PATTERNS OF ν MASSES:

(a bottom-up approach) or

WHAT CAN WE LEARN ON/FROM U_{e3}?

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Goals:

1. To reach a description as accurate as possible (cfr. hadronic sector) of
   - oscillation phenomena
   - $\not{L}$ processes (if measurable)
   - $\not{CP}$ in leptonic sector (if measurable)

2. Use the results in 1. to understand the origin of flavor in a fundamental theory.

Here: mainly 1. [2. → Ramond, Lola, Yanagida, Altarelli]

- need of optimizing the experimental strategy
- need to define some basic theoretical framework.

3 main points enter the theoretical framework

① How many light states partecipate in the oscillations?

$$\left.\begin{array}{l}\text{atm } \nu \\ \text{solar } \nu\end{array}\right\} \text{ do not require more than } (\nu_e, \nu_\mu, \nu_\tau)$$

→ check LSND (mini BOONE)

$$\left\{\begin{array}{l}\nu_e, \nu_\mu, \nu_\tau + \nu_s \\ \nu_e, \nu_\mu, \nu_\tau\end{array}\right.$$

↳ chosen here to proceed.

② Is $\Delta m^2_{sol} \ll \Delta m^2_{atm}$ a good approximation?

$\longleftarrow$ most of the fits to atm use this

In this case:

$$\begin{array}{lcl|l} \text{atm} & \leftrightarrow & \Delta m^2_{atm}, \vartheta_{23}, \vartheta_{13} & |\vartheta_{13}| < 0.2 : \text{ two almost} \\ \text{solar} & \leftrightarrow & \Delta m^2_{sol}, \vartheta_{12}, \vartheta_{13} & \text{decoupled systems} \end{array}$$

$\Delta m^2_{sol} \ll \Delta m^2_{atm}$ supported by present data and essentially due to Clorine

Without Clorine data Δm^2_{sol} up to $10^{-3}\,eV^2$ <u>is allowed</u> (if $\sin^2 2\vartheta_{sol} \simeq 1$)

→ SNO, Borexino, ...

③ $(\nu_e, \nu_\mu, \nu_\tau)$ are light Majorana fermions

mass term: $-\frac{1}{2}\nu^T m_\nu \nu$ $\qquad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$

$$m_\nu^T = m_\nu$$

From theory:
most convenient way to understand

$$|m_\nu| \ll |m_f| \qquad f = u, d, l$$

$-\frac{1}{2}\nu^T m_\nu \nu$ arises from $\qquad \dfrac{(\varphi l)(\varphi l)}{\Lambda}$
after e.w. symmetry breaking: $\qquad$ ↳ scale of $\not L$

$$m_\nu \approx \frac{v^2}{\Lambda}$$

From experiment:

no indication at present

- Oscillations cannot distinguish between pure Dirac and pure Majorana

- $0\nu\beta\beta$ occurs if ν are Majorana

  present sensitivity: $|(m_\nu)_{ee}| < 0.2-0.8\ \text{eV}$

miniBOONE ↘ SNO, Borexino, Kamland, ... ↘ $0\nu\beta\beta$ ↘

If ① + ② + ③, then

$$m_\nu = U\, m_{diag}\, U^T$$

$U \equiv$ mixing matrix in the lept. sector, if charged leptons are diagonal

④ 0^{th} order approximation:

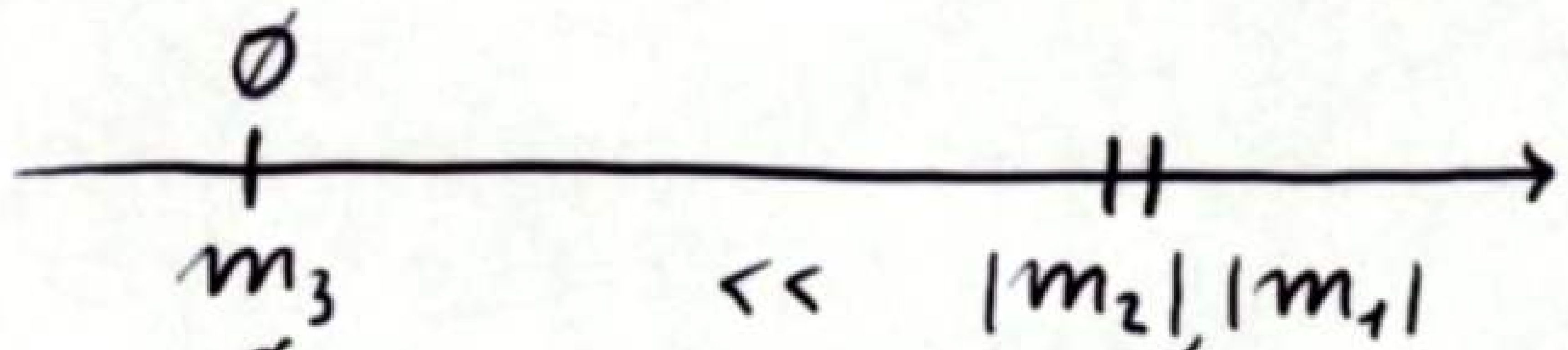
- $\Delta m^2_{atm} \approx 3 \times 10^{-3}\, eV^2$
- $\sin^2 2\vartheta_{atm} \approx 1$
- $\Delta m^2_{sol} \approx 0$
- $U_{e3} \equiv \sin\vartheta_{13} = 0$
- $\delta \approx 0$ → $\cancel{CP}$

} reasonable if ② holds

m^0_ν depends on:

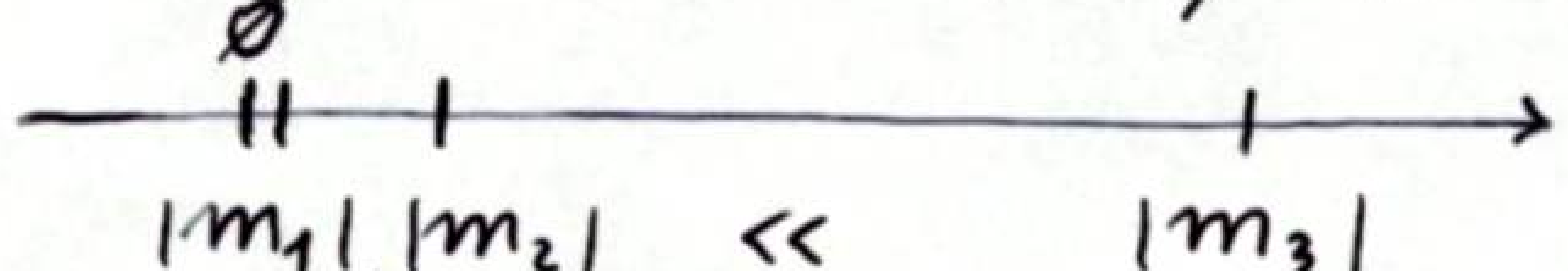
Ⓐ hierarchy among $|m_1|, |m_2|, |m_3|$:

(I)



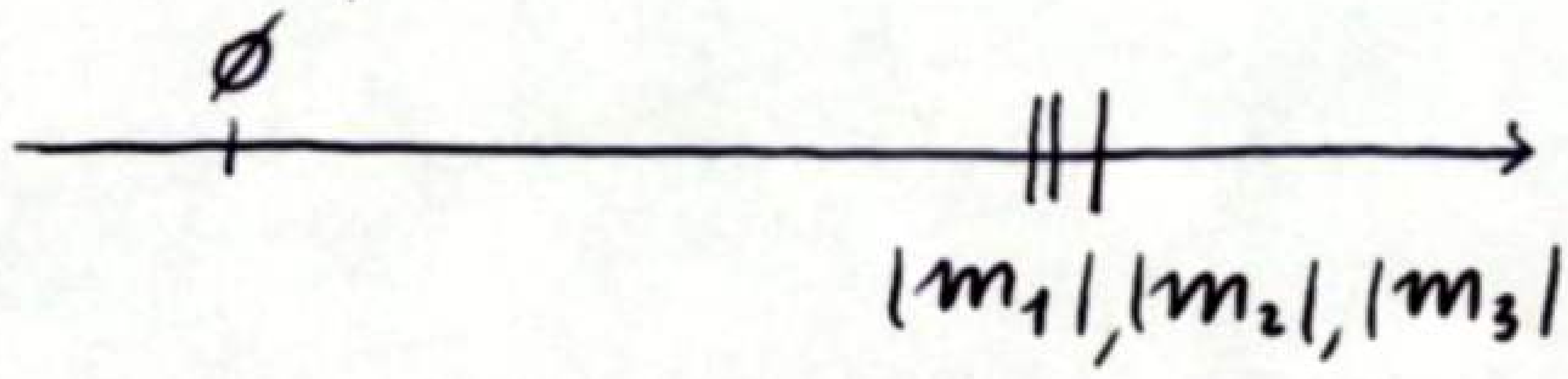
inverse hierarchy

(II)



hierarchical

(III, IV)



degenerate

Ⓑ

$$\begin{cases} \sin^2 2\vartheta_{12} \approx 0.0025 \approx 0 & \text{SAMSW} \\ \sin^2 2\vartheta_{12} \approx O(1) \approx 1 & \text{LAMSW, LOW, VO} \end{cases}$$

SAMSW → disfavored by SNO

TABLE: 0th order TEXTURES

(up to phase redefinitions,
in the basis where e, μ, τ are
diagonal [exception IV])

Overall scale: m

m_{diag}	$\sin^2 2\vartheta_{12} \equiv 1$	$\sin^2 2\vartheta_{12} \equiv 0$	Δm^2_{atm}
(I) $[m, -m, 0]$	$\frac{m}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	—	m^2
(II) $[0, 0, m]$	$\frac{m}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$	↔ SAME	m^2
(III) $[m, -m, m+\delta m]$	$m \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & \frac{1+\delta}{2} & -\frac{1+\delta}{2} \\ 1/\sqrt{2} & -\frac{1+\delta}{2} & \frac{1+\delta}{2} \end{pmatrix}$	—	$2m^2\delta$
(IV) $[m, m, m+\delta m]$	$m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+\delta \end{pmatrix}$	↔ SAME	$2m^2\delta$

Other textures excluded because
- unstable under RGE
- not matching existing models

⑤ Up to now, $U_{e3} \equiv 0$

Can we guess U_{e3} in (I) to (IV)?

Not without additional theoretical input:

- Regard (I) to (IV) as arising in some symmetry limit
- Add to (I), ...(IV) small SB terms to reproduce

$$\begin{cases} \Delta m^2_{sol} \neq 0 & (SA, LA, LOW, VO) \\ \sin^2 2\vartheta_{12} \neq 0 & (SA) \end{cases}$$

- Look to what U_{e3} is generated.

Pay attention to:

5.1 $$U \equiv U_\ell^+ U_\nu$$ in real models

$$\rightarrow U_{e3} \approx -\sin\vartheta_{12}^\ell \sin\vartheta_{atm} + \sin\vartheta_{13}^\nu$$

can be either ν-dominated or ℓ-dominated or both: (ν), (ℓ), (ν/ℓ).

5.2 Results should be taken with great care: they are indications, not theorems

Ultimately this exercise aims to answer the question:

What is the range of U_{e3} favored by current models?

TEXTURE (I):

- Barbieri, Hall, Smith, Strumia Weiner '98

SYMMETRY LIMIT: $(L_e - L_\mu - L_\tau)$

FEATURES: $\vartheta_{23} \sim O(1) \quad \vartheta_{13} = 0$
$\vartheta_{12} = \pi/4$ (sharply) | bimaximal mixing

AFTER SB:

$$m_\nu = \frac{m}{\sqrt{2}} \begin{pmatrix} \eta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{pmatrix}$$

$m_3 \approx m\eta$
$m_1 \approx m(1 + c\eta)$
$m_2 \approx -m(1 - c\eta)$

$\Delta m^2_{atm} \approx m^2$
$\Delta m^2_{sol} \approx O(m^2\eta)$

$$\rightarrow \underbrace{\vartheta^\nu_{12} \approx \vartheta^\ell_{13}}_{(\nu/\ell)} \approx \eta \approx \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = \begin{cases} 10^{-2} & \text{LA} \\ 10^{-4} - 10^{-5} & \text{LOW} \\ 10^{-7} & \text{VO} \end{cases}$$

TEXTURE (II):

SYMMETRY LIMIT:
- anomalous $U(1)$ B. Stech '98
- $U(2)$ - Barbieri, Creminelli, Romanino '99
- $SO(10)$ + HD operators - Sato, Yanagida '97
- $\not R$

- Albright, Babu, Barr '98
- Irges, Lavignac, Ramond '98
- Altarelli, F., Masina '99

...

FEATURES: $\vartheta_{23} \sim O(1)$
$\vartheta_{13} = 0$
ϑ_{12} undetermined (2 0 masses)

well compatible with GUT ($SU(5)$, $SO(10)$) and see-saw. $\vartheta_{23} \sim O(1)$ may originate from ℓ sector

large $(\mu - \tau)$ mixing ↔ large $(s^c - b^c)$ mixing and then transferred to ν.

SYMMETRY BREAKING :

(II.1) SAMSW

$$\begin{cases} U_{e2} \simeq \vartheta_{12}^{\nu} - \vartheta_{12}^{\ell} \cos\vartheta_{atm} \\ U_{e3} \simeq \vartheta_{13}^{\nu} - \vartheta_{12}^{\ell} \sin\vartheta_{atm} \end{cases}$$

(II.1.1) ℓ - dominated

$$\begin{cases} U(2) \\ SO(10) + \text{HD operators} \end{cases}$$

$$U_{e3} \approx U_{e2} \tan\vartheta_{atm} \approx 10^{-2}$$

(II.1.2) ν - dominated

(often in models with U(1))

$$\rightarrow U_{e2} \simeq \vartheta_{12}^{\nu} \qquad U_{e3} \simeq \vartheta_{13}^{\nu} \qquad \frac{\vartheta_{13}^{\nu}}{\vartheta_{12}^{\nu}} \underset{\uparrow}{\approx} \frac{m_2}{m_3} \equiv \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$$

not a precise relation
order-of-magnitude
estimate

$$U_{e3} \simeq \vartheta_{12}^{\nu} \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} \approx 10^{-3} - 10^{-2}$$

(II.2) $\vartheta_{12} \sim$ maximal LAMSW, LOW, VO

$\vartheta_{12} \simeq \vartheta_{12}^{\nu}$ almost always

$$\rightarrow \begin{cases} U_{e2} \simeq \sin\vartheta_{12}^{\nu} - \vartheta_{12}^{\ell} \cos\vartheta_{12}^{\nu} \cos\vartheta_{atm} \\ U_{e3} \simeq \vartheta_{13}^{\nu} - \vartheta_{12}^{\ell} \sin\vartheta_{atm} \end{cases}$$

(II.2.1) ℓ - dominated

$$U_{e3} \approx -\vartheta_{12}^{\ell} \sin\vartheta_{atm} \lesssim \sqrt{\frac{m_e}{m_\mu}} \sin\vartheta_{atm} \approx 0.05$$

(II.2.2) ν - dominated

$$U_{e3} \approx \vartheta_{13}^{\nu} \approx \vartheta_{12}^{\nu} \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} \approx \begin{cases} 10^{-1} & \text{LA} \\ 10^{-2} & \text{LOW} \\ 10^{-3} - 10^{-4} & \text{VO} \end{cases}$$

TEXTURE (III)

SYMMETRY LIMIT: - SB SO(3)

FEATURES:

$$\left.\begin{array}{l}\vartheta_{23} \sim O(1) \\ \vartheta_{13} = 0 \\ \vartheta_{12} = \frac{\pi}{4} \text{ (sharply)}\end{array}\right| \text{bimaximal mixing}$$

- $m_{ee} = 0$ limit from $0\nu\beta\beta$ evaded even for $m \approx (1 \div 2)\,\mathrm{eV}$
- Stability under RGE depends on $\left(\frac{\varepsilon}{\delta}\right)$

$$\varepsilon \approx \frac{y_\tau^2}{16\pi^2} \log\frac{\Lambda}{Q} \approx \left(\frac{10^{-6} \div 10^{-5}}{\cos^2\beta}\right) \text{ in MSSM}$$

$$\delta \approx \frac{\Delta m^2_{atm}}{2m^2} \approx 1.5 \times 10^{-3} \left(\frac{1\mathrm{eV}}{m}\right)^2$$

DEPARTURE FROM 0th APPROX:

Assume that is induced by RGE

$$\left.\begin{array}{l}\vartheta_{12} = \pi/4 \\ \vartheta_{23} \sim O(1)\end{array}\right\} \text{stable} \qquad \begin{array}{l}\Delta m^2_{atm} = 2m^2\delta \\ \Delta m^2_{sol} = m^2\delta\left(\frac{\varepsilon}{\delta}\right)^2\end{array}$$

$$U_{e3} \approx \frac{\varepsilon}{2\delta} = \frac{1}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} = \begin{cases} 10^{-1} & LA \\ 10^{-2} & LOW \\ 10^{-3} - 10^{-4} & VO \end{cases}$$

$$\Delta m^2_{sol} \approx 600\left(\frac{m}{1\mathrm{eV}}\right)^4 \varepsilon^2\ \mathrm{eV}^2$$

$$\rightarrow \varepsilon \lesssim \begin{cases} LA & (10^{-3} \div 10^{-4})\left(\frac{1\mathrm{eV}}{m}\right)^2 & \text{o.k.} \\ LOW & (10^{-4} - 10^{-5})\left(\frac{1\mathrm{eV}}{m}\right)^2 & \rightarrow \tan\beta < 10 \\ VO & 10^{-6} \left(\frac{1\mathrm{eV}}{m}\right)^2 & \rightarrow m \lesssim 0.1\mathrm{eV} \end{cases}$$

TEXTURE(IV)

SYMMETRY LIMIT: $\begin{cases} \text{FLAVOR DEMOCRACY} \\ S_{3L}\times S_{3R}\ ,\ O(3)_L \times O(3)_R \end{cases}$

FEATURES :

charged sector: $$m_f = \frac{m_f^0}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \delta m_f$$

$(f=u,d,\ell)$

→ hierarchy of charged fermion masses
(1 dominant mass × family)

→ small angles in V_{CKM} from cancellation in $U_u^+ U_d$.

ν-sector $$m_\nu = m_\nu^0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta m_\nu$$

→ large angles in $U = U_\ell^+ U_\nu$ $\quad \sin^2 2\vartheta_{atm} = \frac{8}{9}$!

attention to:

- $(m_\nu)_{ee} \sim m_\nu^0 \lesssim (0{,}2-0{,}8)\,eV$ from $0\nu\beta\beta$
- RGE tends to destabilize the texture.

SYMMETRY BREAKING:

on a empirical basis: $$\delta m_\nu = \begin{pmatrix} 0 & \eta & 0 \\ \eta & 0 & 0 \\ 0 & 0 & \delta \end{pmatrix} \rightarrow SAMSW$$

$|\eta| \ll |\delta| \ll m_\nu^0$ $$\delta m_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \delta \end{pmatrix} \rightarrow \begin{cases} LA \\ \boxed{\begin{matrix} LOW \\ VO \end{matrix}} \end{cases}$$

spoiled by running

In any case U_{e3} is ℓ-dominated:

$$U_{e3} \approx \vartheta_{12}^{\ell}\ \underbrace{\sin\vartheta_{atm}}_{\sqrt{2/3}} \equiv \sqrt{\frac{2}{3}}\sqrt{\frac{m_e}{m_\mu}} \simeq 0.05$$

TIMETABLE (very rough ...)

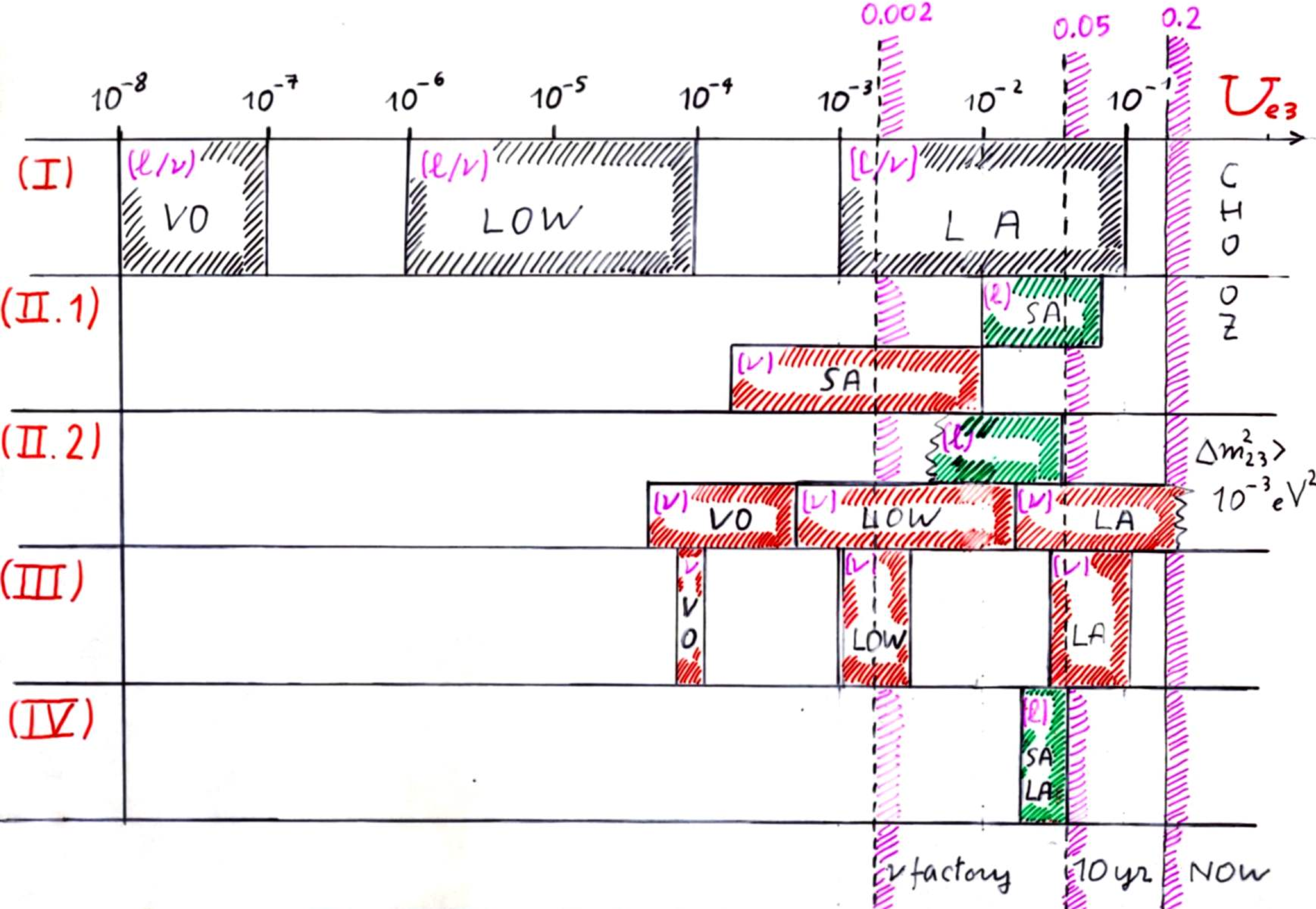
		today	in 10 yr	ν factory
atm	Δm^2_{23} $\sin^2 2\vartheta_{23}$	$\sim 10^{-3}$ eV^2 ~ 1	10% accuracy K2K, MINOS, NGS	5% accuracy
solar	Δm^2_{12} $\sin^2 2\vartheta_{12}$	$\lesssim 10^{-4}$ eV^2 ?	10 - 20% accuracy SNO, Borexino, Kamland	
	$\sin^2 2\vartheta_{13}$	< 0.1	< 0.01 JHF	< 0.00002
	δ_{CP}	?	?	Yes if LA
	Δm_{23}	?	?	accessible
	$\lvert U_{e3} \rvert$	< 0.15	< 0.05	< 0.002

To plot U_{e3} I used :

Δm^2_{sol}	$4\times 10^{-6} - 10^{-5}$	$10^{-5} - 10^{-4}$	$10^{-8} - 10^{-7}$	10^{-10}
$\sin^2 2\vartheta_{sol}$	$10^{-3} - 10^{-2}$	$0.6 - 0.9$	$0.6 - 0.9$	$0.6 - 0.9$
	SAMSW	LAMSW	LOW	VO

When the theoretical estimate suggests:

$$\approx \quad \longrightarrow \quad \text{factor 10 of uncertainty} \equiv \left(\frac{3 \times \text{upper}}{\frac{1}{3} \times \text{lower}} \right)$$


0.002
0.05
0.2
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
U_{e3}
(I)
(ℓ/ν)
VO
(ℓ/ν)
LOW
[ℓ/ν]
LA
CHOOZ
(II.1)
(ℓ)
SA
(ν)
SA
(II.2)
(ℓ)
$\Delta m^2_{23} > 10^{-3} eV^2$
(ν)
VO
(ν)
LOW
(ν)
LA
(III)
ν
VO
(ν)
LOW
(ν)
LA
(IV)
(ℓ)
SA
LA
ν factory
10 yr
NOW

COMMENTS

1. With the planned experiments, we will have access to a <u>significant</u> portion of the U_{e3} parameter space.

2. A knowledge of the absolute spectrum would not help in clarifying U_{e3} and viceversa the value of U_{e3} by itself would not help in discriminate among (I) ÷ (IV)

3. solar ν and U_{e3}

$$U_{e3} \leftarrow \cdots \rightarrow \text{solution to solar } \nu$$

- Values of U_{e3} close to 0.1 seem possible only within LA [I, II.2, III]

- $0.02 \lesssim U_{e3} \lesssim 0.1$ → LA still preferred but also SA [II.1, IV] possible

  <u>Exception</u>: hierarchical, $\sin^2 2\vartheta_\odot \sim 1$ if
  ℓ-dominated

- VO seems related to an unmeasurable U_{e3}. <u>Exception</u>: as before

- $0.001 \lesssim U_{e3} \lesssim 0.02$ difficult region
  SA, LA & LOW possible with ν-dominance preferred