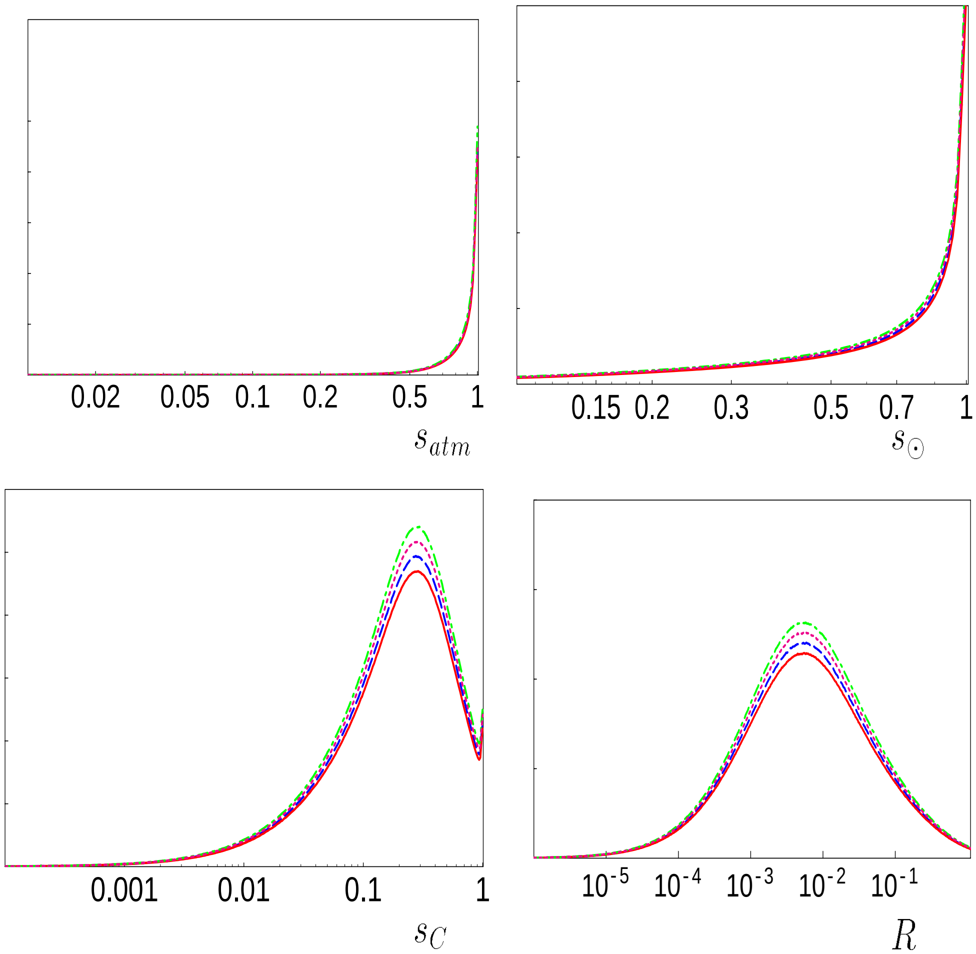
$\Rightarrow$ Conversion $\epsilon \leftrightarrow Y_B$ depends on assumed thermal history of the universe

ϵ and Y_B for FC1-FC4





Neutrino observables for variants of FC3



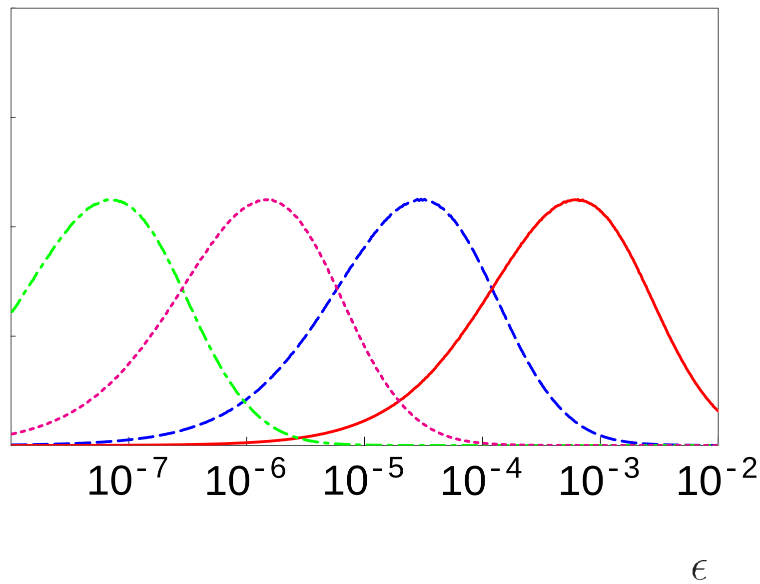
⇒ Variants differ only in l_i, while keeping n_i and σ constant

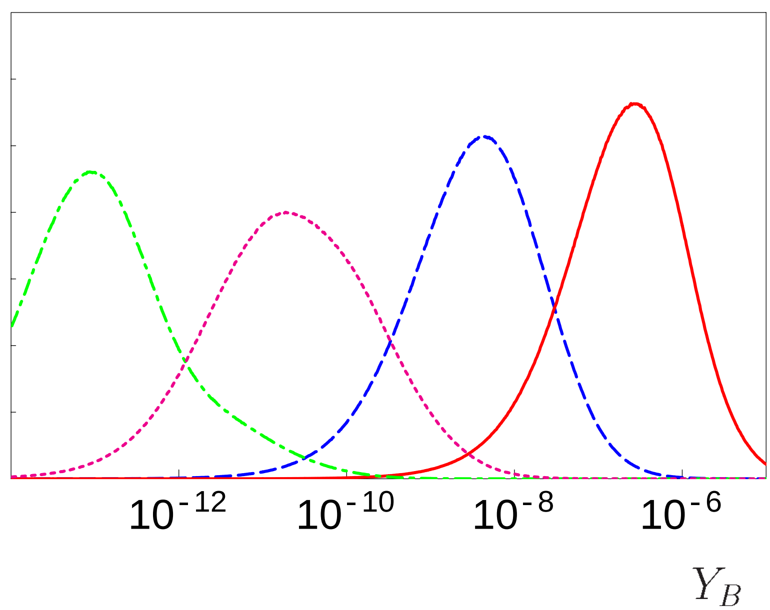
⇒ Keeps low-energy observables unchanged, re-scales Yukawa matrix

Models	l_1	l_2	l_3	n_1	n_2	n_3	σ	Colour:	Factor:
FC3	-1	1	1	1/2	0	-1/2	-1	red	1
FC3a	-2	2	2	1/2	0	-1/2	-1	blue	1.05
FC3b	-3	3	3	1/2	0	-1/2	-1	magenta	1.1
FC3c	-4	4	4	1/2	0	-1/2	-1	green	1.15

Leptogenesis and LA-MSW solution:

Variants of FC3



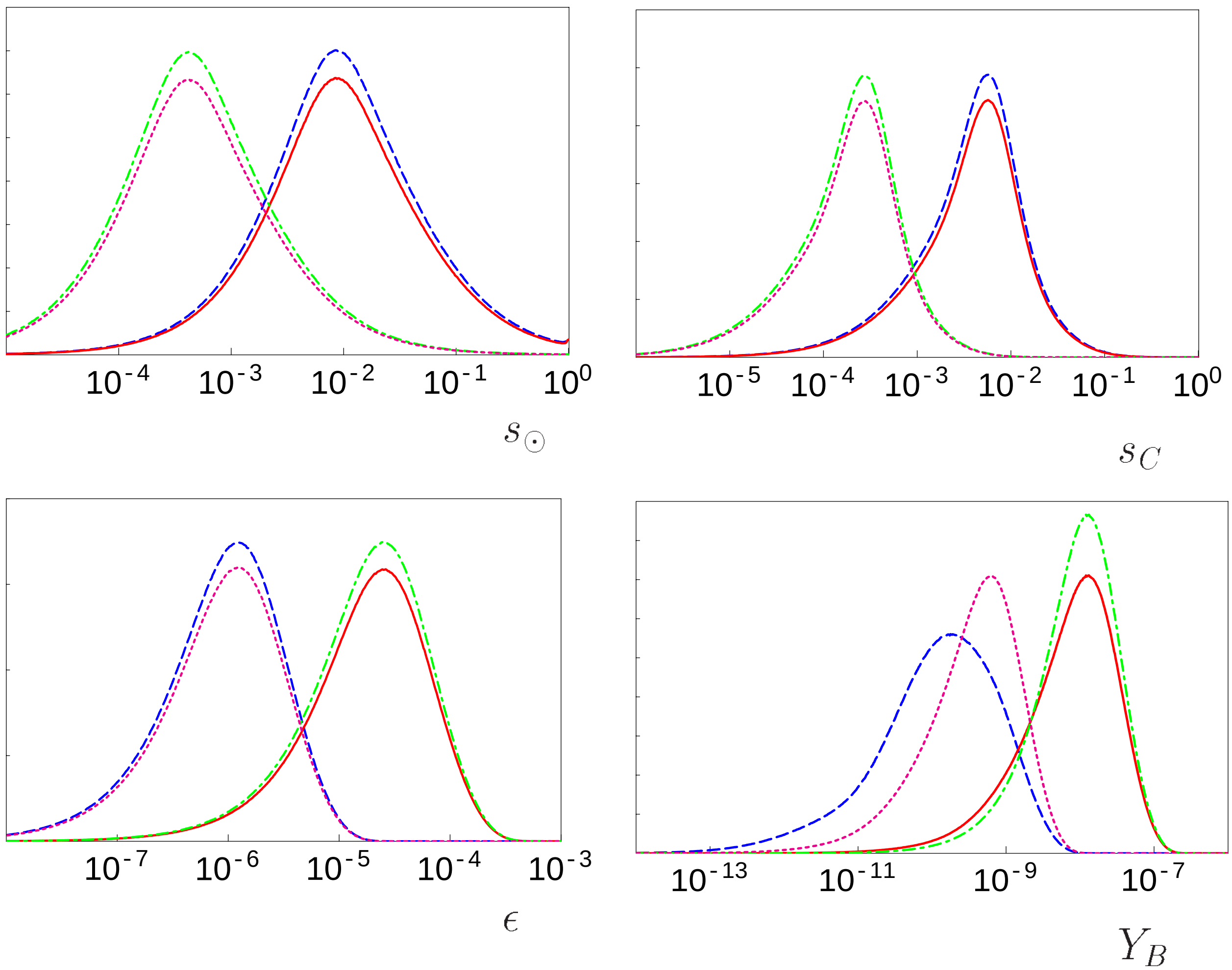


$\Rightarrow$ without specific assumptions about Yukawa matrix,

Leptogenesis independent from low energy observables!

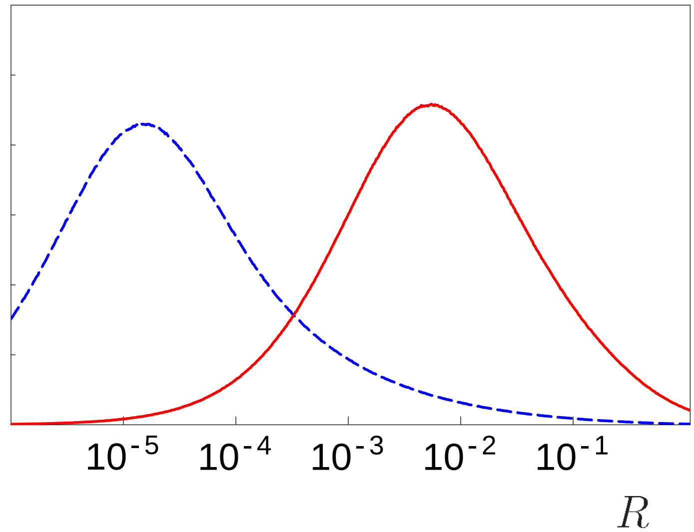
Leptogenesis and SA-MSW solution:

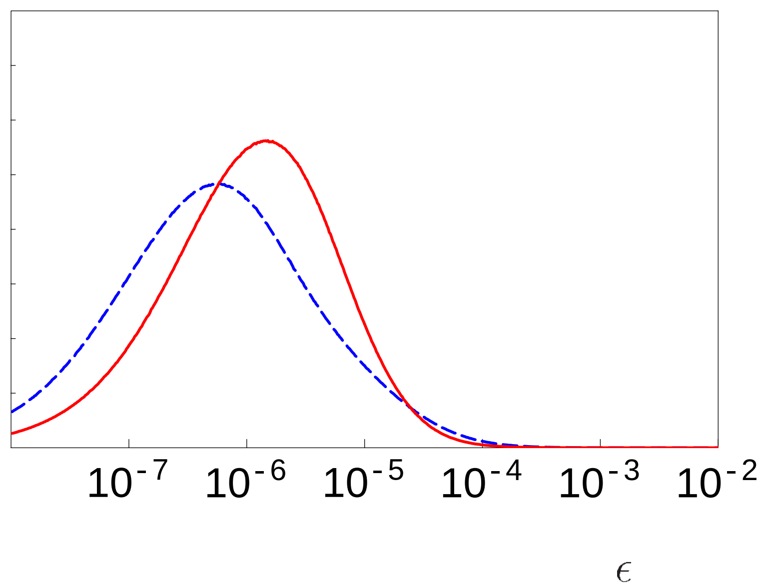
Variants of FC2



Models	l_1	l_2	l_3	n_1	n_2	n_3	σ	Colour:	Factor:
FC2	-3	-1	-1	-3	0	-1	3	red	1
FC2a	-4	-2	-2	-3	0	-1	3	blue	1.1
FC2b	-4	-1	-1	-3	0	-1	3	magenta	1
FC2c	-5	-2	-2	-3	0	-1	3	green	1.1

Leptogenesis and LOW-MSW solution:





a) blue: FC5, defined as
$(l_1, l_2, l_3, n_1, n_2, n_3, \sigma) = (3, -3, -3, 0, -1/2, 1, 1)$

b) for comparison red: FC2b

Summary

$\Rightarrow$ $\mathcal{O}(1)$ coefficients in texture models are very important:
Future progress in texture models will be possible only if these coefficients can be calculated sufficiently accurate

$\Rightarrow$ Without specific assumptions on Yukawa matrix,
Leptogenesis completely independent from low-energy observables

$\Rightarrow$ Leptogenesis can provide information on models otherwise unaccesible